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A Generalized Neutrosophic Exponential Product-Type Estimator under Indeterminate Data Conditions

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Abstract

In classical statistical frameworks, estimating a population mean generally relies on exact observations along with auxiliary information. However, in many practical scenarios, such as financial markets and temperature measurements, data often appear imprecise, uncertain, or expressed in interval form. These characteristics reduce the efficiency and reliability of conventional estimation techniques. To address such challenges, neutrosophic statistics offer a more adaptable and comprehensive framework. This study introduces a neutrosophic exponential product–exponential type estimator aimed at estimating the finite population mean under uncertainty and indeterminacy, while minimizing the Mean Squared Error (MSE). Neutrosophic statistics extend traditional statistical methods by incorporating uncertainty, inconsistency, and incomplete information into the analysis. In this research, a neutrosophic exponential sine-type estimator is constructed by utilizing auxiliary information to estimate the neutrosophic mean of the study variable. The bias and MSE expressions of the proposed estimator are derived using first-order approximation methods. To assess performance, measures such as MSE and Percent Relative Efficiency (PRE) are employed. The results demonstrate that the proposed estimator outperforms several existing neutrosophic estimators. Furthermore, its practical relevance is validated using real-world datasets from medical sales and marketing sectors. Simulation studies are also conducted to verify the theoretical findings, confirming that the proposed estimator provides improved efficiency and reliability under uncertain conditions.

Keywords: Classical statistical techniques, Neutrosophic statistics theory, Exponential product-type estimator, Use of auxiliary information, Mean squared error, Percent relative efficiency.

1 | Introduction

The primary aim of sampling theory is to obtain highly accurate and reliable estimates of the study variable based on sample data. Over time, statisticians have increasingly recognized that incorporating auxiliary information significantly enhances the precision and efficiency of estimation procedures, and therefore it has been widely integrated into survey sampling methodologies. The selection of an appropriate estimator is

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largely determined by the nature and strength of the relationship between the study variable and the auxiliary variable. In particular, ratio estimators are preferred when there exists a strong positive correlation between these variables, as they effectively utilize proportional relationships to improve estimation accuracy. Conversely, product estimators are employed when the relationship is negative, as they are better suited for inverse associations. Through continuous methodological advancements and rigorous statistical evaluation, both ratio and product estimators have established themselves as dependable tools in population estimation. Their performance is notably improved when auxiliary information is properly utilized, leading to greater accuracy, reduced estimation error, and enhanced efficiency in survey sampling applications.

In many estimation problems, various properties of auxiliary variables such as kurtosis and coefficient of variation are taken into account. Incorporating these characteristics into the estimation procedure helps in obtaining more precise and efficient estimators that better capture the complexity and variability of the data. Over time, the development of estimation techniques like ratio, product, and exponential estimators has greatly improved the estimation of population means in survey sampling. Ratio estimators are based on the assumption of a proportional relationship between the study and auxiliary variables. By using auxiliary information, they construct a ratio between these variables, leading to improved estimation accuracy. The combination of sample data and auxiliary information enhances the precision of population mean estimation and reflects both variability and association among variables. Product estimators, on the other hand, use a multiplicative relationship between study and auxiliary variables, offering an alternative method for estimation, particularly useful in cases of inverse relationships. Exponential estimators further extend these ideas by applying exponential transformations to auxiliary variables, providing a more flexible and generalized approach to estimation.

In classical statistics, numerous estimation methods have been devised based on single as well as multiple auxiliary variables. Cochran [1] was the first to utilize auxiliary data to improve the precision of the ratio estimation technique. A ratio estimator is applied when the auxiliary variable is positively correlated with the study variable. The product method of estimation, introduced by Murthy [2], was designed as a parallel to the ratio estimator. The product method of estimation is used when a negative correlation is observed between the study and auxiliary variables. The regression estimator provides more accurate results than the ratio and product estimators when the regression line does not pass through the origin. Sharma et al. [3] introduced exponential ratio-product estimators that incorporate auxiliary information through second-order approximation techniques. Singh and Sharma [4] introduced a set of exponential ratio estimators that utilize two auxiliary variables to estimate the finite population mean. Exponential imputation approaches for estimating the population mean under the assumption of completely random missing data were suggested by Singh and Sharma [4]. Classical statistics operates on datasets where all values are crisp and the measurements involve no ambiguity. But in many situations, the data exhibits feature of uncertainty, and ambiguous interpretation. To tackle such ambiguity, Zadeh [5] pioneered fuzzy logic as his first contribution. In everyday scenarios, we often deal with indeterminate data, for example, temperature readings, decision-making systems, weather predictions, economic trends, etc. In these situations, it becomes essential to develop new methods for managing indeterminate data. To resolve uncertainty and vagueness in data, fuzzy logic is often applied, but it lacks the capacity to address indeterminacy. When faced with this kind of situation, methods based on neutrosophic logic are ideal for managing both unpredictability and indeterminacy. Smarandache [6] proposed neutrosophy, an extension of fuzzy logic that considers both randomness and indeterminacy. Neutrosophic statistics address scenarios with data uncertainty, unlike classical statistics, which are applied when the data is clear and deterministic [7]. When there is no indeterminacy, neutrosophic statistics behaves just like classical statistics, reflecting its role as an expansion of the classical model [7]. Neutrosophic statistics is effective in addressing various real-world challenges, including the study of traffic accidents [8]. A goodness-of-fit method grounded in neutrosophic statistics was developed by Aslam [9]. Neutrosophic statistics, as described in [10], deals with situations where the sample size isn't precisely known. A more sophisticated version of fuzzy sets is the complex fuzzy set, and its broader generalization is known as the complex neutrosophic set. Under the framework of sampling theory, Tahir et al. [11] addressed the estimation of the population mean in the face of data indeterminacy and lack of certainty. An almost unbiased estimator for determining the population mean using neutrosophic information was suggested by Singh et al. [12]. In an effort to enhance population mean estimation, Yadav and Smarandache [13] proposed a generalized method under the neutrosophic framework. In the past few years, the field of neutrosophic statistics has seen major contributions from various scholars. The estimation of population mean under uncertainty has been extensively studied in neutrosophic statistics. In this direction, Vishwakarma and Singh [14] proposed estimators for population mean estimation within a neutrosophic framework. Further advancement was made by Kumar et al. [15], who developed a neutrosophic exponential-type estimator that incorporates both study and auxiliary variables to handle population mean estimation under uncertain conditions. To improve robustness against data irregularities, Alomair and Shahzad [16] introduced neutrosophic Hartley-Ross-type ratio estimators, which are capable of producing reliable estimates even in the presence of outliers and both sensitive and non-sensitive variables. In another contribution, Singh et al. [17] proposed neutrosophic regression-cum-ratio estimators, particularly useful for estimating population means in uncertain environments with applications in medical science. The use of sampling techniques was further extended by Singh and Kumari [18], who developed a neutrosophic ranked set sampling method for estimating population means with applications in demographic studies. Similarly, Singh et al. [19] introduced regression and ratio-based estimators under neutrosophic stratified sampling and evaluated their performance using real climate data. To further enhance estimation accuracy, Singh and Gupta [20] proposed a hybrid neutrosophic estimator combining ratio-cum-product and exponential methods along with two auxiliary variables. Additionally, Singh et al. [21] introduced a class of neutrosophic estimators for mean estimation based on real-world applications. The work of [22] focused on estimation using two neutrosophic auxiliary variables under uncertain conditions, while Singh and Tiwari [23] proposed an improved method utilizing known medians of two auxiliary variables in a neutrosophic framework.

