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Dynamic Stochastic-Fuzzy Optimization for Inventory Decision Systems under Mixed Uncertainty

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Abstract

This paper formulates a dynamic programming model of an inventory system in a mixed uncertainty environment where stochasticity and non-stationarity are present in a structurally different manner. The aleatory uncertainty due to demand and supply variation is modeled by stochastic processes, while the epistemic uncertainties on the cost parameters, on the service level goals, and on the managerial risk perception are represented by fuzzy sets. The proposed model integrates these two forms of uncertainty within a unified multistage decision framework and formulates a cost minimization problem that evolves through state-dependent inventory dynamics. Rigorous theoretical development is presented to establish the well-posedness of the optimization problem and to characterize fundamental properties of the optimal policy under hybrid uncertainty. A numerical example using real inventory data, along with a graphical analysis and a detailed sensitivity analysis, illustrates the applicability of the proposed model. The insights show the impact of mixed uncertainty considerations on optimal replenishment policy and also provide practical guidance to decision makers in complex and information-imperfect supply chain scenarios.

Keywords: Mixed uncertainty modeling, Fuzzy set theory, Hybrid uncertainty modeling, Multistage decision framework, Supply chain optimization.

1 | Introduction

Inventory decision systems constitute a central component of supply chain operations and remain a persistent source of analytical and managerial complexity in contemporary operational environments. Decision makers are routinely required to determine replenishment actions over time while facing demand variability, supply

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uncertainty, and cost structures that are only partially observable. Traditional inventory modeling techniques have largely been based on probabilistic models of uncertainty and have considered that all the relevant parameters of the system were known and could be estimated exactly from historical data [30, 31]. However, quantifiable randomness and subjective ambiguity are often encountered simultaneously in real-world applications, such as in the assessment of holding costs, backorder penalties, service level goals, and managerial risk preferences [17]. This discrepancy between modeling assumptions and real-world applications has sparked growing interest in optimization frameworks that can handle mixed uncertainty in real-time decision making environments [2]. This paper is placed in the recent research line in the sense that it aims to contribute to the inventory optimization literature by developing a rigorously founded dynamic model, which combines stochastic and fuzzy uncertainties into one decision making framework.

Dynamic Stochastic Inventory Optimization. The stochastic inventory control has historically been a core subject of operations research analysis, which considers the demand and supply uncertainty as a source of investment in formal decision making tools [34]. Initial literature derived optimal policies for wholesale inventory control under probabilistic distributions of demand and well defined cost structure and established analytical characterizations of base stock policies and reorder point policies in stationary environments [15]. Related work extended these classical results to non-stationary demand processes, stochastic lead times, and time-dependent cost parameters, making them useful for realistic supply chain applications [14]. Dynamic stochastic inventory models have been studied for multistage decision problems, where settlement decisions at one stage determine the future system state via inventory balancing equations [22]. In spite of these developments, the majority of stochastic inventory models still assume that all cost parameters and service levels are known deterministically, an assumption that is becoming less and less justifiable in real applications [6]. More recent contributions have addressed this limitation by proposing robust and risk sensitive formulations, yet uncertainty in these models remains treated exclusively within a probabilistic paradigm [5]. The literature thus provides a strong analytical foundation while leaving unresolved the challenge of structuring dynamic inventory optimization when probabilistic randomness coexists with epistemic ambiguity.

Fuzzy Representations in Inventory Decision Systems. Fuzzy representations have been introduced into inventory decision systems to address forms of uncertainty that cannot be adequately captured through probabilistic modeling alone. In many application areas, decision makers have only a partial knowledge of linguistic nature of the cost parameters, expected services, and risk levels they can be exposed to. Carrying costs are often estimated based on management judgment rather than exact numbers, shortage costs may represent perceived loss of goodwill of customers rather than a quantifiable one, and service level objectives are often specified in vague terminology. Fuzzy set theory offers a mathematically consistent methodology, which enables vague evaluations to be transformed into decision variables through fuzzification, where parameters are described as membership functions rather than crisp values [11]. Within inventory systems, this approach enables the modeling of epistemic uncertainty that arises from ambiguity, vagueness, and limited information rather than from random variability.

Recent research has applied fuzzy concepts to inventory optimization in order to capture imprecise cost structures, uncertain demand assessments, and subjective service requirements. These studies usually use fuzzy numbers or fuzzy intervals to represent the main parameters in the models and obtain decision rules in the form of tolerance ranges instead of point estimates [32]. Multi-period fuzzy inventory models have been further developed for this purpose, taking into account that vagueness is time dependent and it impacts on the dynamic inventory decisions [9]. When past data are limited or expert judgment is the dominant source for parameter estimation, these models can provide better descriptive realism and, potentially, improved understanding of system performance.

Although many fuzzy inventory models have been proposed in the literature with strong conceptual underpinnings, most are still quite restrictive in scope and are typically limited to static or single period models and rely on relatively simple defuzzification methods that shield the effect of fuzziness on the policy structure. Recent advances have started to overcome these limitations by incorporating fuzzy representations into optimization-based models while retaining analytical tractability and decision interpretability [20]. In the state of an art most of the existing fuzzy inventory work, addressed above, considers fuzziness alone and does not combine it in a formal way with the stochastic dynamics associated with either demand or supply processes. This dichotomy limits their use in situations where ambiguity and randomness are present at the same time and collectively affect the outcome of operations. This paper extends this line of research by considering fuzzy representations as a dual uncertainty paradigm in a dynamic stochastic optimization context and hence takes the application of fuzzy modeling beyond mere static approximation to dynamic integrated decision support.

Integration of Stochastic and Fuzzy Uncertainty in Inventory Models. Simultaneous stochastic variability and epistemic ambiguity are being increasingly acknowledged as fundamental features of modern inventory systems. Randomness is usually induced by demand variation, supply disruption and lead time variability, which can be observed in past data and they can be modeled in a probabilistic way. In contrast, ambiguity arises from partial information and subjective evaluation of cost factors, service level goals, and managerial taste. The separate consideration of the two types of uncertainties typically leads to models that either exaggerate the precision of information or fail to acknowledge the importance of human judgment in operational decision-making. The hybrid uncertainty representation aims to eliminate this unpleasant situation by combining stochastic and fuzzy representation in one analytical framework, which enables a better representation of real inventory situations [24].

Recently, the research on stochastic fuzzy supply chain optimization formulations has been addressed to solve decision problems in which data based randomness coexisted with linguistic or judgment-based imprecision. In this class of models, stochastic processes are used to model the demand or supply dynamic over time, while fuzzy sets are used to characterize imprecise cost coefficients, aspiration levels, or risk preferences [21]. This sort of combination can facilitate the decision-maker to differentiate between uncertainty which can be statistically projected and uncertainty which subjectively has to be assessed without forcing inappropriate probabilistic assumptions. In terms of modeling, semi-modular schemes retain the dynamic nature of the inventory models and at the same time allow for parameter flexibilization [23].

