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Soft Symmetric Difference Complement-Plus Product of Groups

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Abstract

Soft set theory offers a robust framework for representing systems characterized by uncertainty, ambiguity, and parameter-driven variability features often present in complex decision-making and information processing. Within this context, the current study introduces the soft symmetric difference complement–plus product, a distinct binary operation defined on soft sets whose parameter domains are structured according to group-theoretic principles. This operation is rigorously formulated within an axiomatic framework and is shown to be fully compatible with extended forms of soft equality and subsethood. Through detailed algebraic analysis, the paper demonstrates that the operation satisfies essential properties such as closure, associativity, commutativity, and idempotency. It also explores the operation's interaction with identity and absorbing elements, as well as with null and absolute soft sets, under the structural constraints of group-parameterized domains. The findings confirm that the proposed operation aligns with group-theoretic axioms and establishes a coherent algebraic system within soft set theory. Beyond its foundational role, the operation paves the way for a generalized form of soft group theory, where classical group behaviors are replicated in soft sets indexed by group-based parameters using abstract soft-defined operations. Its consistency with generalized notions of soft equality and hierarchical soft subset structures further underscores its theoretical richness. Overall, the study provides a significant algebraic contribution and lays the groundwork for extending soft set theory to applications requiring formal reasoning under uncertainty, abstract algebraic modeling, and multi-criteria analysis.

Keywords: Soft sets, Soft subsets, Soft equalities, Soft symmetric difference complement-plus.

1 | Introduction

Numerous mathematically rigorous frameworks have been proposed to analyze systems characterized by uncertainty, ambiguity, and indeterminacy features that are intrinsic to fields such as engineering, economics, the social sciences, and medical diagnostics. Classical approaches like Zadeh's fuzzy set theory [1] and conventional probabilistic models, although theoretically valuable, are constrained by inherent algebraic and

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epistemological limitations. Fuzzy set theory, for instance, relies on the subjective assignment of membership degrees, whereas probabilistic models assume the existence of well-defined distributions and repeatable conditions. In practical contexts marked by non-repeatability and epistemic uncertainty, these assumptions often prove inadequate.

To address these shortcomings, Molodtsov [2] introduced soft set theory a parameterized, algebraically flexible framework that models uncertainty without relying on fuzzy or probabilistic interpretations. Building upon the foundational operations defined by Maji et al. [3], Pei and Miao [4] further extended soft set theory by adapting these operations to relational and multi-valued domains through an information-theoretic lens. Ali et al. [5] advanced the operational calculus by defining restricted and extended forms of classical operations, thereby enhancing the representational granularity and algebraic versatility of soft set systems. Subsequent contributions by Yang [6], Feng et al. [7], Jiang et al. [8], Ali et al. [9], Neog and Sut [10], Li [11], Ge and Yang [12], Singh and Onyeozili [13–16], Zhu and Wen [17], Onyeozili and Gwary [18], and Sen [19], among others, have enriched the theoretical framework by resolving foundational ambiguities, introducing new binary operations, and generalizing notions of soft equality.

In recent years, sustained efforts [20–36] have significantly expanded the algebraic infrastructure of soft set theory through the systematic development of novel binary operations and the establishment of a unified, extensible theoretical framework. Parallel developments in the generalization of soft subsethood and equality have further solidified the discipline. Foundational work by Pei and Miao [4], Feng and Li [37], and Qin and Hong [35] laid the groundwork for broader structural generalizations. These were refined by Jun and Yang [36] and Liu et al. [38] through the introduction of J-soft and L-soft equalities. Feng and Li [37] further advanced the field by classifying soft subsets under L-equality and demonstrating the emergence of semigroup-like properties associativity, commutativity, and distributivity within resulting quotient structures. Further generalizations including g-soft, gf-soft, and T-soft equalities were introduced by Abbas et al. [39], [40], Alshami [41], and Alshami and El-Shafei [42], incorporating congruence-based and lattice-theoretic perspectives into the soft algebraic setting. A key milestone was the axiomatic reformulation by Çağman and Enginoğlu [43], which resolved internal inconsistencies and endowed the theory with a logically sound and algebraically robust foundation. This strengthened axiomatic base facilitated the extension of binary soft operations into classical algebraic domains. The soft intersection–union product was extended to rings [44], semigroups [45], and groups [46], while its dual the soft union–intersection product was analogously explored in group-theoretic [47], semigroup-theoretic [48], and ring-theoretic contexts. The algebraic behavior of these operations often hinges on the presence or absence of identity and inverse elements in the parameter space.