In the present study, we propose a neutrosophic product-exponential type estimator for estimating the finite population mean under uncertainty. The structure of this paper is organized as follows: Section 1 presents the introduction, Section 2 describes neutrosophic notation and symbols, Section 3 reviews existing estimators, Section 4 introduces the proposed estimator along with derivations of bias and Mean Squared Error (MSE) using first-order approximation, Section 5 provides theoretical efficiency comparison, Section 6 presents a real-life application based on COVID-19 data and simulation analysis, Section 7 discusses empirical and simulation results, Section 8 presents detailed discussion of findings, and Section 9 concludes the study.

2 | Neutrosophic Terminology

A number of neutrosophic observation types have been developed, involving numerical values that may exist within an uncertain interval (a,b) [7]. Different approaches are available to express a neutrosophic number in interval form. According to [11] and [13], it is represented as $Z_N = Z_L + Z_U I_N$, with I_N belonging to the range $[I_L, I_U]$. In the neutrosophic variable Z_N, the emblems Z_L and Z_U signify lower and upper limits, respectively, The indeterminacy degree is represented by I_N , which ranges from 0 to 1. The i^{th} neutrosophic observation of y is given by $y_N(i) \in [y_L, y_U]$, and the corresponding auxiliary variable x is denoted as $x_N(i) \in [x_L, x_U]$. The sample means of the neutrosophic variables y_N and x_N is $y_N(i) \in [y_L, y_U]$ and $x_N(i) \in [x_L, x_U]$ respectively. $\overline{Y}_N(i)$ and $\overline{X}_N(i)$ be the population means of the study and auxiliary variables, respectively. The neutrosophic coefficient of variation of the study variable y_N and auxiliary variable x_N is given by $C_{yN} \in [C_{yL}, C_{yU}]$ and $C_N \in [C_{xL}, C_{xU}]$ and $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$ is the neutrosophic correlation coefficient between the study variable y_N and x_N auxiliary variable, respectively. Neutrosophic relative errors for the sample means of the study and auxiliary variables are reperesented as y_N and x_N are represented as. $e_{0\mathcal{N}} = \frac{\bar{y}_N - \overline{Y}_N}{\overline{Y}_N}$ and $e_{1\mathcal{N}} = \frac{\bar{x}_N - \overline{X}_N}{\overline{X}_N}$, respectively. To compute the bias and MSE of the estimator.

$$\bar{y}_N = (1 + e_{0N})\overline{Y}_N \text{ and } \bar{x}_N = (1 + e_{1N})\overline{X}_N,$$

$$E(e_{0N}) = E(e_{1N}) = 0;\ E(e_{0N}^2) = f_{\mathcal{N}} C_{yN}^2, E(e_{1N}^2) = f_N C_{xN}^2, E(e_{0N}e_{1N}) = f_N \rho_{yxN} C_{yN} C_{xN},$$

where

$e_{0\mathcal{N}} \in [e_{0L}, e_{0U}], e_{1\mathcal{N}} \in [e_{1L}, e_{1U}]$, and $e_{0N}e_{1N} \in [e_{1L}e_{1U}, e_{0L}e_{0U}]$,

$C_{yN}^2 = \frac{S_{yN}^2}{\bar{Y}_N^2}, \; C_{xN}^2 = \frac{S_{xN}^2}{\bar{X}_N^2}$,

$C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$ and $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$,

$f_{\mathcal{N}} = \frac{N_N - n_N}{n_N N_N}$, where $f_N \in [f_L, f_U]$,

$S_{yxN} = \rho_{yxN} C_{yN} C_{xN}$,

$S_{yN}^2 \in [S_{yL}^2, S_{yU}^2], S_{xN}^2 \in [S_{xL}^2, S_{xU}^2], S_{yxN} \in [S_{yxL}, S_{yxU}], \rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$,

The *Fig. 1* provides a comprehensive illustration of the parameter estimation process within the broader framework of statistical inference. It begins by distinguishing between estimation and hypothesis testing, and then focuses on the estimation pathway. From there, the process is divided into two major approaches: the classical method and the neutrosophic method. In the classical path, the identification of study and auxiliary variables is followed by an assessment of their correlation, which determines the use of ratio or product-type estimators. In contrast, the neutrosophic path incorporates uncertainty in the data through simple random sampling and evaluates the relationship between variables under moderate to high correlation, ultimately leading to the application of a product exponential estimator.