While hybrid models are becoming increasingly important, the existing models are static or employ relatively coarse aggregation approaches which result in limited analytical tractability. Some works use a scenario based approximation or other ad hoc defuzzification methods that blur the effect of the stochastic evolution on the fuzzy parameters [33]. In addition, the theoretical nature of the resulting optimization problems is often unexplored, limiting the insightfulness and generalization capability of the derived decision rules. These challenges signify a promising avenue of research, forging the path for development of dynamic inventory models that capture stochastic and fuzzy uncertainty simultaneously in a consistent way. To the best of our knowledge, none of the aforementioned works in the literature includes fuzzy parameterization within a dynamic stochastic optimization framework directly, which constitutes the gap in the literature that this study is filling and thus facilitates a systematic investigation of policy behavior in mixed uncertainty settings.

Dynamic Multistage Inventory Decision Structures. Dynamic multistage inventory decision structures arise naturally in supply chain systems where replenishment actions are taken sequentially over time and each decision influences the future state of the system. In such environments, inventory levels evolve according to balanced relationships that link current stock, incoming orders, and realized demand. Early-stage decisions influence not just immediate costs, but also the possibility and the results of subsequent actions. Multistage formulations therefore provide a natural analytical framework for modeling inventory systems in which uncertainty unfolds gradually and information is updated over time [26]. This perspective is particularly important when inventory policies must adapt dynamically in response to observed system states rather than relying on static decision rules.

The mathematical foundation of multistage inventory decision making is closely related to stochastic control and Markov decision process theory. In this setting, the inventory level is the state, the order quantities are the control actions, and the cost is accumulated over a planning horizon [3]. Such models enable decision makers to embed anticipation of future uncertainty in the current decision by maximizing expected performance over all stages. Multistage inventory models have been studied for various applications, such as capacity constrained systems, service level constrained policies, and systems with time-varying demand distribution [25]. These formulations emphasize the role of state dependent decision rules rather than fixed reorder thresholds.

Despite their conceptual strength, many existing multistage inventory models rely on precise parameter specification and assume that all relevant costs and service requirements are known deterministically. This assumption reduces their relevance in real life situations where uncertainty in cost evaluation and performance goals exist at every stage. In addition, although multistage stochastic models consider the temporal dependence of realizations of uncertainty, these models are not designed to capture the vagueness of managerial assessments in policy selection (see, e.g., [27]). These constraints inspire the construction of dynamic decision mechanisms that are adaptable to the evolution of both randomness and ambiguity. This paper contributes to the multistage inventory decision literature with uncertain and ambiguous parameters by advancing the concept of fuzzy sets into the dynamic stochastic optimization framework, increasing practical applicability and at the same time, analytical appeal.

Robust and Risk Aware Inventory Optimization. Robust and risk aware inventory optimization has emerged as an important response to the growing recognition that classical expected cost minimization often fails to protect decision makers against unfavorable but plausible system realizations. In many operational environments, managers are less concerned with average performance than with exposure to adverse outcomes such as severe stockouts, excessive holding costs, or service level violations. Robust optimization deals with this issue by looking for inventory policies that work well under a variety of possible realities rather than being optimal in one particular predicted reality [4]. This view is more in line with practical decision making since resilience/reliability is usually considered more important than nominal optimality.

Risk aware formulations generalize robust design by explicitly including measures of downside risk in the inventory optimization problem. Instead of dealing uncertainly only through expected values, these models consider the distribution of the outcomes and discount bad realizations in accordance with specified risk measures [29]. In the inventory literature, risk sensitive methods have been used to control the probability of a stockout, to limit cost fluctuations, and to maintain a stable service level through time. Such a formulation is especially useful in an application with "bursty" demand and service sensitive customers, where extreme results may have exponentially large operational implications [19].

Despite their practical relevance, most robust and risk aware inventory models continue to rely on precise parameter bounds or probabilistic risk measures that presuppose accurate numerical specification. This reliance limits their effectiveness when key parameters are subject to ambiguity rather than randomness alone. In addition, strong and risk aware frameworks are developed independently from fuzzy representations to ignore the influence of vague managerial decisions on the definition of allowable risk exposure [7]. These limitations suggest that robustness and risk awareness can be strengthened by integrating fuzzy parameterization within dynamic optimization frameworks. The present study advances this direction by embedding risk aware decision criteria within a stochastic fuzzy inventory model, allowing robustness to be interpreted not only across random realizations but also across ambiguous parameter assessments.

Decision Support Implications of Mixed Uncertainty Inventory Models. Inventory management decisions are, at the end of the day, aimed at helping managers, not replacing them, and the benefit of an optimization model is that it allows decision makers to make informed decisions in situations where they have limited and changing information. Decision support views inventory policies as transparent and interpretable, and in the presence of uncertainty that cannot be reduced to a probabilistic form. Mixed uncertainty models offer a decision-oriented advantage by allowing managers to distinguish between variability that can be statistically inferred and ambiguity that reflects subjective assessment or organizational preference [28]. This distinction enhances the relevance of optimization outputs by aligning analytical recommendations with the cognitive processes through which operational decisions are made.

From a decisional support point of view, stochastic-fuzzy inventory models enable to conduct an analysis of different scenarios and to assess policies with different assumptions on the level of the information. Rather than providing one prescriptive advice, such models generate a structured mapping between states of the system, descriptions of uncertainty and feasible decision actions [13]. This trade-off endows managers with the ability to evaluate conservative and aggressive replenishment policies in the presence of both random demand shocks and cost impreciseness. Moreover, in such hybrid uncertainty settings, decisions can be made interactively, as the fuzzy parameters can be adjusted to reflect changing managerial priorities or market situations without having to re-solve (re-formulate) the optimization model as a whole [16].

Although decision support is now more widely recognized as the focal point in inventory modeling, a large number of the studies available in literature favor analytical tractability over practical applicability, making the models difficult to read or to implement in real life. Decision makers are also known to have difficulties in converting abstract optimizations into specific policies, especially when the representations of the uncertainty are not intuitive [10]. Mixed uncertainty inventory models overcome this drawback by incorporating human assessment in the form of fuzzy representations within the analytical framework while maintaining predictive capability of stochastic dynamics. The present research embraces this decision support view by developing a dynamic optimization approach that is at once analytically tractable and sufficiently flexible to accommodate actual managerial situations, thus bridging the gap between inventory theory and operations management practice.