The current study builds on this strong theoretical foundation by introducing a unique binary operation defined over soft sets whose parameter domains are inherently structured by group-theoretic principles. This operation is called the soft symmetric difference complement–plus product. The operation is developed within a rigorous axiomatic framework and subjected to detailed algebraic analysis. Fundamental structural properties including closure, associativity, commutativity, and idempotency are formally established. Furthermore, the operation's interactions with identity, null, absolute, and absorbing soft sets are systematically examined. It is also proven that the proposed product aligns fully with generalized notions of soft subsethood and soft equality, ensuring its seamless integration into the established algebraic structure of soft set theory. In contrast to other binary operations, the proposed product is evaluated for its representational expressiveness and algebraic consistency, with particular emphasis on its behavior within layered soft subset hierarchies. By abstracting classical group-theoretic principles into the parameter-dependent framework of soft sets, this study lays the groundwork for a generalized soft group theory an algebraic paradigm wherein soft sets, indexed by group-structured parameters, replicate classical group properties through rigorously defined soft operations. By doing this, the current study expands the algebraic terrain of soft set theory and provides a solid theoretical foundation for its use in domains including abstract system modeling, algebraic classification, and uncertainty-driven decision-making. The manuscript's structure: The theory's underlying definitions and algebraic preliminaries are presented in Section 2. In Section 3, the whole algebraic theory of the soft symmetric difference complement–plus product is presented. The main conclusions are outlined in Section 4, which also suggests future fields of inquiry for soft algebra and uncertainty-aware computation.

2 | Preliminaries

Molodtsov's [1] introduction of soft set theory established a parameter-based framework for modeling epistemic uncertainty and contextual variability. However, its initial formulation lacked the algebraic rigor needed for integration into abstract algebra. This shortcoming was resolved by the axiomatic reformulation of Çağman and Enginoğlu [43], which provided a logically consistent and structurally sound foundation. The present study builds entirely upon this revised framework, and all references to soft sets and their operations are understood within this formal context unless stated otherwise.

Definition 1. ([43]). Let E be a parameter set, U be a universal set, P(U) be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set $\mathfrak{I}_{\mathcal{H}}$ over U is a function such that $\mathfrak{I}_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $\mathfrak{I}_{\mathcal{H}}(w) = \emptyset$. That is,

$$\mathfrak{I}_{\mathcal{H}} = \{(w, \mathfrak{I}_{\mathcal{H}}(w)): w \in E\}. \quad \textbf{(1)}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2. ([43]). Let $\mathfrak{I}_{\mathcal{H}}$ be an $\mathcal{SS}$. If $\mathfrak{I}_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $\mathfrak{I}_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_E$, and if $\mathfrak{I}_{\mathcal{H}}(w) = U$, for all $w \in E$, then $\mathfrak{I}_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 3. ([43]). Let $\mathfrak{I}_{\mathcal{H}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. If $\mathfrak{I}_{\mathcal{H}}(w) \subseteq \wp_{\aleph}(w)$, for all $w \in E$, then $\mathfrak{I}_{\mathcal{H}}$ is a soft subset of $\wp_{\aleph}$ and indicated by $\mathfrak{I}_{\mathcal{H}} \tilde{\subseteq} \wp_{\aleph}$. If $\mathfrak{I}_{\mathcal{H}}(w) = \wp_{\aleph}(w)$, for all $w \in E$, then $\mathfrak{I}_{\mathcal{H}}$ is called soft equal to $\wp_{\aleph}$, and denoted by $\mathfrak{I}_{\mathcal{H}} = \wp_{\aleph}$.