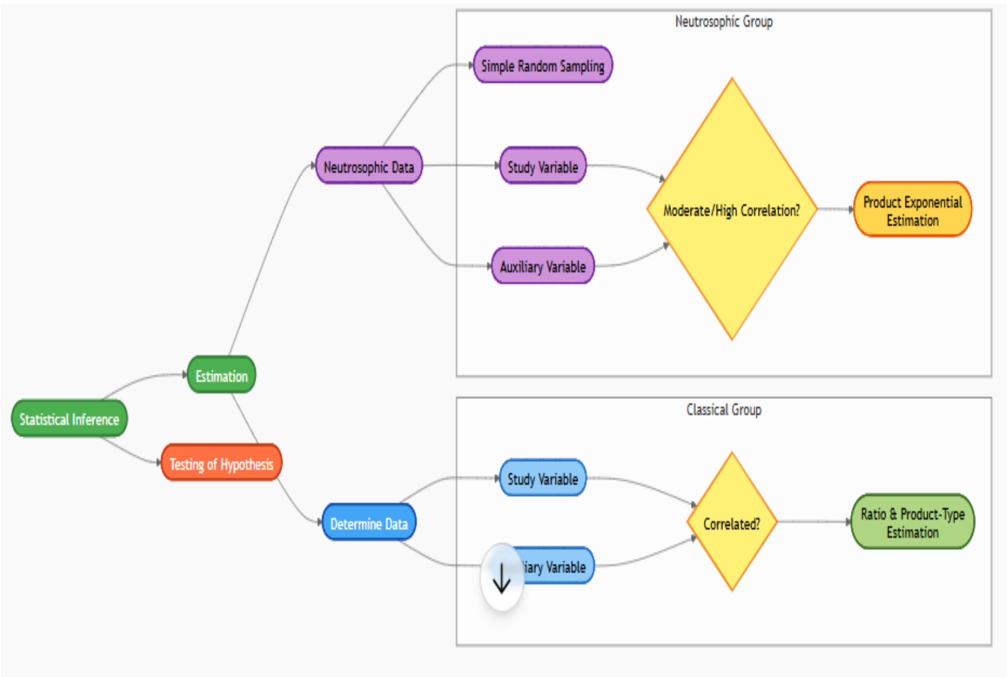

Fig. 1. Flow diagram illustrating the process of parameter estimation.

3 | Existing Estimators

The neutrosophic population mean $(\bar{Y}_N)$ is estimated using the corresponding neutrosophic sample mean.

$$t_{1N} = \bar{y}_N. \tag{1}$$

The variance associated with the estimator $\bar{Y}_N$ is defined by the following expression.

$$V(t_{1N}) = f_N \bar{Y}_N^2 C_{yN}^2, \text{ where } t_{1N} \in [t_{1L}, t_{1U}]. \tag{2}$$

Using the approach developed in [2], the neutrosophic product estimators can be expressed as follows.

$$t_{2N} = \bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N}\right). \quad \textbf{(3)}$$

The bias and MSE expressions corresponding to the estimator t_{2N} are given as follows.

$$\text{Bias }(t_{2N}) = f_N \bar{Y}_N \rho_{yxN} C_{yN} C_{xN}, \quad \textbf{(4)}$$

$$\text{MSE }(t_{2N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + C_{xN}^2 + 2\rho_{yxN} C_{yN} C_{xN}\right), \text{ where } t_{2N} \in [t_{2L}, t_{2U}]. \quad \textbf{(5)}$$

A neutrosophic ratio estimator for the estimation of a finite population mean was proposed in [11] and is defined as follows.

$$t_{3N} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right). \quad \textbf{(6)}$$

Under a first-order approximation, the bias and MSE of estimator $t_{3\mathcal{N}}$ are derived as given below.

$$\text{Bias }(t_{3N}) = f_N \bar{Y}_N \left(C_{xN}^2 - \rho_{yxN} C_{yN} C_{xN}\right). \quad \textbf{(7)}$$

$$\text{MSE }(t_{3N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + C_{xN}^2 - 2\rho_{yxN} C_{yN} C_{xN}\right), \text{ where } t_{3N} \in [t_{3L}, t_{3U}]. \quad \textbf{(8)}$$

Building on the work of [24], Tahir et al. [11] introduced a neutrosophic ratio estimator utilizing the coefficient of variation associated with the neutrosophic variable X as

$$t_{4\mathcal{N}} = \bar{y}_N \left(\frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}}\right). \quad \textbf{(9)}$$

Using a first-order approximation, the respective bias and MSE of estimator $t_{4\mathcal{N}}$ are obtained as follows.

$$\text{Bias}(t_{4N}) = f_N \bar{Y}_N \left(\lambda_{1N}^2 C_{xN}^2 - \lambda_{1N} \rho_{yxN} C_{yN} C_{xN}\right). \quad \textbf{(10)}$$

$$\text{MSE}(t_{4N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + C_{xN}^2 \lambda_{1N}^2 - 2\lambda_{1N} \rho_{yxN} C_{yN} C_{xN}\right), \quad \textbf{(11)}$$

where, $\lambda_{1N} = \frac{\bar{X}_N}{\bar{X}_N + C_{xN}}$ and $t_{4N} \in [t_{4L}, t_{4U}]$.

Motivated by Bahl and Tuteja [25], Tahir et al. [11] introduced the following neutrosophic exponential ratio-type estimator.

$$t_{5\mathcal{N}} = \bar{y}_N \exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{x}_N + \bar{X}_N}\right). \quad \textbf{(12)}$$

Below are the expressions for the bias and MSE associated with estimator $t_{5\mathcal{N}}$.

$$\text{Bias}(t_{5\mathcal{N}}) = f_N \bar{Y}_N \left(\frac{3}{8} C_{xN}^2 - \frac{1}{2} \rho_{yxN} C_{yN} C_{xN}\right). \quad \textbf{(13)}$$

$$\text{MSE}(t_{5N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} - \rho_{yxN} C_{yN} C_{xN}\right), \text{ where } t_{5N} \in [t_{5L}, t_{5U}]. \quad \textbf{(14)}$$

Motivated by Cochran [1], Jerajuddin and Kishun [26] introduced the neutrosophic exponential product-type estimator given below.

$$t_{6N} = \bar{y}_N \exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{x}_N + \bar{X}_N}\right). \quad \textbf{(15)}$$

The associated expressions for the bias and MSE are given as follows.

$$\mathrm{Bias}(t_{6N}) = f_N \overline{Y}_N \left(\frac{3}{8} C_{xN}^2 + \frac{1}{2} \rho_{yxN} C_{yN} C_{xN}\right). \tag{16}$$

$$\mathrm{MSE}(t_{6\mathcal{N}}) = f_N \overline{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} + \rho_{yxN} C_{yN} C_{xN}\right), \text{ where } t_{6\mathcal{N}} \in [t_{6L}, t_{6U}]. \tag{17}$$

A neutrosophic ratio estimator that utilizes the population correlation coefficient was introduced by Yadav and Smarandache [13].

$$t_{7N} = \bar{y}_N \left(\frac{\overline{X}_N + \rho_{yxN}}{\bar{x}_N + \rho_{yxN}}\right). \tag{18}$$

The analytical expressions of the bias and MSE corresponding to estimator t_{7N} are shown below.

$$\mathrm{Bias}(t_{7\mathcal{N}}) = f_N \overline{Y}_N \left(\lambda_{2\mathcal{N}}^2 C_{xN}^2 - \lambda_{2N} \rho_{yxN} C_{yN} C_{xN}\right). \tag{19}$$

$$\mathrm{MSE}(t_{7N}) = f_N \overline{Y}_N^2 \left(C_{yN}^2 + \lambda_{2N}^2 C_{xN}^2 - 2\lambda_{2N} \rho_{yxN} C_{yN} C_{xN}\right), \tag{20}$$

where, $\lambda_{2N} = \frac{\overline{X}_N}{\overline{X}_N + \rho_{yxN}}$ and $t_{7N} \in [t_{7L}, t_{7U}]$.