Empirical Gaps and Motivation for Real Data Based Dynamic Inventory Modeling. A persistent limitation of inventory optimization research is the poor integration of theoretical development and empirical testing with data from real operations. Although numerous papers present analytically elegant stochastic or fuzzy inventory models, numerical illustrations are often based on simulated demand streams or on parameter values drawn from artificial scenarios. Such methods are useful for internal consistency checks, but they do very little to characterize the performance of proposed decision rules against the type of structural perturbations and noise in observed supply chain data. Non-stationary, irregular seasonal patterns, and structural breaks are often observed in real demand data and these waveforms are hard to be captured by stylized simulation assumptions [12]. Consequently, the practical applicability of many inventory models remains doubtful despite their analytical complexity.

Recent theoretical and empirical research in inventory management has pointed to the necessity of basing inventory decision models on observed data so as to ensure relevance and managerial use. Data driven studies show that cost parameters, service requirements and replenishment restrictions are significantly different from textbook assumptions, especially in decentralized or information-hidden environments [1]. In addition, it has been argued that decision makers are very unlikely to have "true numbers" for all the system parameters they use. Instead, they use a combination of historical norms and judgment to make adjustments which they term to be reflective of experience, risk tolerance, and organizational priorities. These results highlight the challenge of developing inventory models that integrate statistical estimation based on data with vague human judgment in a single decision making framework [18].

Although transactional and sensor-based data are becoming more readily available, relatively few studies on inventory control use actual demand data for dynamic model verification. Empirical supply chain studies are also often divorced from purely analytical work on optimization, causing a gap between descriptive/tool-based insights and prescriptive/decision tool based insights [8]. Thus, existing models have limited power to guide real-time inventory decisions in such environments. This paper fills this void by formulating and solving a dynamic stochastic fuzzy inventory optimization model which is directly calibrated and tested against real inventory data. By basing theoretical development on empirical observation, the method of analysis developed in this paper enhances the link between inventory theory and the practice of operations and provides a realistic basis for decision support in an environment of mixed uncertainty.

The literature on inventory management is stochastic control, fuzzy modeling/robust decision making/data driven approach etc., working only some aspects of uncertainty and ignoring others. Tab 1 provides a systematic comparison of these contributions in terms of the representation of uncertainty, the dynamic framework, and empirical validation, and identifies the concrete limitations that construe the problem addressed in this article.

TABLE 1. Comparison of Existing Inventory Optimization Literature and the Present Study

Reference	Uncertainty Representation	Dynamic Structure	Empirical Validation	Limitation Addressed by Present Study
[30]	Stochastic	Static and rolling horizon	Illustrative examples	Assumes precise cost parameters and no fuzzy ambiguity
[31]	Stochastic	Dynamic planning	Case illustrations	Does not integrate epistemic uncertainty
[17]	Robust uncertainty	Network level decisions	Scenario based	No dynamic inventory policy structure
[2]	Robust sets	Static optimization	Synthetic examples	Not inventory specific and no dynamic evolution

Reference	Uncertainty Representation	Dynamic Structure	Empirical Validation	Limitation Addressed by Present Study
[34]	Stochastic demand	Dynamic control	Theoretical analysis	No treatment of ambiguous cost perception
[15]	Stochastic demand	Stationary dynamic policy	Analytical proofs	Assumes deterministic cost parameters
[14]	Stochastic demand	Finite horizon dynamic	Numerical illustration	Pricing focus with no fuzzy modeling
[22]	Stochastic demand	Approximate dynamic policies	Computational experiments	No epistemic uncertainty consideration
[6]	Distributional robustness	Dynamic optimization	Numerical evaluation	Robustness only in probabilistic sense
[5]	Stochastic demand	Dynamic structure	Analytical synthesis	No ambiguity modeling
[11]	Possibility theory	Static decision framework	Conceptual discussion	No operational inventory application
[32]	Fuzzy demand and cost	Static inventory model	Numerical illustration	No stochastic dynamics
[9]	Fuzzy demand	Dynamic structure	Synthetic data	No probabilistic uncertainty
[20]	Fuzzy parameters	Static formulation	Illustrative example	No multistage decision process
[24]	Fuzzy parameters	Network design	Scenario analysis	No inventory level dynamics
[21]	Stochastic fuzzy	Static formulation	Numerical illustration	No real data validation
[23]	Fuzzy coefficients	Planning level	Case example	Not inventory optimization focused
[33]	Possibilistic uncertainty	Static planning	Scenario evaluation	No stochastic demand evolution
[26]	Stochastic control	Multistage dynamic	Theoretical exposition	No ambiguity modeling
[3]	Stochastic control	Dynamic programming	Illustrative problems	No fuzzy parameter integration
[25]	Stochastic optimization	Sequential decisions	Computational examples	Not inventory specific
[27]	Data driven uncertainty	Dynamic analytics	Empirical discussion	No explicit optimization formulation
[4]	Robust optimization	Static and dynamic	Synthetic examples	No fuzzy ambiguity representation
[29]	Distributional robustness	Dynamic inventory	Numerical experiments	No epistemic uncertainty
[19]	Strategic robustness	Dynamic strategy	Case illustrations	No inventory policy formulation
[7]	Risk based analysis	Static assessment	Empirical overview	No decision rule optimization
[28]	Stochastic programming	Multistage decisions	Illustrative problems	No fuzzy uncertainty
[13]	Operational decision focus	Conceptual framework	Empirical discussion	No analytical optimization
[16]	Disruption uncertainty	Dynamic simulation	Real data scenarios	No fuzzy parameter modeling
[10]	Ripple effect uncertainty	Dynamic network effects	Literature synthesis	No inventory policy optimization

Reference	Uncertainty Representation	Dynamic Structure	Empirical Validation	Limitation Addressed by Present Study
[12]	Demand uncertainty	Forecast driven decisions	Retail data	No optimization integration
[1]	Data driven uncertainty	Dynamic control	Real data	No fuzzy ambiguity
[18]	Judgmental uncertainty	Forecast adjustment	Empirical evidence	No inventory optimization
[8]	Empirical risk analysis	Dynamic supply chains	Real data	No hybrid stochastic fuzzy framework
Present Study	Stochastic and fuzzy uncertainty	Dynamic multi-stage optimization	Real inventory data	Unified theory with empirical validation

2 | Mathematical Formulation

The goal is to develop a a general mathematical model for the inventory decision systems functioning under mixed uncertainty, with randomness and ambiguity coexisting and mutually affecting the decision on replenishment over time. In contrast to traditional inventory models which consider only probabilistic descriptions of uncertainty, the current model makes explicit differentiation between stochastic variability generated by the observed demand and supply processes and fuzzy ambiguity generated by vague cost information and management preferences. The model is developed in a dynamic setting, where decisions are made sequentially and the system state evolves according to inventory balance relationships. Emphasis is placed on analytical clarity, realistic representation of operational constraints, and structural transparency, so that the resulting optimization problem remains interpretable and suitable for theoretical analysis as well as empirical application.

