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Algebraic Properties in Hybrid Cosine-Jaccard Measure for Neutrosophic Set

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Abstract

The recent hybrid cosine-Jaccard measure for neutrosophic sets proposed to handle uncertainty, indeterminacy, and inconsistency in the complex data more effectively. The three main properties: boundedness, symmetry, and reflexivity have been established; therefore, algebraic characteristics should be explored in depth for the establishment of advanced computational frameworks. This paper is the extension of the theoretical foundation by introducing monotonicity and a distributive-like property over weighted combinations. Mathematical proof is provided for each property, followed by numerical examples to demonstrate their validity and computational consistency. This research establishes a theoretical framework for a hybrid cosine-Jaccard measure for neutrosophic sets, facilitating its facilitating reliable implementation in intricate algebraic operations related to neutrosophic multi-criteria decision-making, clustering algorithms, and information retrieval systems.

Keywords: Hybrid cosine-Jaccard, Neutrosophic set, Similarity measures, Algebraic properties.

1 | Introduction

In complex real-world decision-making scenarios, there are many uncertainties and inaccuracies exist in the complexity of a data set where it is necessary to have advanced mathematical frameworks that can capture the robustness of information processing [1]. Fuzzy set theory has overcome complexities and uncertainties by quantifying the extent to which elements belong to a set [2], [3]. Nevertheless, the classical crisp sets and the basic fuzzy sets often fail to encompass the entire range of vagueness and ambiguity characteristic of human judgment and expert systems. To address these constraints, neutrosophic sets are an extension of fuzzy set where it contains the degrees of truth, indeterminacy, and falsity [4], [5]. These three membership functions make neutrosophic sets more useful when making complicated decisions [6]. Neutrosophic sets are different from earlier extensions like intuitionistic fuzzy sets because they allow for the independence of truth, indeterminacy, and falsity memberships. These earlier sets only deal with incomplete information and not inherent indeterminacy or inconsistency [7]. As a result, creating similarity measures within the neutrosophic

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framework is very important to show how similar neutrosophic sets are. This is useful for real-life application such as pattern recognition, clustering, and decision support systems [1]. The extension of algebraic properties in hybrid similarity measures, exemplified by the cosine-Jaccard measure for neutrosophic sets, provides a more sophisticated methodology for assessing the relationships among uncertain data elements [6], [8], [7]. These advanced similarity measures are crucial for tackling decision-making problems that involve uncertain and inconsistent information, which are prevalent in fields like medical diagnosis and artificial intelligence [1], [9].

In this paper, we investigate two main extension of algebraic properties in hybrid cosine-Jaccard similarity measure for neutrosophic set: monotonicity and linearity in weights. This theoretical framework is important so that it can prove the consistency. Monotonicity is an important property in the hybrid cosine-Jaccard measure for neutrosophic sets. This is to reinforce the three main property: symmetry, boundedness and reflexivity to ensure the reliability of the measure and logical consistency when comparing sets. In the neutrosophic set, it is vital to highlight the membership functions as truth (T), indeterminacy (I), and falsity (F) memberships; monotonicity guarantees that the similarity score behaves predictably. For instance, it is said that two neutrosophic sets become more alike when their truth memberships increase simultaneously, their indeterminacies align more closely, and their falsities decrease in a correlated manner. Therefore, the hybrid similarity measure must also increase or, at a minimum, but not decrease. This monoticity property is crucial because a similarity measure that violates monotonicity can yield error results; for instance, two sets might be objectively more alike than another pair but receive a lower similarity score, rendering the measure untrustworthy for practical decision-making. The hybrid cosine-Jaccard measure strengthens of both cosine and Jaccard so that can handle both the vector-like and set-like aspects of neutrosophic information. Enforcing monotonicity ensures that the robustness in hybrid approach. This property is applicable in real-life application such as pattern recognition, clustering, and multi-criteria decision-making, where the correctness of the similarity ordering directly impacts the quality and validity of the final results. Without monotonicity, the measure lacks a foundational guarantee of its logical soundness.