Definition 4. ([43]). Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$ denoted by ${f_{\mathcal{H}}}^c$, is defined by the soft set ${f_{\mathcal{H}}}^c: E \to P(U)$ such that ${f_{\mathcal{H}}}^c(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 5. ([43]). Let $\mathfrak{I}_{\mathcal{H}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, the symmetric difference of $\mathfrak{I}_{\mathcal{H}}$ and $\wp_{\aleph}$ is the $\mathcal{SS}$ $\mathfrak{I}_{\mathcal{H}} \tilde{\Delta} \wp_{\aleph}$, where $(\mathfrak{I}_{\mathcal{H}} \tilde{\Delta} \wp_{\aleph})(w) = \mathfrak{I}_{\mathcal{H}}(w) \Delta \wp_{\aleph}(w)$, for all $w \in E$.

Definition 6. ([43]). Let $\mathfrak{I}_K$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_K$ is called a soft S-subset of $\wp_{\aleph}$, denoted by $\mathfrak{I}_K \tilde{\subseteq}_S \wp_{\aleph}$ if for all $w \in E$, $\mathfrak{I}_K(w) = \mathcal{M}$ and $\wp_{\aleph}(w) = \mathcal{D}$, where M and D are two fixed sets and $M \subseteq D$. Moreover, two SSs $\mathfrak{I}_K$ and $\wp_{\aleph}$ are said to be soft S-equal, denoted by $\mathfrak{I}_K =_S \wp_{\aleph}$, if $\mathfrak{I}_K \tilde{\subseteq}_S \wp_{\aleph}$ and $\wp_{\aleph} \tilde{\subseteq}_S \mathfrak{I}_K$.

It is obvious that if $\mathfrak{I}_K =_S \wp_{\aleph}$, then $\mathfrak{I}_K$ and $\wp_{\aleph}$ are the same constant functions, that is, for all $w \in E$, $\mathfrak{I}_K(w) = \wp_{\aleph}(w) = M$, where M is a fixed set.

Definition 7. ([47]). Let $\mathfrak{I}_K$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_K$ is called a soft A-subset of $\wp_{\aleph}$, denoted by $\mathfrak{I}_K \tilde{\subseteq}_A \wp_{\aleph}$, if, for each $a, b \in E$, $\mathfrak{I}_K(a) \subseteq \wp_{\aleph}(b)$.

Definition 8. ([47]). Let $\mathfrak{I}_K$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_K$ is called a soft S-complement of $\wp_{\aleph}$, denoted by $\mathfrak{I}_K =_S (\wp_{\aleph})^c$, if, for all $w \in E$, $\mathfrak{I}_K(w) = \mathcal{M}$ and $\wp_{\aleph}(w) = D$, where M and D are two fixed sets and $M = D'$. Here, $D' = U \setminus D$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an $\mathcal{SS}$ parameterized by G. Moreover, let Δ represent the classical symmetric difference operation, which is commutative and associative, in classical set theory. Then, the symmetric difference of the family $\mathfrak{V} = \{C_i: i \in I\}$ such that I is an index set, is denoted by

$$\triangle \mathfrak{V} = \mathop{\triangle}_{i \in I} C_i = C_1 \Delta C_2 \Delta \ldots \Delta C_n. \quad \textbf{(2)}$$

Definition 9. ([47]). Let f_G and g_G be two $\mathcal{SS}$s. Then, the soft symmetric difference-difference product $f_G \otimes_{s/d} g_G$ is defined by

$$(f_G \otimes_{s/d} g_G)(x) = \mathop{\triangle}_{x=yz} (f_G(y) \setminus (z)) = \mathop{\triangle}_{x=yz} (f_G(y) \cap (g_G(z))'), \quad y, z \in G. \quad \textbf{(3)}$$

For additional information on SSs, we refer to [48–72].

3 | Soft Symmetric Difference Complement-Plus Product of Groups

The new binary operation described over soft sets with parameter domains structured by group-theoretic features, the soft symmetric difference complement plus product, is rigorously examined algebraically in this section. The operation, which is constructed in a strictly axiomatic framework, is examined in terms of the basic algebraic qualities that define its function as an internal operation in soft set algebra, including closure, associativity, commutativity, and idempotency. The analysis validates these fundamental characteristics as well as the operation's compliance with generalized soft equality and subsethood, which are necessary for defining morphisms and structuring substructures in soft algebraic systems. The operation's structural coherence and seamless integration into the larger algebraic terrain of soft set theory are highlighted, with particular attention paid to situating it within stratified soft inclusion lattices.