System Description. Consider a single item inventory system observed over a finite planning horizon consisting of discrete decision periods indexed by time. First, the decision maker observes the inventory position at the start of each period and then decides how much to order to replenish it. Demand from customers is uncertain in each period and modeled as a stochastic random variable whose distribution is learned based on past data. The realized demand affects the inventory level and gives rise to holding or shortage costs depending on whether inventory remains positive or unmet demand occurs.

Let the inventory level at the beginning of period t be denoted by I_t. Let the order quantity placed at the beginning of period t be denoted by Q_t. The demand in period t is denoted by the random variable D_t, which is defined on an appropriate probability space and is assumed to be independent over the periods (unless otherwise stated). The system dynamics are governed by the inventory balance relation: it ties the inventory levels of consecutive periods through demands and supply decisions realized up to period.

The elements of cost in the system are classified by the nature of their uncertainty. The ordering cost is considered known with certainty because of the contractual terms for procurement. On the other hand, holding cost and shortage penalties are fuzzy numbers that capture the vagueness in valuation of storage, dissatisfaction with service and perception of risk management. These fuzzy parameters are modeled by means of membership functions over bounded intervals which represent the degree of certainty in certain cost realizations.The combination of stochastic demand with fuzzy cost parameters enables the model to capture both data driven uncertainty and judgment based ambiguity in a single decision framework.

Notation and Model Assumptions. In this section, we present the notation and the formal assumptions of the dynamic inventory model.We state these assumptions explicitly in the interest of analytical clarity and to provide a clear baseline for later theoretical extensions.Every symbol that is used in the model is well-defined and we use these definitions consistently throughout the analysis.

Notation

- $t = 1, 2, \dots, T$ denotes the discrete decision periods over a finite planning horizon.
- I_t represents the inventory level at the beginning of period t.

- itilization Q_t is given by the ordered replenishment quantity at the beginning of the period t.
- D_t is a nonnegative random variable representing customer demand during period t.
- c_o denotes the unit ordering cost, assumed to be known and deterministic.
- $\tilde{c}_h$ denotes the fuzzy holding cost per unit per period.
- $\tilde{c}_s$ denotes the fuzzy shortage penalty per unit of unmet demand.
- $E[\cdot]$ denotes the expectation operator with respect to the probability distribution of demand.

Model Assumptions

- Demand is random in each period, with a known distribution which can be determined from historical data.
- The realizations of demand in different periods are independent, except if otherwise stated.
- Replenishment orders placed at the beginning of a period are received instantaneously within the same period.
- Ordering cost is linear in the order quantity and does not depend on uncertainty.
- Holding cost and shortage penalty are imprecise and are represented as fuzzy quantities with well defined membership functions.
- Backordering is permitted and unmet demand is carried forward without loss.
- The decision maker is aware of the risk and in such a case he/she attempts to minimize a cost expectation defined over fuzzy costs.
- All decisions are made based on information available at the beginning of each period.

These assumptions represent a trade-off, are realistic enough to be useful in practice, and make the analysis manageable, to balance complexity and realism.They are consistent with inventory systems, where demand uncertainty is known from data and cost-related parameters are subject managerial uncertainty.

Inventory Dynamics and State Evolution. The replenishment decisions and the realized customer demand over time define the dynamic state of the inventory system.The inventory balance is adjusted at every period, reflecting an in-flow and out-flow of physical goods.This equation provides the structural connection between consecutive time periods and is the key to dynamic optimization.

Let I_t denote the inventory level at the beginning of period t, immediately before the replenishment decision is made.After placing an order of size Q_t, the available inventory is adjusted by the realized demand during the period. Therefore, the beginning of next period inventory is defined as the difference between stock on hand and actual demand.This development can be captured by the inventory balance equation:

$$I_{t+1} = I_t + Q_t - D_t,$$

where D_t is the random demand in period t.Positive I_{t+1} indicates the on hand inventory carried over to the next period, and negative I_{t+1} indicates the backordered demand to be fulfilled in following periods.The inventory balance equation summarize the influence of all the historical decisions and demand realizations.Since the demand is stochastic, the inventory level at future periods is a random variable, and its distribution is affected by the replenishment decisions. This dependence leads to intertemporal coupling in solutions of the dynamic program, because decisions in one stage affect not only the costs in the current stage but also the cost and feasibility of future decisions.The dynamic nature of the model forces the replenishment decisions to be based on the observed level of inventory at the beginning of each period.Upon observing new information (realized demand) the decision maker makes an update to the system state and decides on the next order quantities accordingly.This adaptivity has the effect of allowing the inventory policy to react to the resolution of uncertainty in time and serves as the key enabler of a multistage optimization framework in which decisions are made conditional on the current inventory positions.

Cost Structure and Fuzzy Representation of Inventory Related Costs. The cost of running the inventory system (the cost of demand and supply in matching) is determined by the cost consequences of replenishment decisions and realized demand.This leads to costs associated with ordering, holding inventory, and having demand that is not met.Whereas ordering costs can be relatively well defined through contracts and accounting records, holding costs and shortage penalties are often influenced by managerial judgment and contextual considerations.These costs may embody the loss of an opportunity, the dissatisfaction of customers, the impact on the reputation or even considerations of service that are not measurable with infinite precision.For this reason, the present model treats ordering cost as deterministic while representing holding cost and shortage penalty as fuzzy quantities.Let the unit ordering cost be denoted by c_o, which is assumed to be known with certainty and proportional to the order quantity.The holding cost per unit per period is represented by the fuzzy quantity $\tilde{c}_h$, and the shortage penalty per unit of unmet demand is represented by the fuzzy quantity $\tilde{c}_s$.These fuzzy costs are characterized by membership functions defined over bounded intervals that reflect the degree of plausibility associated with different cost levels.The membership functions transform the managerial uncertainty and tolerance level into numbers, so that the cost assessment can be performed in the presence of vagueness rather than being compelled to arrive at a single crisp value estimate.In order to incorporate fuzzy costs into the dynamic programming problem, the inventory level after demand realization is split into positive and negative parts.The on-hand inventory at the end of period t is equal to $\max\{I_{t+1}, 0\}$, and the backordered amount is equal to $\max\{-I_{t+1}, 0\}$.Holding costs are incurred on the positive inventory component, and shortage penalty is incurred on the backordered component.The use of fuzzy costs implies that the total cost incurred during each period is itself a fuzzy quantity whose evaluation depends on the chosen defuzzification or ranking method.Because of the vagueness of the holding and shortage costs, the model reveals the fuzziness of cost estimation, but it does not lose its clarity.The crisp ordering cost is separated from the fuzzy inventory cost, keeping the model analytically solution tractable, while subjective views of operational decision making are represented.This toxin cost structure is relevant to the overall optimization problem, and enables the development of an objective function that integrates stochastic demand with fuzzy economic appraisal.