Besides, the linearity property in hybrid cosine-Jaccard measure for neutrosophic is best investigated where it examines the accuracy and interpretability of similarity computation in decision-making complexity that involves uncertainties [10]. Algebraic linearity property ensures that the hybrid similarity measure is consistent and has a predictable scale with regard to the weights provided such that the analysis will be trustworthy in various applications such as pattern recognition and medicine diagnosis [6], [11]. Moreover, the property also supports the development of algorithms advancement for multi-criteria decision-making processes under neutrosophic hesitant fuzzy linguistic environments [10]. The intrinsic property helps enable the development of more sophisticated similarity measures by supporting the decomposition and reconstruction of sophisticated similarity judgments, thereby making them simpler for mathematics computation.

The paper is organized into different sections. Section 2 provides the preliminaries related to neutrosophic sets and similarity measures. Section 3 presents the extension of algebraic properties in hybrid cosine-Jaccard measure for neutrosophic set with proof. Later, Section 4 the illustrates numerical examples. Finally, the conclusion and future research are provide.

2 | Preliminaries

This section provides some preliminaries of the core concept and necessary definitions for neutrosophic sets and similarity measures.

Definition 1 ([12], [13]). A neutrosophic set A is defined as an object having the form:

$$A = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in N \},$$

where N is the universe of discourse, x is a generic element in N, $T(x)$, $I(x)$, and $F(x)$ are functions that define the degree of membership (truth), indeterminacy, and non-membership (falsity) of the element x to the set A.

Definition 2 ([1], [10]). For neutrosophic set A and set B, we need to define S_c, cosine similarity measures in X where the truth (T), indeterminacy (I), and falsity (F) components. The formula is given by:

$$S_C(A,B) = \frac{\Sigma_{x\in X}[T_A(x)T_B(x)+ I_A(x)I_B(x)+ F_A(x)F_B(x)]}{\sqrt{\Sigma_{x\in X}[T_A(x)^2+ I_A(x)^2+ F_A(x)^2]}\cdot\sqrt{\Sigma_{x\in X}[T_B(x)^2+ I_B(x)^2+ F_B(x)^2]}}. \quad (1)$$

Definition 3 ([14]). For neutrosophic set A and set B, we need to define S_J, Jaccard similarity measures in X where the truth (T), indeterminacy (I), and falsity (F) components and the formula is given by:

$$S_J(A,B) = \frac{\Sigma_{x\in X}[\min(T_A(x),T_B(x)) + \min(I_A(x),I_B(x)) + \min(F_A(x),F_B(x))]}{\Sigma_{x\in X}[\max(T_A(x),T_B(x)) + \max(I_A(x),I_B(x)) + \max(F_A(x),F_B(x))]}. \quad (2)$$

Definition 4 ([15]). Let a hybrid cosine-Jaccard similarity measure between two neutrosophic sets A and B as follows:

$$S_{HCJ}(A,B) = \alpha\cdot S_{cosine}(A,B) + \beta\cdot S_{Jaccard}(A,B), \quad (3)$$

where $S_{cosine}(A,B)$ is the cosine similarity measure between A and B, and $S_{Jaccard}(A,B)$ is the Jaccard similarity measure between A and B; α and β are weights such that $\alpha+\beta=1$ and $\alpha,\beta\in[0,1]$. These weights determine the relative importance of the cosine and Jaccard components according to the decision maker.

Let A and B be the neutrosophic sets defined on a universal set X. The hybrid Cosine-Jaccard similarity measure $S_{HCJ}(A,B)$ between A and B can be decomposed into three independent components corresponding to the truth (T), indeterminacy (I), and falsity (F) membership. By integrating the weighted sum of Cosine-Jaccard similarity for each neutrosophic component can be represented as follows:

$$S_{HCJ}(A,B) = w_T\cdot S^T_{HCL}(A,B) + w_I\cdot S^I_{HCJ}(A,B) + w_F\cdot S^F_{HCJ}(A,B), \quad (4)$$

where $w_T, w_I, w_F\in[0,1]$ such that the weight, $w_T+w_I+w_F=1$. If the weights are equal, *Eq. (4)* is simplified as follow:

$$S_{HCJ}(A,B) = \left(\frac{1}{3}\right)\left[S^T_{HCJ}(A,B) + S^I_{HCJ}(A,B) + S^F_{HCJ}(A,B)\right].$$

3 | Theorems

In this section, we show the extension properties for hybrid cosine-Jaccard similarity measure for neutrosophic set: monotonicity and linearity in weights. These properties are important to reinforce the three main properties [15] and to enhance the interpretability and prediction.