Motivated by Singh et al. [27], the neutrosophic ratio estimator t_{8N} was proposed by employing ratio estimators for the population mean based on the selected sample size.

$$t_{8N} = \bar{y}_N \left(\frac{\overline{X}_N + n_N}{\bar{x}_N + n_N}\right). \tag{21}$$

The bias and MSE corresponding to the neutrosophic ratio estimator $t_{8\mathcal{N}}$ are expressed below.

$$\mathrm{Bias}(t_{8N}) = f_N \overline{Y}_N \left(\lambda_{3N}^2 C_{xN}^2 - \lambda_{3N} \rho_{yxN} C_{yN} C_{xN}\right). \tag{22}$$

$$\mathrm{MSE}(t_{8N}) = f_N \overline{Y}_N^2 \left(C_{yN}^2 + \lambda_{3N}^2 C_{xN}^2 - 2\lambda_{3N} \rho_{yxN} C_{yN} C_{xN}\right), \tag{23}$$

where, $\lambda_{3N} = \frac{\overline{X}_N}{\overline{X}_N + \rho_{yxN}}$ and $t_{8N} \in [t_{8L}, t_{8U}]$.

Inspired by the methodology in [28], Tahir et al. [11] developed the neutrosophic modified exponential ratio estimator in the following manner.

$$t_{9N} = \bar{y}_N \exp\left(\frac{(a_n \overline{X}_N + b_n) - (a_n \bar{x}_N + b_n)}{(a_n \overline{X}_N + b_n) + (a_n \bar{x}_N + b_n)}\right). \tag{24}$$

Here, a_n and b_n signify the auxiliary neutrosophic parameters.

Employing a first-order approximation, the bias and MSE corresponding to the neutrosophic exponential ratio estimator t_{9N} are presented as follows.

$$\mathrm{Bias}\ (t_{9N}) = \left(\frac{3}{8} \lambda_{4N}^2 C_{xN}^2 - \frac{1}{2} \lambda_{4N} \rho_{yxN} C_{yN} C_{xN}\right). \tag{25}$$

$$\mathrm{MSE}(t_{9N}) = f_N \overline{Y}_N^2 \left(C_{yN}^2 + \lambda_{4N}^2 C_{xN}^2 - 2\lambda_{4N} \rho_{yxN} C_{yN} C_{xN}\right), \tag{26}$$

where, $\lambda_{4N} = \frac{a_n \overline{X}_N}{2(a_n \overline{X}_N + b_n)}$ and $t_{9N} \in [t_{9L}, t_{9U}]$.

4 | Proposed Product-Exponential Neutrosophic Estimator

In survey sampling, auxiliary information is extensively used to enhance the precision of estimators for population parameters. Traditional estimators, including ratio, product, and exponential types, rely on the assumption that the data are exact and free from uncertainty. However, in practical scenarios, data are often incomplete or imprecise due to measurement errors, missing responses, or vague information. To overcome these challenges, Neutrosophic Set Theory, proposed by Smarandache [6], offers a comprehensive approach for explicitly handling indeterminacy. This theory integrates truth, falsity, and indeterminacy components, making it well-suited for analyzing uncertain and inconsistent data. Inspired by the contributions of Solanki et al. [29], who developed improved estimators utilizing auxiliary information in the classical framework, we generalize their approach to the neutrosophic setting. In particular, we introduce a neutrosophic estimator for the population mean given by

$$t^*_{(\alpha,\delta)N} = \bar{y}_N \left[2 - \left(\frac{\bar{x}_N}{\bar{X}_N}\right)^{\alpha} \exp\left\{\delta \frac{(\bar{x}_N - \bar{X}_N)}{(\bar{x}_N + \bar{X}_N)}\right\}\right], \tag{27}$$

where, $\bar{y}_N, \bar{x}_N$ and $\bar{X}_N$ represent neutrosophic sample and population means, and α and δ is suitably chosen scalars.

The proposed estimator aims to improve efficiency by combining the strengths of ratio-type and exponential estimators while accounting for uncertainty inherent in the data. The inclusion of neutrosophic components allows for a more flexible and realistic modeling of practical sampling situations where indeterminacy cannot be ignored.

Table 1. Distinct members of the proposed estimator class.

Case	α	δ	Resulting Estimator $\left(t^*_{(\alpha,\delta)N}\right)$	Name/Type
1	0	0	$t^*_{(0,0)N} = \bar{y}_N$	Sample neutrosophic mean estimator
2	1	0	$t^*_{(1,0)N} = \bar{y}_N \left[2 - \left(\frac{\bar{x}_N}{\bar{X}_N}\right)\right]$	Neutrosophic ratio-type estimator
3	-1	0	$t^*_{(-1,0)N} = \bar{y}_N \left[2 - \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\right]$	Neutrosophic product-type estimator
4	0	1	$t^*_{(0,1,)N} = \bar{y}_N \left[2 - \exp\left\{\frac{(\bar{x}_N - \bar{X}_N)}{(\bar{x}_N + \bar{X}_N)}\right\}\right]$	Neutrosophic exponential-type estimator
5	1	1	$t^*_{(1,1)N} = \bar{y}_N \left[2 - \left(\frac{\bar{x}_N}{\bar{X}_N}\right) \exp\left\{\frac{(\bar{x}_N - \bar{X}_N)}{(\bar{x}_N + \bar{X}_N)}\right\}\right]$	Neutrosophic ratio-typeexponential estimator
6	1	-1	$t^*_{(1,-1)N} = \bar{y}_N \left[2 - \left(\frac{\bar{x}_N}{\bar{X}_N}\right) \exp\left\{-\frac{(\bar{x}_N - \bar{X}_N)}{(\bar{x}_N + \bar{X}_N)}\right\}\right]$	Neutrosophic modified ratio-type estimator

The *Eq. (27)* can be expressed in terms of error components and the Right-Hand Side (RHS) is presented as follows.

$$t^*_{(\alpha,\delta)N} = (1 + e_{0N})\bar{Y}_N \left[2 - \left(\frac{(1 + e_{1N})\bar{X}_N}{\bar{X}_N}\right)^{\alpha} \exp\left(\delta \frac{(1 + e_{1N})\bar{X}_N - \bar{X}_N}{(1 + e_{1N})\bar{X}_N + \bar{X}_N}\right)\right]. \tag{28}$$

$$t^*_{(\alpha,\delta)N} = (1 + e_{0N})\bar{Y}_N \left[2 - (1 + e_{1N})^{\alpha} \exp\left(\delta \frac{(1 + e_{1N})\bar{X}_N - \bar{X}_N}{(1 + e_{1N})\bar{X}_N + \bar{X}_N}\right)\right]. \tag{29}$$