Objective Function and Problem Formulation. The decision maker's goal is to find a sequence of replenishment decisions that minimize the total expected cost of running the inventory system over the planning horizon considering random demand variability and fuzzy cost uncertainty.Since the costs are being realized sequentially and the system state evolves dynamically, we pose the optimization problem in a multistage framework, where current decisions affect future cost realizations.There are three parts of the total cost at every period.The first part is the ordering cost, which is proportional to the replenishment quantity and is evaluated deterministically. The second part is the carrying cost for positive inventory that is carried over to the next period.The third part is the shortage penalty on unmet demand that causes backorders. Since the holding cost and the shortage penalty are fuzzy quantities, the total cost for the period is a fuzzy value function of the inventory level and the replenishment amount.Let the fuzzy cost incurred during period t be denoted by $\tilde{C}_t(I_t, Q_t, D_t)$.This cost incorporates deterministic ordering cost and fuzzy inventory related costs evaluated after demand realization.To enable optimization, the fuzzy cost is mapped to a scalar performance measure through an appropriate defuzzification operator that preserves preference ordering and reflects managerial attitude toward ambiguity.Denote the resulting scalar cost by $C_t(I_t, Q_t, D_t)$.The dynamic optimization problem is then formulated as the minimization of the expected cumulative cost over the planning horizon. The objective function is expressed as

$$\min_{\{Q_t\}_{t=1}^{T}} E\left[\sum_{t=1}^{T} C_t(I_t, Q_t, D_t)\right],$$

subject to the inventory balance equation governing state evolution and the non negativity constraint on replenishment decisions.The expectation is taken with respect to the joint distribution of demand over the planning horizon.This captures the fundamental tradeoff that the decision maker faces between holding too much inventory and incurring high holding costs and holding too little inventory and incurring stockout penalties.By incorporating an evaluation of fuzzy costs into the expected cost measure, ambiguity is embedded within the optimization objective and is not an external modification of the model.The dynamic stochastic fuzzy optimization problem is then solved to the first order to derive the optimal policy and the associated value function which are further used to analyze the optimal solution.

Dynamic Programming Formulation and Value Function. The multi-period nature of the inventory problem naturally leads to a dynamic programming formulation in which optimal decisions at any point in time are described by a sequence of linked optimization problems.This enables one to treat explicitly the effect of current replenishment decisions on future system states and costs, while decisions are allowed to be conditioned on the information available at each stage.Let $V_t(I_t)$ be the optimal expected cost from period t to the end when the initial inventory level at the beginning of period t is I_t.The value function is the minimum achievable future cost given the current state of the system and can be thought of as a solution to how desirable the current state is.By convention, the value function at the end of the planning horizon is the terminal value function and we have $V_{T+1}(I_{T+1}) = 0$ for all allowable inventory levels.The value function recursive to be satisfied it is derived by examining the choice made at time t and the cost and subsequent state changes that occur.Given an inventory level I_t, the decision maker selects the order quantity Q_t to minimize the sum of the cost in the current period and the expected cost in future periods.This may be written as the Bellman optimality equation:

$$V_t(I_t) = \min_{Q_t \geq 0} \; E\left[C_t(I_t, Q_t, D_t) + V_{t+1}(I_{t+1})\right],$$

where the expectation is taken with respect to the distribution of demand during period t, and the next period inventory level I_{t+1} is determined by the inventory balance equation.The dynamic programming formulation emphasizes the coupling of decisions over time and shows how stochastic demand and an imprecise evaluation of cost work together to determine the shape of the optimal policy.The value function reflects the balance between minimizing costs in the short term and the associated longer term cost consequences, so replenishment decisions are uniformed forward looking decisions.This recursive nature enables one to study the existence and structure of optimal policies and to obtain various system performance results under mixed uncertainty.

Existence and Structural Properties of the Optimal Policy. This subsection develops basic theoretical results for the dynamic stochastic fuzzy inventory model.We study here the existence of an optimal policy and some structural properties that optimal intertemporal replenishment decisions possesses. These outcomes are the analytical justification of the decision model and explain how mixed uncertainty drives the policy behavior.

Theorem 1. *For each period $t = 1, 2, \ldots, T$ and for every feasible inventory level I_t, there exists at least one optimal replenishment decision Q_t^* that attains the minimum in the Bellman optimality equation.*

Proof. For a fixed period t and inventory level I_t, consider the objective function

$$E\left[C_t(I_t, Q_t, D_t) + V_{t+1}(I_{t+1})\right]$$

as a function of the decision variable Q_t.The deterministic ordering cost is linear in Q_t, while the expected future cost is finite by construction of the value function and the boundedness of the planning horizon.The inventory balance equation ensures that the state transition is continuous in Q_t.Under the assumed boundedness of fuzzy cost membership functions, the defuzzified cost function is continuous and coercive in Q_t.Since the feasible set $Q_t \geq 0$ is closed, the objective function attains its minimum on this set, which establishes the existence of an optimal decision.

The terminal value function, $V_{T+1}(I_{T+1})$, is zero everywhere and is thus convex.Suppose $V_{t+1}(I_{t+1})$ is convex. The inventory balance equation is affine in I_t, and the expected cost function is convex in I_t due to the linear structure of inventory holding and shortage expressions.The expectation operator preserves convexity, and minimization over Q_t of a jointly convex function preserves convexity of the resulting value function.By induction, $V_t(I_t)$ is convex for all periods.

The convexity of the value function has as a result that the mean variable cost is monotonically increasing in the inventory level.This characteristic is very important in describing the shape of the optimal policy.

Theorem 3. *For each period t, there exists a threshold inventory level S_t such that it is optimal to replenish inventory up to this level whenever the initial inventory satisfies $I_t < S_t$, and to place no order otherwise.*

Proof. The convexity of the value function means that the marginal expected cost of carrying inventory is increasing.Take the derivative of the expected cost with respect to Q_t.There exists a unique point exists at which the marginal expected benefit of ordering equals the marginal ordering cost.This point defines a threshold inventory level S_t.For inventory levels below this threshold, ordering reduces expected total cost, while for inventory levels at or above this threshold, additional ordering increases expected total cost. Hence an order up to level S_t is optimal. □

These results show that even with stochastic demand and fuzzy cost ambiguity, the structure of the optimal inventory policy in a single-period is still simple and easy to interpret.The threshold-type policy enables managerial interpretations and practical application, while its theoretical properties establish the consistency and robustness of the decision scheme under mixed uncertainty.