3.1 | Monotonicity Theorem

Let A, B and C be three single-valued neutrosophic sets defined by a finite universal set $X=\{x_1,x_2,\dots,x_n\}$.

Suppose that for every element $x\in X$, the components of B are "closer" to those of A than the components of C are. Formally, we assume the following conditions hold for all $x\in X$:

I. Truth-Monotonicity: $|T_A(x)-T_B(x)|\leq|T_A(x)-T_C(x)|$.

II. Indeterminacy-Monotonicity: $|I_A(x)-I_B(x)|\leq|I_A(x)-I_C(x)$.

III. Falsity-Monotonicity: $|F_A(x)-F_B(x)|\leq|F_A(x)-F_C(x)$.

Furthermore, we assume that the membership values are non-negative and bounded, where $T,I,F\in[0,1]$.

Then, the Hybrid Cosine-Jaccard similarity measure is monotonic, meaning the similarity between A and B is greater than or equal to the similarity between A and C:

$$S_{HCJ}(A,B) \geq S_{HCJ}(A,C),$$

where S_{HCJ} is defined with fixed weights $\alpha, \beta \in [0,1]$such that $\alpha + \beta = 1$.

Proof: Firstly, from the cosine similarity for neutrosophic set is defined as follows:

$$S_C(A,B) = \frac{\Sigma_{i=1}^{n}(T_A\, T_B + I_A\, I_B + F_A\, F_B)}{\sqrt{\Sigma_{i=1}^{n}(T_A^2 + I_A^2 + F_A^2)} \cdot \sqrt{\Sigma_{i=1}^{n}(T_B^2 + I_B^2 + F_B^2)}}.$$

Let us consider the difference in a single component, say Truth, for an element x. The given condition $|\, T_A - T_B\,| \leq |\, T_A - T_C\,|$ implies that $(T_A - T_B)^2 \leq (T_A - T_C)^2$.

Consider the squared Euclidean distance between the entire neutrosophic vectors A and B:

$$d_E^2(A,B) = \Sigma_i{}^{=1n}[(T_A - T_B)^2 + (I_A - I_B)^2 + (F_A - F_B)^2].$$

$$d_E^2(A,C) = \Sigma_i{}^{=1n}[(T_A - T_C)^2 + (I_A - I_C)^2 + (F_A - F_C)^2].$$

From the given *Conditions (1)-(3),*

$$(T_A - T_B)^2 + (I_A - I_B)^2 + (F_A - F_B)^2 \leq (T_A - T_C)^2 + (I_A - I_C)^2 + (F_A - F_C)^2.$$

Summing over all n elements in X

$$d_E^2(A,B) \leq d_E^2(A,C).$$

By improving the relationship between Euclidean distance and Cosine Similarity, it is known that for any two vectors u and v:

$$d_E^2(u,v) = \|u\|^2 + \|v\|^2 - 2\|u\|\|v\|\cos(\theta).$$

where cos (θ) is the cosine similarity.

Applying this to the cosine similarity for neutrosophic sets

$$d_E^2(A,B) = \|A\|^2 + \|B\|^2 - 2\|A\|\|B\|S_C(A,B).$$

$$d_E^2(A,C) = \|A\|^2 + \|C\|^2 - 2\|A\|\|C\|S_C(A,C).$$

We know $d_E^2(A,B) \leq d_E^2(A,C)$.

Thus,

$$\|A\|^2 + \|B\|^2 - 2\|A\|\|B\|S_C(A,B) \leq \|A\|^2 + \|C\|^2 - 2\|A\|\|C\|S_C(A,C).$$

$$S_C(A,B) \leq S_C(A,C).$$

Thus, under the given distance conditions and the assumption of comparable magnitudes for B and C, the Cosine Similarity for neutrosophic set is monotonic. □

Secondly, the Jaccard similarity for neutrosophic set is defined as followed:

$$S_J(A,B) = \frac{\Sigma_{i=1}^{n}[\min(T_A,T_B) + \min(I_A,I_B) + \min(F_A,F_B)]}{\Sigma_{i=1}^{n}[\max(T_A,T_B) + \max(I_A,I_B) + \max(F_A,F_B)]}.$$

In the case for minimum, $\min(T_A, T_B) \geq \min(T_A, T_C)$. For instance,

I. If $T_A > T_C$, then $T_B \geq T_C$, so $\min(T_A, T_B) \geq \min(T_A, T_C) = T_C$.

II. If $T_A < T_C$, then $T_B \leq T_C$, so $\min(T_A, T_B) \geq \min(T_A, T_C) = T_A$. Therefore, symmetric argument holds.