On solving the expressions on the RHS of the *Eq. (29)*, we get

$$t^*_{(\alpha,\delta)N} = (1 + e_{0N})\bar{Y}_N \left[2 - (1 + e_{1N})^{\alpha} \exp\left(\delta \frac{e_{1N}}{2 + e_{1N}}\right)\right]. \tag{30}$$

$$t^*_{(\alpha,\delta)N} = (1 + e_{0N})\bar{Y}_N \left[2 - (1 + e_{1N})^{\alpha} \exp\left(\delta \frac{e_{1N}}{2 + e_{1N}}\right)\right]. \tag{31}$$

$$t^*_{(\alpha,\delta)N} = (1 + e_{0N})\bar{Y}_N \left[2 - (1 + e_{1N})^{\alpha} \exp\left\{\frac{\delta e_{1N}}{2}\left(1 + \frac{e_{1N}}{2}\right)^{-1}\right\}\right]. \tag{32}$$

We assume that $|e_{1N}| < 1$, so that $(1+e_{1N})^{\alpha}$, $\exp\left\{\frac{\delta e_{1N}}{2}\left(1+\frac{e_{1N}}{2}\right)^{-1}\right\}$ and $\left(1+\frac{e_{1N}}{2}\right)^{-1}$ are expandable. Now, on expanding the RHS of *Eq. (32)*, the expression becomes

$$t^*_{(\alpha,\delta)N} = (1+e_{0N})\overline{Y}_N\left[\begin{array}{c} 2-\left\{1+\alpha e_{1N}+\frac{\alpha(\alpha-1)}{2}e_{1N}^2\right\}\times \\ \left\{1+\frac{\delta e_{1N}}{2}\left(1+\frac{e_{1N}}{2}\right)^{-1}+\frac{\delta^2 e_{1N}^2}{8}\left(1+\frac{e_{1N}}{2}\right)^{-2}\right\}\end{array}\right]. \tag{33}$$

$$t^*_{(\alpha,\delta)N} = (1+e_{0N})\overline{Y}_N\left[\begin{array}{c} 2-\left\{1+\alpha e_{1N}+\frac{\alpha(\alpha-1)}{2}e_{1N}^2\right\}\times \\ \left\{1+\frac{\delta e_{1N}}{2}\left(1-\frac{e_{1N}}{2}+\frac{e_{1N}^2}{4}-\cdots+\frac{\delta^2 e_{1N}^2}{8}e_{1N}^2(1-e_{1N})\right)\right\}\end{array}\right]. \tag{34}$$

$$t^*_{(\alpha,\delta)N} = (1+e_{0N})\overline{Y}_N\left[2-\left\{1+\frac{(2\alpha+\delta)}{2}e_{1N}+\frac{(2\alpha+\delta)(2\delta+\delta-2)}{8}e_{1N}^2\right\}\right]. \tag{35}$$

$$t^*_{(\alpha,\delta)N} = \overline{Y}_N\left[\left\{\begin{array}{c}1+e_{0N}-\frac{(2\alpha+\delta)}{2}(e_{1N}+e_{0N}e_{1N})-\frac{(2\alpha+\delta)(2\delta+\delta-2)}{8} \\ (e_{1N}^2+e_{0N}e_{1N}^2)-..\end{array}\right\}\right]. \tag{36}$$

By neglecting all e's terms of degree greater than two in *Eq. (36)*, we get the following result.

$$t^*_{(\alpha,\delta)N} \cong \overline{Y}_N\left[1+e_{0N}-\frac{(2\alpha+\delta)}{2}\left\{e_{1N}+e_{0N}e_{1N}-\frac{(2\delta+\delta-2)}{4}e_{1N}^2\right\}\right]. \tag{37}$$

Now Subtracting the population mean $(\overline{Y}_N)$ from both sides of *Eq. (37)*, we obtain the following expression.

$$t^*_{(\alpha,\delta)N}-\overline{Y}_N \cong \overline{Y}_N\left[1+e_{0N}-\frac{(2\alpha+\delta)}{2}\left\{e_{1N}+e_{0N}e_{1N}-\frac{(2\delta+\delta-2)}{4}e_{1N}^2\right\}\right]-\overline{Y}_N. \tag{38}$$

We get

$$t^*_{(\alpha,\delta)N}-\overline{Y}_N \cong \overline{Y}_N\left[e_{0N}-\frac{(2\alpha+\delta)}{2}\left\{e_{1N}+e_{0N}e_{1N}-\frac{(2\delta+\delta-2)}{4}e_{1N}^2\right\}\right]. \tag{39}$$

Upon taking expectations of both sides of *Eq. (39)*, the resulting expression gives the bias of the class of estimators $t^*_{(\alpha,\delta)N}$ at the first level of approximation.

$$\text{Bias}\left(t^*_{(\alpha,\delta)N}\right) = f_N\overline{Y}_N C_{xN}^2\left[-\frac{(2\alpha+\delta)}{2}\left\{\frac{(2\delta+\delta-2)}{4}+k\right\}\right], \tag{40}$$

where, $k=\left(\frac{\rho C_{yN}}{C_{xN}}\right)$.

By squaring both sides of *Eq. (40)* and neglecting higher-order e_s terms (above degree two), we get the following result.

$$\left(t^*_{(\alpha,\delta)N}-\overline{Y}_N\right)^2 \cong \overline{Y}_N\left[e_{0N}^2+\frac{(2\alpha+\delta)}{2}\left\{\frac{(2\alpha+\delta)}{4}e_{1N}^2-2e_{0N}e_{1N}\right\}\right]. \tag{41}$$

Taking the expectation of both sides of *Eq. (41)*, we obtain the MSE of $t^*_{(\alpha,\delta)N}$ to the first degree of approximation.

$$\mathrm{MSE}\left(t^*_{(\alpha,\delta)N}\right) = f_N \overline{Y}_N^2 \left[C_{yN}^2 + \frac{(2\alpha+\delta)}{4} C_{xN}^2\{(2\alpha+\delta) - 4k\}\right]. \quad \textbf{(42)}$$

This is minimum when

$$(2\alpha+\delta) = k.$$

Hence, the resulting minimum MSE for the proposed estimator $t^*_{(\alpha,\delta)N}$ is given by the following expression.

$$\mathrm{minMSE}\left(t^*_{(\alpha,\delta)N}\right) = f_N C_{yN}^2 (1-\rho^2). \quad \textbf{(43)}$$

5 | Efficiency Comparison

The comparison of efficiency between existing estimators and a proposed estimator plays a crucial role in statistical research, as it provides insight into the relative performance of different estimation techniques. This process involves examining key performance indicators, including variance and MSE, to evaluate how well each estimator approximates the true population parameter. Through both theoretical derivations and numerical illustrations, the proposed estimator can be assessed against established methods, thereby demonstrating its advantages, limitations, and potential applicability in various practical scenarios.