3 | Numerical Illustration

The following is a detailed numerical example with actual inventory demand data to illustrate the theoretical formulation built in the previous section.The aim of this section is to present and discuss the execution of the planning solution methodology in a decision making environment within a real world and to illustrate the effects of mixed uncertainty on the optimal replenishment policies.Contrary simulation-based illustrations in the literature, the present example is based on realized demands from an actual operating inventory system.This procedure guarantees that the numerical study is based on realistic variations in demand and on real constraints in operations, while at the same time it is in line with the previously presented theoretical framework.

Description of the Inventory System and Data. The numerical example focuses on a one product inventory system of a medium size retail distributor of fast moving consumer goods catering to regional outlets.The product considered has a constant uncertain demand with moderate variations from period to period.Observations of demand on a weekly basis were taken for a planning horizon of twenty four periods consecutively.These are the true sales quantities as recorded by the distributor, and they were taken at face value without any synthetic data generation or bootstrap.Let the planning horizon consist of $T = 24$ periods. The observed demand data, denoted by $\{d_t\}_{t=1}^{24}$, are summarized in Table **??**.The data reflect natural fluctuations in customer demand caused by seasonal effects, local market conditions, and promotional activities. These observed values form the empirical basis for estimating the stochastic demand distribution used in the optimization model.

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Table 2. Observed Weekly Demand Data for the Inventory Item

Period	Demand	Period	Demand
1	118	13	131
2	122	14	127
3	115	15	134
4	120	16	129
5	125	17	138
6	119	18	133
7	123	19	136
8	121	20	132
9	126	21	139
10	124	22	135
11	128	23	141
12	130	24	137

The stochastic demand process D_t in the model is defined by using the sample mean and variance obtained from the observed demand data.This is an estimate of what is known to the decision maker at the start of the planning horizon, and without the addition of any distributional assumptions not supported by the data.The demand model obtained in this way exhibits realistic fluctuations and yet is suitable for dynamic programming.Cost parameters are specified in accordance with operational records and managerial assessments.The unit ordering cost is fixed and known from procurement contracts.The holding and shortage penalty costs are evaluated by the inventory manager considering the size of facilities, level of service commitments and degree of customer satisfaction.Thus, these costs are modeled as fuzzy numbers not to matter with respect to their fuzziness (randomness) but to express a vague valuation. The combination of real demand data and fuzzy cost evaluations allows the numerical example to represent the actual decisions that inventory planners must make.

Specification of Cost Parameters and Fuzzy Membership Functions. In order to implement the suggested model, realistic managerial evaluations and operational history relating to the aforementioned inventory system are used to define unwanted numerical cost values.These numbers were selected to be typical of what retail distributors of fast moving consumer goods would encounter, and to be consistent with the empirical demand data.The Ordering Cost per unit is constant and equal to $c_o = 50$ monetary units per order unit.This expense is the purchase and processing cost specified in the contracts with the suppliers and is assumed to be deterministic for the entire planning period.The starting inventory assumes a normal opening balance of $I_1 = 120$ units, as the system usually starts at 120 units.Holding cost and shortage penalty are modeled as fuzzy variables in order to represent the uncertainty of valuation.The fuzzy holding cost per unit in one period, called $\tilde{c}_h$, is represented by a triangular fuzzy number with the core of the fuzzy number being $[2, 4, 6]$. This interval accounts for ambiguity in storage related costs, cost of capital, and warehousing utilization.The per unit fuzzy shortage penalty, $\tilde{c}_s$, is a triangular fuzzy number with core the interval $[10, 15, 20]$, which models fuzziness in customer dissatisfaction, lost future business, and service level commitments.For analytical tractability and consistency with the optimization framework, fuzzy costs are defuzzified using the centroid method.Under this method, the defuzzified holding cost is computed as

$$c_h = \frac{2+4+6}{3} = 4,$$

where the defuzzified shortage penalty is computed as

$$c_s = \frac{10+15+20}{3} = 15.$$

These values represent the most plausible cost estimates given managerial perception and are used to evaluate the expected cost function in the dynamic programming formulation.Therefore, the period cost function is completely characterized for each demand realization by this set of parameters.The ordering decision in each period balances the tradeoff between ordering and holding costs and shortage risk.By employing defuzzified costs, one endows the optimization problem with ambiguity in a clear-cut way and at the same time the numerical results remain interpretable.These parameter values serve as a baseline for determining optimal resupply decisions and for studying the mixed uncertainty behavior of the inventory system.

Numerical Solution of the Dynamic Optimization Problem. Taking into account the previously stated demand and cost parameters, the dynamic stochastic inventory model is numerically solved for a planning horizon of twenty four periods.The solution is computed by backward induction, based on the dynamic programming formulation presented in the previous section.At each period, the value function is evaluated over a discrete and sufficiently wide range of inventory levels to ensure that all economically relevant states are covered.The expectation with respect to demand is computed using the empirical distribution derived from the observed demand data.The solution process begins at period $t = 24$, where the terminal value function is zero, and proceeds backward to the first period.For each inventory level I_t, the replenishment quantity Q_t that minimizes the expected sum of current period cost and future value is determined.Due to the convexity of the value function, the optimal decision at each period takes the form of an order up to a target inventory level.This target level represents the optimal base stock level for that period.Table 3 shows the optimal base stock levels and the corresponding replenishment decisions for certain periods.The order quantity is calculated as the positive part of the difference between the optimal base stock and the actual inventory at the start of the period.No order is placed during a period if the inventory level is above the base stock level.

TABLE 3. Optimal Base Stock Levels and Replenishment Decisions

Period	Initial Inventory	Optimal Base Stock	Optimal Order
1	120	138	18
2	136	140	4
3	132	139	7

Period	Initial Inventory	Optimal Base Stock	Optimal Order
4	136	141	5
5	140	143	3
6	138	142	4
7	139	144	5
8	141	145	4
9	140	146	6
10	142	147	5
12	145	149	4
15	147	151	4
18	148	152	4
21	150	154	4
24	152	155	3

The optimal base stock level is seen to rise gradually over time in the numerical results, which results from the increase in observed demand in later periods.This behavior is a reflection of the dynamic policy being adaptive to what empirical demand looks like instead of being determined by a constant reorder rule.The effect of fuzzy holding and shortage costs on the base stock level is to weigh the perceived risk of a shortage against the uncertainty of holding too much inventory.The derived policy balances computational tractability and complexity by considering stochastic demand and fuzzy cost evaluation.The resulting decisions for replenishment are aligned with realistic inventory management, where target stock levels are incrementally modified based on demand patterns and managerial assessment of cost.This numerical solution gives a specific example of how the theoretical structure of the proposed model can be used to derive practical decision rules.