In the case for maximum, max $(T_A, T_B) \leq$ max (T_A, T_C). For example,

I. If $T_A > T_C$, then $T_B \leq T_A$, so max $(T_A, T_B) \leq$ max $(T_A, T_C) = T_A$.

II. If $T_A < T_C$, then $T_B \leq T_C$, so max $(T_A, T_B) \leq$ max $(T_A, T_C) = T_C$.

Thus, a symmetric argument holds.

The same logic applies to the Indeterminacy and Falsity components.

For every element $x \in X$, the numerator can be written as

$$\min(T_A, T_B) + \min(I_A, I_B) + \min(F_A, F_B) \geq \min(T_A, T_C) + \min(I_A, I_C) + \min(F_A, F_C),$$

while the denominator can be written as

$$\max(T_A, T_B) + \max(I_A, I_B) + \max(F_A, F_B) \leq \max(T_A, T_C) + \max(I_A, I_C) + \max(F_A, F_C).$$

Then

$$S_J(A, B) \geq S_J(A, C).$$

The Jaccard Similarity for neutrosophic set is monotonic under the given conditions. □

The hybrid measure is a convex combination of the two monotonic measures:

$$S_{HCJ}(A, B) = \alpha \cdot S_C(A, B) + \beta \cdot S_J(A, B).$$

$$S_{HCJ}(A, C) = \alpha \cdot S_C(A, C) + \beta \cdot S_J(A, C).$$

From the monotonicity of cosine similarity and Jaccard similarity for neutrosophic set:

$$S_C(A, B) \geq S_C(A, C), \text{and } S_J(A, B) \geq S_J(A, C).$$

Since the weights α and β are non-negative,

$$\alpha \cdot S_C(A, B) \geq \alpha \cdot S_C(A, C).$$

$$\beta \cdot S_J(A, B) \geq \beta \cdot S_J(A, C).$$

By adding these two inequalities together yields the final result:

$$\alpha \cdot S_C(A, B) + \beta \cdot S_J(A, B) \geq \alpha \cdot S_C(A, C) + \beta \cdot S_J(A, C).$$

$$\therefore S_{HCJ}(A, B) \geq S_{HCJ}(A, C). \blacksquare$$

3.2 | Linearity in Weights Theorem

The hybrid cosine-Jaccard similarity measure $S_{HCJ}(A, B)$ is linear with respect to the weights α, β, w_T, w_I, and w_F. For any two sets of the weights (α_1, β_1) and (α_2, β_2), and for any scalar $\lambda \in [0,1]$ can be written as followed:

$$S_{HCJ}(A, B)^{(\lambda\alpha_1 + (1-\lambda)\alpha_2, \lambda\beta_1 + (1-\lambda)\beta_2)} = \lambda \cdot S_{HCJ}(A, B)^{(\alpha_1, \beta_1)} + (1 - \lambda) \cdot S_{HCJ}(A, B)^{(\alpha_2, \beta_2)}.$$

Therefore, linearity property holds for the component weights w_T, w_I, w_F.

Proof: Let (α_1, β_1) and (α_2, β_2) be two pairs of weights such that $\alpha_1 + \beta_1 = 1$ and $\alpha_2 + \beta_2 = 1$. Let $\lambda \in [0,1]$. We define a new pair of weights as

$$\alpha' = \lambda\alpha_1 + (1 - \lambda)\alpha_2, \qquad \beta' = \lambda\beta_1 + (1 - \lambda)\beta_2.$$

Then

$$\alpha' + \beta' = \lambda(\alpha_1 + \beta_1) + (1-\lambda)(\alpha_2 + \beta_2) = \lambda \cdot 1 + (1-\lambda) \cdot 1 = 1.$$

Thus, (α', β') is a valid weighting pair.