$V(t_{1N}) - \mathrm{minMSE}\left(t^*_{(\alpha,\delta)N}\right) > 0$, or $f_N \overline{Y}_N^2 C_{yN}^2 - f_N C_{yN}^2 (1-\rho^2)$.

6 | Empirical Study

6.1 | Numerical Study

In this section, the datasets used for empirical evaluation are introduced. Dataset 1, shown in *Table 2*, is used for the baseline analysis and includes the initial observations required to study the key relationships between the existing and proposed estimators. Dataset 2, presented in *Table 4*, is applied for extended analysis to verify the consistency and robustness of the results across a wider data structure.

Data Set 1: The empirical data on product sales and medical marketing were obtained from [30]. Medical practitioners gathered information on customer networking and product sales during two separate two-month intervals: the pre-lockdown period (January 1-March 1, 2021) and the lockdown period (September 1-November 1, 2022). The dataset, which contains uncertainty, is divided into two parts representing customer networking and sales variables for both pre-pandemic and post-pandemic phases. In this study, the pre-pandemic customer networking percentage is treated as the auxiliary variable, while the pre-pandemic sales percentage is considered the study variable.

Y_N: Pre-pandemic sales percentage

X_N: Pre-pandemic networking percentage

Table 2. Statistical description of Data set 1.

Prameters	Data set 1
N_n	[30,30]
n_N	[9,9]
$\overline{Y}_N$	[37.0033,37.3266]
$\overline{X}_N$	[34.1600,34.4500]
S_{yN}	[12.3153,12.3233]
S_{xN}	[15.0789,15.0891]
C_{yN}	[0.3328,0.3301]
C_{xN}	[0.4414,0.4380]
ρ_{yxN}	[0.8776,0.8775]

Percent Relative Efficiency (PRE) is a statistical measure used to evaluate and compare how well different estimators perform when estimating a population parameter. It is expressed as a percentage and calculated using the ratio of the variance of a standard estimator to that of the estimator being assessed. A larger PRE indicates greater efficiency, meaning the estimator produces more accurate results with less variability.

$$\mathrm{PRE}(\mathrm{Estimators}) = \frac{\mathrm{MSE}(\mathrm{t}_{1\mathrm{N}})}{\mathrm{MSE}(\mathrm{Existing\ and\ Proposed\ Estimators})} \times 100.$$

The MSE and PRE outcomes for Data Set 1 are presented in *Table 3*, while *Table 5* provides the corresponding results for Data Set 2. MSE is used to assess the accuracy of estimators, with smaller values indicating greater precision. In contrast, PRE measures how well the proposed estimator performs compared to a standard one, where higher values denote better efficiency. Both datasets incorporate indeterminacy arising from incomplete or uncertain information, which is addressed through Neutrosophic Statistics, allowing for more adaptable treatment of uncertainty. Additionally, *Figs. 2* and *3* illustrate the lower and upper bounds of MSE, highlighting variations in performance under uncertain conditions.

Table 3. The MSE and PRE values of the proposed estimator along with various existing estimators for Data Set 1.

Estimator	MSE	$\mathbf{I_N}$	PRE
t_{1N}	[11.7076, 55.4415]	[0, 0.0011]	[100,100]
t_{2N}	[54.4685, 89.8383]	[0, 0.0015]	[21.4942, 61.7125]
t_{3N}	[5.0169, 68.6044]	[0, 0.0029]	[233.3634, 80.8133]
t_{4N}	[29.0354, 78.5532]	[0, 0.0020]	[40.3218, 70.5782]
t_{5N}	[3.8535, 56.0780]	[0, 0.0012]	[303.8204, 98.8650]
t_{6N}	[28.5793, 66.6949]	[0, 0.0014]	[40.9653, 83.1270]
t_{7N}	[4.7559, 68.4508]	[0, 0.0034]	[246.1690, 80.9947]
t_{8N}	[3.4460, 62.0306]	[0, 0.0049]	[339.7478, 89.3776]
t_{9N}	[3.8535, 56.0780]	[0, 0.0012]	[303.8204, 98.8650]
$\mathbf{t}^*_{(\alpha,\delta)\mathrm{N}}$	[0.00198,0.00195]	[0,0.0052]	[591.0987,284.3126]