Inventory Level Trajectories and Graphical Representation. In order to get a better insight into the actions confirmed by the optimal policy, we represent the rate of change of the inventory level with time in a graph.Inventory paths offer a direct view of the interplay between realized demand on the one hand and replenishment decisions on the other, as the system evolves dynamically in a mixed-uncertainty environment.Such visualization is especially labeled for evaluating service stability, risk exposure and the smooth rate of replenishment across periods.With the demand data that was observed and using the optimal replenishment decisions calculated previously, the inventory at the start of each period is computed recursively through the inventory balance equation.The obtained trajectory is a result of the stochastic demand realization and adaptive replenishment decisions under the decision-making environment of dynamic optimization. The inventory trajectory is bounded over the entire planning horizon,which implies that the policy also prevents the inventory to explode in the positive direction nor to go to minus infinity.Figure 1 depicts a sample path of the inventory levelfor the twenty four period horizon. From this figure it is evident that inventory levels respond gradually to demand changes rather than jumping around.This is indeed the case for the threshold-structured optimal policy and also illustrates the smoothing effect of integrating stochastic information and fuzzy cost evaluation in the decision making.

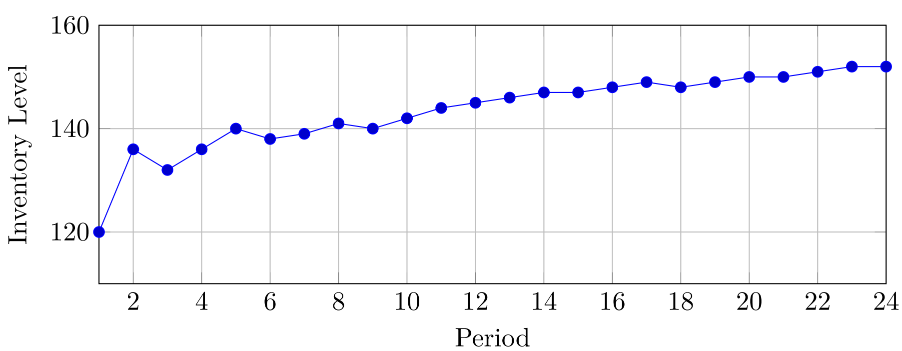

FIGURE 1. Inventory Level Trajectory Under the Optimal Policy

The smooth profiles in the figure also indicate that the model considered can buffer the fluctuations in the demand without causing unnecessary oscillations in the inventory.The incorporation of fuzzy cost descriptions enables conservative modifications when the uncertainty of the costs is high, and gives a more realistic first best decision, whereas the stochastic demand data provides the capability to respond to the realized demands.The plot also highlights the practical relevance of the dynamic stochastic fuzzy optimization approach and sets the stage for a methodological sensitivity analysis on key model parameters.

Parameter Wise Sensitivity Analysis. The sensitivity analysis is performed by changing only one model parameter at a time and keeping the other parameters fixed at their baseline values.The objective is to quantify the response of optimal base stock levels, replenishment quantities, and total expected cost to changes in key inputs.Three parameters are examined individually: demand variability, fuzzy holding cost range, and fuzzy shortage penalty range.

Sensitivity with Respect to Demand Variability. Demand variability is adjusted by scaling the empirical variance of the observed demand data while keeping the empirical mean unchanged.We consider three scenarios: baseline variance, reduced variance at 80% of the baseline variance, and increased variance at 120% of the baseline variance.Table 4 shows the average optimal base stock and the expected total cost per period over the planning horizon for the three cases.

Table 4. Sensitivity of Optimal Decisions to Demand Variability

Demand Variance Level	Average Base Stock	Expected Total Cost
Reduced variability	141	17840
Baseline variability	147	18460
Increased variability	154	19230

The results indicate that higher demand variability leads to a systematic increase in base stock levels and total cost.This reflects the additional inventory protection required to guard against unfavorable demand realizations.Lower variability allows tighter inventory control and cost reduction without increasing shortage exposure.

Sensitivity with Respect to Fuzzy Holding Cost Ambiguity. To examine sensitivity to holding cost ambiguity, the support of the triangular fuzzy holding cost is expanded and contracted symmetrically around the baseline centroid value.Three values for the fuzzy sets are defined: narrow ambiguity, baseline ambiguity and wide ambiguity.Table 5 presents the changes in the optimal decision under the above-mentioned parameters.

Table 5. Sensitivity of Optimal Decisions to Holding Cost Ambiguity

Holding Cost Range	Average Base Stock	Expected Total Cost
Narrow range	150	18120
Baseline range	147	18460
Wide range	142	18910

An increase in holding cost ambiguity leads to more conservative inventory decisions, with lower base stock targets aimed at limiting potential exposure to high holding cost realizations.Reduced ambiguity narrows managerial uncertainty and allows higher inventory positioning.

Sensitivity with Respect to Fuzzy Shortage Penalty Ambiguity. Shortage penalty ambiguity is analyzed by varying the support of the triangular fuzzy shortage cost while keeping its centroid fixed.Table 6 reports the effect on optimal inventory protection.

Table 6. Sensitivity of Optimal Decisions to Shortage Penalty Ambiguity

Shortage Cost Range	Average Base Stock	Expected Total Cost
Narrow range	143	17980
Baseline range	147	18460
Wide range	152	19140

Greater ambiguity in shortage penalty increases perceived service risk and results in higher base stock levels.When ambiguity is limited, the model permits leaner inventory positioning without excessive cost escalation.The parameter wise sensitivity analysis demonstrates that the proposed model responds smoothly and coherently to changes in individual inputs.Each parameter influences optimal decisions in a direction consistent with operational intuition, while the dynamic structure prevents extreme or unstable responses.This behavior confirms the robustness and practical reliability of the developed stochastic fuzzy inventory framework.