Now, consider the hybrid measure under these weights:

$$S_{HCJ}(A,B)^{(\alpha\prime,\beta\prime)} = \alpha' \cdot S_C(A,B) + \beta' \cdot S_J(A,B).$$

$$= (\lambda\alpha_1 + (1-\lambda)\alpha_2) \cdot S_C(A,B) + (\lambda\beta_1 + (1-\lambda)\beta_2) \cdot S_J(A,B).$$

$$= \lambda\big(\alpha_1 S_C(A,B) + \beta_1 S_J(A,B)\big) + (1-\lambda)\big(\alpha_2 S_C(A,B) + \beta_2 S_J(A,B)\big).$$

$$= \lambda \cdot S_{HCJ}(A,B)^{(\alpha_1,\beta_1)} + (1-\lambda) \cdot S_{HCJ}(A,B)^{(\alpha_2,\beta_2)}.$$

This completes the proof for the linearity in the hybrid weights α and β. Thus, the component weights w_T, w_I, w_F can be derived in the similar way.

Next, let (w_T^1, w_I^1, w_F^1) and (w_T^2, w_I^2, w_F^2) be two sets of component weights, where the total weight is 1. For $\lambda \in [0,1]$, we define the weights as

$$w_T' = \lambda w_T^1 + (1-\lambda) w_T^2, w_I' = \lambda w_I^1 + (1-\lambda) w_I^2, w_F' = \lambda w_F^1 + (1-\lambda) w_F^2.$$

Then

$$S_{HCJ}(A,B)^{(w_T',w_I',w_F')} = w_T' \cdot S_{HCJ}^T(A,B) + w_I' \cdot S_{HCJ}^I(A,B) + w_F' \cdot S_{HCJ}^F(A,B).$$

$$= \lambda\big(w_T^1 S_{HCJ}^T + w_I^1 S_{HCJ}^I + w_F^1 S_{HCJ}^F\big) + (1-\lambda)\big(w_T^2 S_{HCJ}^T + w_I^2 S_{HCJ}^I + w_F^2 S_{HCJ}^F\big).$$

$$= \lambda \cdot S_{HCJ}(A,B)^{(w_T^1,w_I^1,w_F^1)} + (1-\lambda) \cdot S_{HCJ}(A,B)^{(w_T^2,w_I^2,w_F^2)}.$$

Therefore, the measure is also linear in the component of weights. ■

This linear in weight theorem allows decision makers to adjust weights such as giving the importance to truth in medical diagnosis and have predictable results. It also helps to combine decision maker's opinions.

4 | Numerical Examples

In this section, we demonstrate two numerical examples to show the monotonicity and linearity in weights theorem from the previous section.

Example 1 (monotonicity). By using the example of the previous paper [15].

Let A and B be neutrosophic sets over $U = \{u\}$:

$$A = (T_A, I_A, F_A) = (0.5, 0.3, 0.2).$$

$$B = (T_B, I_B, F_B) = (0.6, 0.2, 0.1).$$

$$C = (T_C, I_C, F_C) = (0.4, 0.5, 0.3).$$

Besides, we also consider the sum weight of truth (T), indeterminacy (I), and falsity (F) is equal to 1 and $\alpha = \beta = 0.5$.

Step 1. Calculating the cosine similarity and Jaccard similarity Cosine Similarity:

$$S_C(A,B) = \frac{[(0.5)(0.6)+(0.3)(0.2)+(0.2)(0.1)]}{[\sqrt{0.5^2+0.3^2+0.2^2}\cdot\sqrt{0.6^2+0.2^2+0.1^2}]} = \frac{0.38}{0.6164\times0.6403} \approx 0.9628.$$

$$S_C(A,C) = \frac{[(0.5)(0.4)+(0.3)(0.5)+(0.2)(0.3)]}{[\sqrt{0.5^2+0.3^2+0.2^2}\cdot\sqrt{0.4^2+0.5^2+0.3^2}]} = \frac{0.41}{(0.6164\times0.7071)} \approx 0.9407.$$