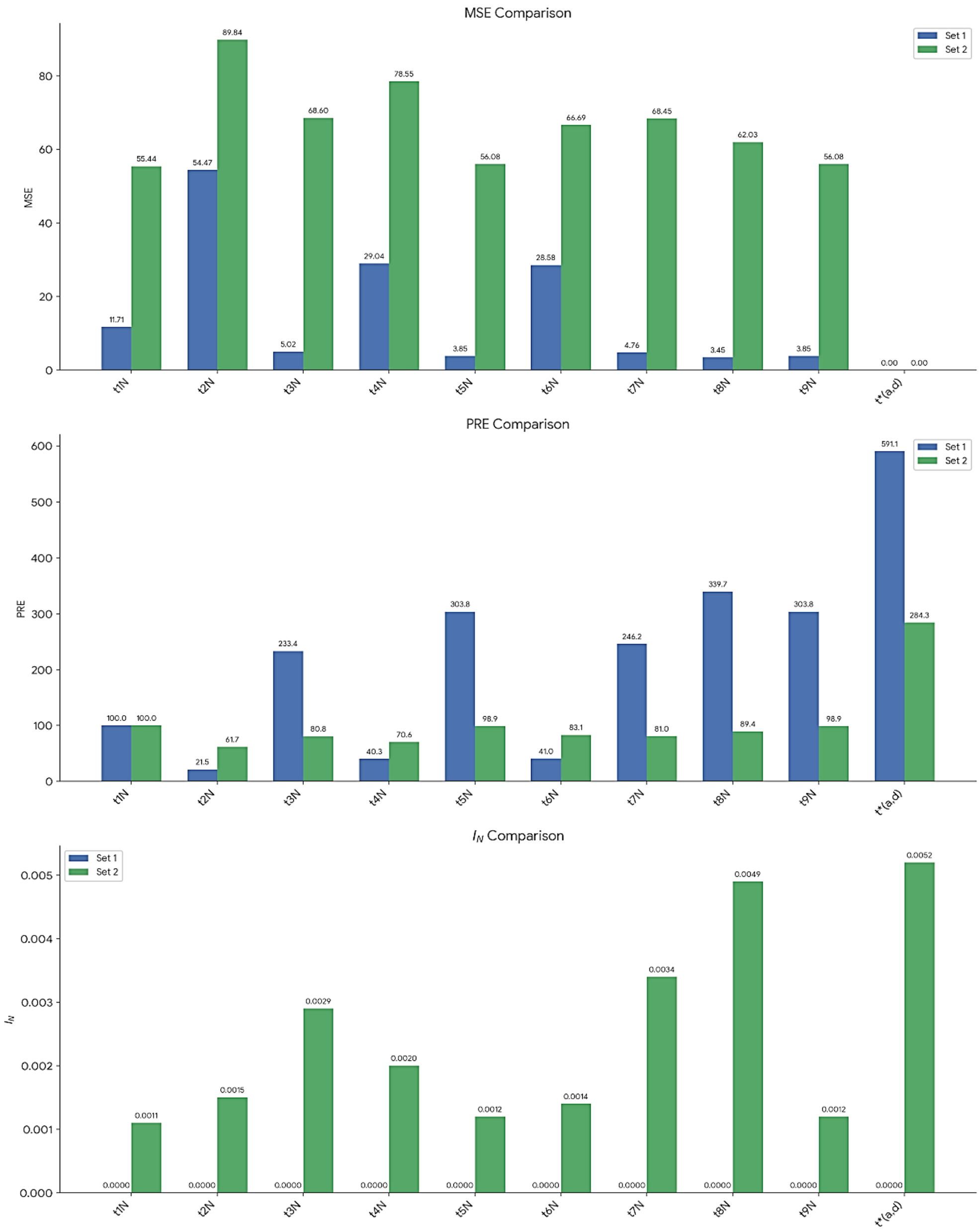

Fig. 2. The graphical representation shows the comparative performance of MSE, PRE, and indeterminacy values for different estimators.

Data set 2: Following a similar approach to [31], the second dataset uses the percentage of networking after the pandemic as auxiliary information, while the post-pandemic sales percentage is considered the study variable. In the neutrosophic framework, the study variable is denoted by Y_N, where Y_N represents the minimum sales percentage observed from either the pre- or post-pandemic period. The associated auxiliary variable is expressed as Y_N. The relevant study parameters are listed in *Table 1*.

Y_N: Post-pandemic sales percentage.

X_N: Post-pandemic networking percentage.

Table 4. Statistical description of Data set 2.

Parameters	Population 2
N_n	[30,30]
n_N	[9,9]
$\overline{Y}_N$	[34.7266, 40.040]]
$\overline{X}_N$	[34.7266, 34.9033]
S_{yN}	[12.2704, 26.7004]
S_{xN}	[15.2288, 15.2452]
C_{yN}	[0.3533, 0.6668]
C_{xN}	[0.4385, 0.4367]
ρ_{yxN}	[0.8508, 0.1462]

Table 5. The MSE and PRE values of the proposed estimator along with various existing estimators for Data Set 2.

Estimator	MSE	I_N	PRE
t_{1N}	[11.7987,11.8657]	[0,0.6888]	[100,100]
t_{2N}	[60.0026,60.0748]	[0,0.03738]	[9.6576, 19.6493]
t_{3N}	[5.0767,5.2004]	[0,0.05303]	[231.9272, 231.5202]
t_{4N}	[32.6348,31.7282]	[0,0.8313]	[37.1339, 37.0997]
t_{5N}	[3.2648,3.2169]	[0,0.6512]	[362.5786, 362.5587]
t_{6N}	[26.5298,65.6839]	[0,0.9204]	[38.4060, 38.3954]
t_{7N}	[4.65557,67.3507]	[0,0.0933]	[248.4722, 247.8971]
t_{8N}	[3.3370,64.0605]	[0,0.9212]	[385.4255, 383.9444]
t_{9N}	[3.7686,55.0978]	[0,0.9370]	[362.5786, 362.5587]
$\mathbf{t}^*_{(\alpha,\delta)N}$	[0.02412,39.3048]	[0,0.9620]	[488.9909,308.9823]