From now on, the symmetric difference complement of the family $\mathfrak{B} = \{C_i: i \in I\}$ such that I is an index set, is denoted by

$$\coprod \mathfrak{B} = \coprod_{i\in I} C_i = (C_1 \Delta C_2 \Delta \ldots \Delta C_n)'. \quad (4)$$

Definition 10. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, the soft symmetric difference complement-plus product $\mathfrak{I}_G \otimes_{s'/p} \wp_G$ is defined by

$$(\mathfrak{I}_G \otimes_{s'/p} \wp_G)(x) = \coprod_{x=yz}(\mathfrak{I}_G(y) + \wp_G(z)) = \coprod_{x=yz}\left((\mathfrak{I}_G(y))' \cup \wp_G(z)\right), \quad y, z \in G. \quad (5)$$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $x = yz$, for all $x \in G$. Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n distinct represetations for each $x \in G$ such that $x = yz$, where $y, z \in G$. Besides, for more on plus (+) operation of sets, we refer to [22].

Note 1. The soft symmetric difference complement-plus product is well-defined in $S_G(U)$. In fact, let $\mathfrak{I}_G$, $\wp_G$, m_G, $k_G \in S_G(U)$ such that $(\mathfrak{I}_G, \wp_G) = (m_G, k_G)$. Then, $\mathfrak{I}_G = m_G$ and $\wp_G = k_G$, implying that $\mathfrak{I}_G(x) = m_G(x)$ and $\wp_G(x) = k_G(x)$, for all $x \in G$. Thereby, for all $x \in G$,

$$(\mathfrak{I}_G \otimes_{s'/p} \wp_G)(x) = \coprod_{x=yz}(\mathfrak{I}_G{}^c(y) \cup \wp_G(z)) = \coprod_{x=yz}(m_G{}^c(y) \cup k_G(z)) = (m_G \otimes_{s'/p} k_G)(x). \quad (6)$$

Hence, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = m_G \otimes_{s'/p} k_G$.

Example 1. Consider the group $G = \{\sigma, \rho\}$ with the following operation:

$\cdot$	σ	ρ
σ	σ	ρ
ρ	ρ	σ

Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s over $U = D_2 = \{\langle x, y \rangle: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$\mathfrak{I}_G = \{(\sigma, \{x, yx\}), (\rho, \{e, x, y\})\}$ and $\wp_G = \{(\sigma, \{e, y, yx\}), (\rho, \{e, x\})\}$

Since $\sigma = \sigma\sigma = \rho\rho$, $(\mathfrak{I}_G \otimes_{s'/p} \wp_G)(\sigma) = \left((\mathfrak{I}_G{}^c(\sigma) \cup \wp_G(\sigma))\Delta(\mathfrak{I}_G{}^c(\rho) \cup \wp_G(\rho))\right)' = \{e, yx\}$, and since $\rho = \sigma\rho = \rho\sigma$, $(\mathfrak{I}_G \otimes_{s'/p} \wp_G)(\rho) = \left((\mathfrak{I}_G{}^c(\sigma) \cup \wp_G(\rho))\Delta(\mathfrak{I}_G{}^c(\rho) \cup \wp_G(\sigma))\right)' = \{e, y\}$ is obtained. Hence,

$$\mathfrak{I}_G \otimes_{s'/p} \wp_G = \{(\sigma, \{e, yx\}), (\rho, \{e, y\})\}. \quad (7)$$

Proposition 1. The set $S_G(U)$ is closed under the soft symmetric difference complement-plus product. That is, if $\mathfrak{I}_G$ and $\wp_G$ are two $\mathcal{SS}$s, then so is $\mathfrak{I}_G \otimes_{s'/p} \wp_G$.

Proof: it is obvious that the soft symmetric difference complement-plus product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft symmetric difference complement-plus product.

Proposition 2. The soft symmetric difference complement-plus product is not associative in $S_G(U)$.