Jaccard Similarity:

$$S_J(A,B) = \frac{[\min(0.5,0.6)+\min(0.3,0.2)+\min(0.2,0.1)]}{[\max(0.5,0.6)+\max(0.3,0.2)+\max(0.2,0.1)]} = \frac{(0.5+0.2+0.1)}{(0.6+0.3+0.2)} \approx 0.7273.$$

$$S_J(A,C) = \frac{[\min(0.5,0.4)+\min(0.3,0.5)+\min(0.2,0.3)]}{[\max(0.5,0.4)+\max(0.3,0.5)+\max(0.2,0.3)]} = \frac{(0.4+0.3+0.2)}{(0.5+0.5+0.3)} \approx 0.6923.$$

Step 2. Using the formula of hybrid cosine-Jaccard similarity measures By applying the hybrid Cosine-Jaccard Similarity measures:

$$S_{HCJ}(A,B) = (0.5 \times 0.9628) + (0.5 \times 0.7273) \approx 0.84505.$$

$$S_{HCJ}(B,A) = (0.5 \times 0.7273) + (0.5 \times 0.9628) \approx 0.84505.$$

$$S_{HCJ}(A,C) = (0.5 \times 0.940) + (0.5 \times 0.6923) \approx 0.8162.$$

Step 3. Check the monotonicity

$$S_C(A,B) \approx 0.9628 > S_C(A,C) \approx 0.940.$$

$$S_J(A,B) \approx 0.7273 > S_J(A,C) \approx 0.6923.$$

$$S_{HCJ}(A,B) \approx 0.8451 > S_HCJ(A,C) \approx 0.8162.$$

Example 2 (Linearity in Weights). Given the two neutrosophic sets are as followed:

$$A = (0.6, 0.3, 0.2).$$

$$B = (0.5, 0.4, 0.3).$$

Besides, we also assume the component similarities have been computed as followed:

$$S^T_{HCJ}(A,B) = 0.9, \qquad S^T_{HCJ}(A,B) = 0.6, \qquad S^T_{HCJ}(A,B) = 0.7.$$

Step 1. Calculating the weight.

If the weight given are

$$w_1 = (w_T = 0.5, \qquad w_I = 0.3, \qquad w_F = 0.2).$$

$$w_2 = (w_T = 0.2, \; w_I = 0.5, \qquad w_F = 0.3).$$

then

$$S_{HCJ}(A,B)^{w_1} = 0.5 \cdot 0.9 + 0.3 \cdot 0.6 + 0.2 \cdot 0.7 = 0.77.$$

$$S_{HCJ}(A,B)^{w_2} = 0.2 \cdot 0.9 + 0.5 \cdot 0.6 + 0.3 \cdot 0.7 = 0.69.$$

let $\lambda = 0.4$. The estimating weights are

$$w' = 0.4 \cdot w_1 + 0.6 \cdot w_2 = (0.32, 0.42, 0.26).$$

The measure with w' is:

$$S_{HCJ}(A, B)^{w'} = 0.32 \cdot 0.9 + 0.42 \cdot 0.6 + 0.26 \cdot 0.7 = 0.722.$$

Step 2. Verifying the linearity

$$\lambda \cdot S^{w_1} + (1 - \lambda) \cdot S^{w_2} = 0.4 \cdot 0.77 + 0.6 \cdot 0.69 = 0.722.$$

5 | Conlusion

This research investigates the extensive algebraic properties: monotonicity and linearity in weights in hybrid cosine-Jaccard measure for neutrosophic set enhances the consistency with the three main properties: symmetry, boundedness and reflexivity and also increase the quality of decision-making processes. This theoretical framework can also help to optimize practical applications which give better accuracy and dependable evaluations in ambiguous contexts. Monotonicity theorem helps to enhance the establishment of three main properties: boundedness, symmetry, and reflexivity. Besides, the linearity in weight theorems also allows weights adjustment based on different real-life application. Thus, the future research should aim for creating more advanced algorithms that utilize these characteristics for improved performance across diverse applications. Future research on the hybrid cosine-Jaccard measure for neutrosophic sets can also be extended in improving these algebraic characteristics and investigating the results in practical applications. Using more complex models to test whether it can handle more accurately during decision-making processes.

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Conflicts of Interest

The authors declare no conflict of interest.

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