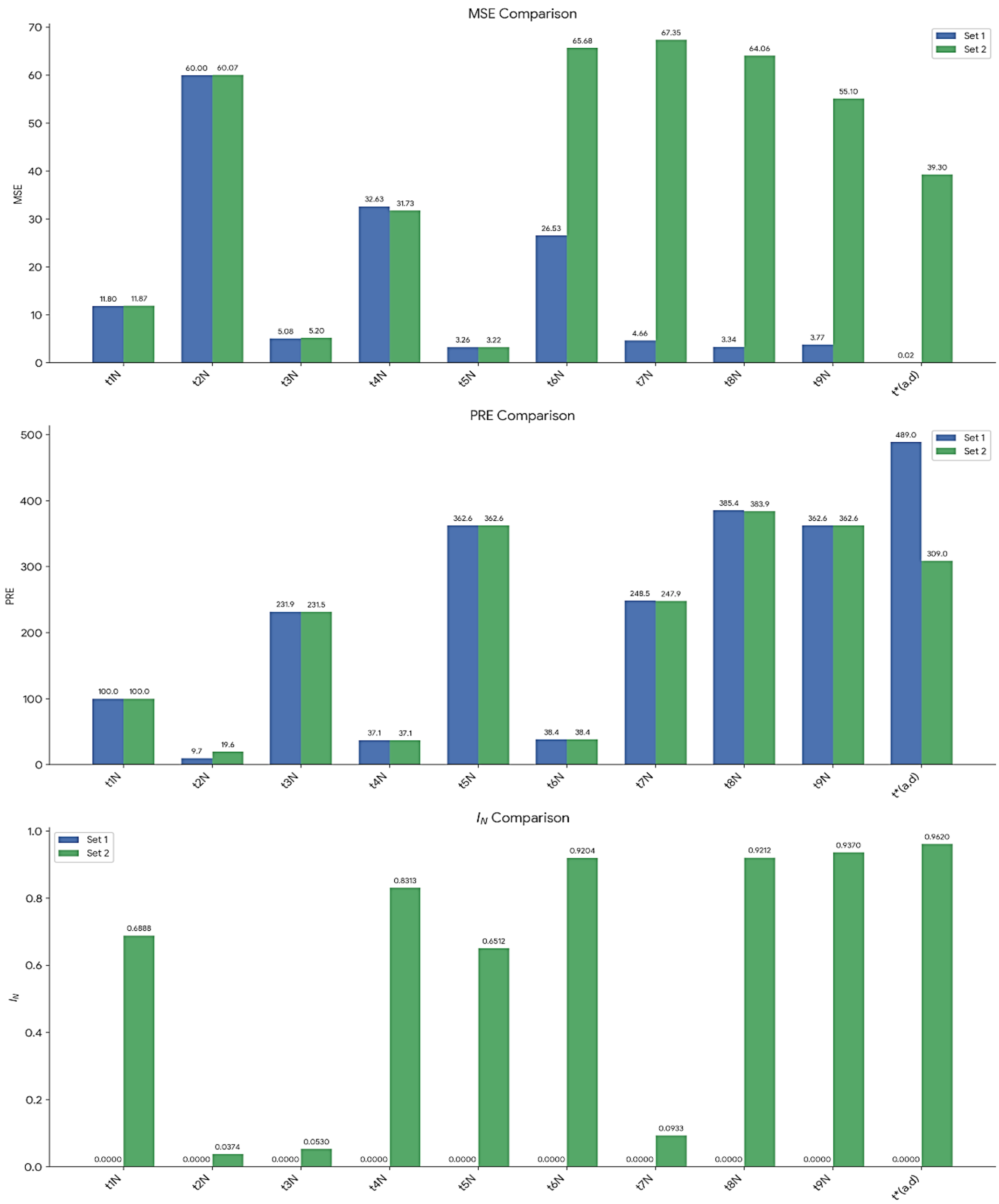

Fig. 3. The graphical results present a comparative analysis of MSE, PRE, and indeterminacy measures.

7 | Results and Discussion

The empirical evaluation of the proposed neutrosophic exponential product-type estimator $t^*_{(\alpha,\delta)N}$ is conducted using two real-world datasets characterized by interval-valued observations and inherent indeterminacy. The performance assessment is carried out through a comprehensive comparison with existing neutrosophic estimators using key statistical measures, namely MSE, Indeterminacy measure I_N, and PRE. *Table 3* For Data Set 1, the proposed estimator demonstrates outstanding performance by achieving the lowest MSE values (0.00198, 0.00195) among all competing estimators. This significant reduction in error clearly reflects its superior accuracy in estimating population parameters under uncertain conditions. In addition, the PRE values (591.0987, 284.3126) are remarkably higher than those of traditional ratio, product, and exponential-type estimators, confirming its strong efficiency advantage. Although estimators such as t_{5N}, t_{8N} and t_{9N} also perform reasonably well, none match the performance level of the proposed estimator.

Furthermore, the indeterminacy measure I_N remains within acceptable limits, indicating that the estimator effectively manages uncertainty without introducing instability.

Data Set 2 (*Table 5*), the estimator continues to exhibit robust performance even under increased variability and weaker correlation structures. The MSE values (0.02412, 39.3048) remain competitive, while the PRE values (488.9909, 308.9823) demonstrate substantial efficiency improvements over existing estimators. Even when compared with moderately performing estimators such as t_{5N} and t_{8N}, the proposed estimator consistently maintains superiority. The indeterminacy values further confirm its reliability in handling vague and incomplete data. Overall, graphical representations strongly support the numerical results, clearly illustrating that the proposed estimator consistently outperforms existing methods. The simultaneous reduction in MSE and enhancement in PRE highlight the effectiveness of integrating neutrosophic theory with exponential product-type structures.

The graphical results in *Fig. 2* support the computed outcomes, showing that the proposed estimator achieves the minimum MSE trajectory and maximum PRE trajectory among all estimators. Additionally, it maintains stable and low indeterminacy values, reflecting controlled uncertainty. Similarly, *Fig. 3* demonstrates through comparative bar plots that the proposed estimator consistently surpasses all competing estimators in terms of all performance measures, especially efficiency. In conclusion, the proposed estimator provides a highly efficient, stable, and robust approach for parameter estimation under uncertainty, making it highly suitable for real-world applications in fields such as healthcare analytics, economics, and social sciences.

8 | Conclusion

This research presents a neutrosophic product-exponential type estimator developed for estimating the finite population mean in the presence of uncertainty and indeterminacy. The proposed estimator effectively incorporates auxiliary information, which significantly improves the efficiency and reliability of estimation. The statistical characteristics of the estimator, particularly bias and MSE, are derived using a first-order approximation technique, which provides a simplified yet effective theoretical framework for analysis. The empirical evaluation of the estimator is supported through graphical representations such as bar charts and comparative performance plots, which clearly demonstrate that the proposed estimator consistently outperforms traditional classical estimation techniques. This superiority is especially evident in situations involving imprecise, vague, or interval-valued data. The enhanced performance is primarily attributed to the estimator's ability to incorporate neutrosophic uncertainty, a feature that classical statistical methods are not equipped to handle effectively. The development of the methodology is based on several standard assumptions. It assumes that sampling is performed using simple random sampling without replacement that strongly correlated auxiliary variables are available, and that neutrosophic observations are correctly expressed in interval form. It is also assumed that population parameters are finite, well-defined, and stable, and that first-order approximation is sufficiently accurate for deriving theoretical expressions.

Despite its advantages, the proposed estimator has certain limitations. Its applicability is mainly confined to simple random sampling and may require significant modifications to be used in more complex sampling designs. The efficiency of the estimator is highly dependent on the strength of correlation between study and auxiliary variables, and its performance may deteriorate when this relationship is weak or nonlinear. Additionally, reliance on first-order approximation may lead to reduced accuracy in situations involving high variability or strong nonlinearity. The method also does not explicitly address issues such as missing data or measurement errors. Furthermore, neutrosophic interval computations may increase computational complexity, particularly when dealing with large-scale datasets.

In future research, the proposed neutrosophic exponential product-type estimator can be extended to incorporate multiple auxiliary variables, nonlinear functional relationships, and datasets that are incomplete, skewed, or highly irregular. Its application is encouraged across diverse fields such as economics, agriculture, mathematics, biology, and social sciences. Further exploration may also extend its applicability to advanced statistical areas including control charts, reliability analysis, inferential procedures, nonparametric estimation, and hypothesis testing.

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Data Availability

All the data are available in this paper.

Conflict of Interest

The authors declare no conflict of interest.

Consent for Publication

The authors confirm consent for the publication of this work

Ethics Approval and Consent to Participate

The authors confirm that this research did not involve human participants or animal subjects.

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