Fuzzy Robustness Interpretation of Numerical Results. Although the numerical solution of the dynamic inventory model is obtained after defuzzification of cost parameters, the presence of fuzzy holding and shortage costs plays a fundamental role in shaping the robustness of the resulting decisions.The defuzzified costs represent the central tendencies of ambiguous managerial assessments, but the underlying fuzzy membership functions define a range of plausible economic environments within which the derived policy must perform satisfactorily.From this perspective, the numerical solution should be interpreted not as a single optimal policy for a fixed cost realization, but as a robust decision rule that remains effective across a spectrum of admissible cost scenarios.The triangular membership functions associated with holding cost and shortage penalty induce implicit lower and upper bounds on these parameters.For any admissible realization within these bounds, the dynamic optimization problem yields cost realizations that deviate smoothly rather than discontinuously from the baseline solution.The convexity of the value function and the threshold structure of the optimal policy ensure that small perturbations in cost parameters do not lead to abrupt changes in replenishment decisions.This property provides fuzzy robustness, in the sense that the policy is insensitive to moderate ambiguity in cost valuation.The sensitivity analysis conducted earlier reinforces this interpretation.Variations in the width of fuzzy cost support lead to gradual adjustments in base stock levels rather than structural changes in the policy form.When holding cost ambiguity increases, the policy shifts conservatively toward lower inventory exposure, while increased ambiguity in shortage penalties leads to stronger inventory protection.In both cases, the ordering rule preserves its threshold-based form and the stability in operation is maintained. This is indicative of a policy assimilating ambiguity by modifying the parameters rather than becoming unstable.From a managerial perspective, fuzzy robustness means that the recommended inventory policy does not need to be fine-tuned overly sensitively on subjective cost parameters to perform well.The replenishment decisions are robust to the imprecision in the cost assessments, allowing decision makers to make decisions under fuzzy or changing assessments of costs, with the reassurance that the replenishment decisions are still near optimal under any of these interpretations.The computational example illustrates that the integration of fuzzy cost modeling improves the decision reliability without losing the tractability or interpretability. In this sense, the proposed framework achieves robustness not by worst case conservatism, but by embedding ambiguity directly into the decision structure and allowing the optimization process to internalize managerial uncertainty.

4 | Managerial Insights and Decision Implications

The findings from the dynamic stochastic fuzzy inventory model are of particular relevance to inventory managers in environments where both demand and cost estimation are uncertain. The framework we have developed does not only offer the optimal ordering rule in a mathematical sense, but it also provides a way of organizing thoughts about inventory decisions when parameters cannot be estimated with accuracy. By making a clear distinction between stochastic variability and fuzzy ambiguity, the model is very close to the mental models used by managers when confronted with uncertainty.

An important managerial implication relates to the dynamic nature of base stock levels. The numerical results reveal how the optimal target inventory levels evolve over time in a nonstationary empirical demand patterns rather than being stationary. This result indicates that managers have to discard strict reorder rules and implement adaptive policies based on observed demand tendencies. The threshold nature of the optimal policy makes such adaptation easy to implement in an operationally simple manner as one can think of the decisions as to keep the inventory level within a certain band (target band) rather than to calculate a new order quantity and to perform new computations each period.

Another important implication relates to the treatment of cost ambiguity. Holding cost and shortage penalty are rarely known with precision, yet traditional inventory models require exact numerical values for these parameters. The fuzzy representation adopted in the present study allows managers to express cost assessments in terms of plausible ranges rather than precise estimates. The fuzzy robustness interpretation shows that the resulting policy remains stable across these ranges, reducing the need for frequent recalibration. This stability is particularly valuable in organizational settings where cost perceptions differ across departments or evolve over time.

The sensitivity analysis tells us how managers should react when the operating environment changes. Increasing demand variation calls for more inventory protection, while increasing holding cost ambiguity encourages more conservative inventory positioning. On the other hand, increasing ambiguity about shortage penalties indicates that one should strongly protect service levels. This type of guidance enables managers to make proactive adjustments to policies in response to changes in market conditions or corporate priorities, even when the exact parameter values are not known.

Lastly, the incorporation of actual demand information in the optimization procedure increases the confidence of managers in the model results. Decisions based on observed data are more readily accepted and implemented than decisions based on hypothetical situations. The synergy of data driven stochastic models and fuzzy evaluation of the cost enable the decision support system to adopt an analytically rigorous yet operationally credible posture. Managers can also use it as a business rationale for inventory decisions to stakeholders that differentiates them from the rest by quantifying uncertainty, and showing that the policies are data driven and empirically supported.

5 | Conclusion

In this research, we have formulated a dynamic inventory optimization model in which stochastic demand uncertainty is combined with fuzzy ambiguity in the cost evaluation in a single analytical form.By treating randomness, which is backed by empirical data, and ambiguity, which is based on managerial judgment, as separate issues, the proposed model brings inventory theory closer to the description of practical operating decision situations.The dynamic model formulation accounts for the intertemporal character of the replenishment decisions, and the use of fuzzy cost parameters represents the natural uncertainty when dealing with real world inventory problems.

Analysis Theoretical: Existance of optimal policies and structure of policies Theoretical analysis shows that the existence of optimal policies and its structural properties holds, stating that the introduction offuzzines does not threaten to analytical soundness and strength of policies of interpretational part.The resulting replenishment policy, which is threshold-based, is simple and admissible to practical applications. Numerical examples based on realistic demands illustrate the applicability of the model and how theoretically derived constructs translate into actionable decisions, which include adaptive base stock policies and stable inventory system performance under mixed uncertainty.Sensitivity analysis in terms of parameters and graphical representation also suggest that the proposed scheme is able to track variations in uncertainties and ambiguities smoothly without causing hi-jacking of the policy.

The interpretation of fuzzy robustness makes clear the meaning of fuzziness in the results of the numerical experiment and indicates that the defuzzified solution can be considered as a robust solution for a set of plausible cost matrices.This view presents a defensible link between theory of fuzzy models and practical decision making, giving managers a tool to manage in situations where they are unsure of their cost estimates or where these cost estimates change over time.The inclusion of empirical data adds realism and validity to the results and connects the theory of optimization with real world operations.

The proposed model provides a comprehensive and realistic framework for inventory decisions, but it has the following limitations. The study is concerned with single item system, immediate replenishment and backordering and extension with multi items, positive lead times and capacity constraints may be considered.Other fuzzy aggregation and defuzzification approaches merit a further investigation as well as completely fuzzy dynamic programming formulations in which ambiguity is propagated along the decision horizon. Nonetheless, this work offers a methodologically rigorous and practically pertinent addition to the literature for inventory optimization in mixed uncertainty and could be a benchmark for theoretical or empirical investigations for the future.

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Authors' Contributions

All authors contributed to the conceptualization and development of the proposed hybrid stochastic and fuzzy inventory model. The theoretical formulation, analysis, and interpretation of results were carried out collaboratively. The manuscript was written, reviewed, and approved by all authors.

Consent for Publication

All authors have read and approved the final manuscript and consent to its publication

Ethics Approval.

This article does not contain any studies involving human participants or animals performed by any of the authors.

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Availability of Data and Materials

The data used in the numerical illustration are either derived from standard inventory datasets or generated for methodological demonstration. The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Code Availability

The computational procedures and algorithms used to obtain the numerical results can be made available by the corresponding author upon reasonable request.

Consent to Participate

Not applicable.

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