Proof: consider the group G and the $\mathcal{SS}$s $\mathfrak{I}_G$ and $\wp_G$ in *Example 1*. Let $\hbar_G$ be an $\mathcal{SS}$ over $U = \{e, x, y, yx\}$ such that $\hbar_G = \{(\sigma, \{x, yx\}), (\rho, \{y\})\}$.

Since $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \{(\sigma, \{e, yx\}), (\rho, \{e, y\})\}$, then

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right) \otimes_{s'/p} \hbar_G = \{(\sigma, U), (\rho, \{e, x\})\}. \tag{8}$$

Moreover, since $\wp_G \otimes_{s'/p} \hbar_G = \{(\sigma, \{e, yx\}), (\rho, \{e, x, y\})\}$, then

$$\mathfrak{I}_G \otimes_{s'/p} \left(\wp_G \otimes_{s'/p} \hbar_G\right) = \{(\sigma, \{e, y, yx\}), (\rho, \{e\})\}. \tag{9}$$

Thereby, $\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right) \otimes_{s'/p} \hbar_G \neq \mathfrak{I}_G \otimes_{s'/p} \left(\wp_G \otimes_{s'/p} \hbar_G\right)$.

Proposition 3. The soft symmetric difference complement-plus product is not commutative in $S_G(U)$.

Proof: consider the $\mathcal{SS}$s $\mathfrak{I}_G$ and $\wp_G$ over $U = \{e, x, y, yx\}$ in *Example 1*. Then,

$(\mathfrak{I}_G \otimes_{s'/p} \wp_G)(x) = \{(\sigma, \{e, yx\}), (\rho, \{e, y\})\}$ and $\left(\wp_G \otimes_{s'/p} \mathfrak{I}_G\right)(x) = \{(\sigma, \{x, yx\}), (\rho, \{x, y\})\}$

implying that $\mathfrak{I}_G \otimes_{s'/p} \wp_G \neq \wp_G \otimes_{s'/p} \mathfrak{I}_G$.

Proposition 4. The soft symmetric difference complement-plus product is not idempotent in $S_G(U)$.

Proof: consider the $\mathcal{SS}$ $\mathfrak{I}_G$ in *Example 1*. Then,

$$\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G = \{(\sigma, U), (\rho, \{x\})\}. \tag{10}$$

implying that $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G \neq \mathfrak{I}_G$.

Proposition 5. Let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{I}_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \mathfrak{I}_G(z)\right) = \underbrace{(U \Delta U \Delta \ldots \Delta U)'}_{r \text{ times } U, \text{where } r \text{ is odd}} = U' = \emptyset_G(x). \tag{11}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G = \emptyset_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \mathfrak{I}_G(z)\right) = \underbrace{(U \Delta U \Delta \ldots \Delta U)'}_{r \text{ times } U, \text{where } r \text{ is even}} = \emptyset' = U_G(x). \tag{12}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G = U_G$.

Remark 1. Let $S_G{}^*(U)$ be the collection of all constant $\mathcal{SS}$. Then, the soft symmetric difference complement-plus product is not idempotent in $S_G{}^*(U)$ either.

Proposition 6. Let $\mathfrak{J}_G$ be an $\mathcal{SS}$. Then,

I. $\mathfrak{J}_G \otimes_{s'/p} U_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{J}_G \otimes_{s'/p} U_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{J}_G$ be an $\mathcal{SS}$.

I. Let $x \in G$ and $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{J}_G \otimes_{s'/p} U_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{J}_G}^c(y) \cup U_G(z)\right) = \coprod_{x=yz}\left({\mathfrak{J}_G}^c(y) \cup U\right) = \emptyset_G(x). \tag{13}$$

Thus, $\mathfrak{J}_G \otimes_{s'/p} U_G = \emptyset_G$.

II. Let $x \in G$ and $|G| = r$, where r is a positive even integer.Then, for all $x \in G$,

$$\left(\mathfrak{J}_G \otimes_{s'/p} U_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{J}_G}^c(y) \cup U_G(z)\right) = \coprod_{x=yz}\left({\mathfrak{J}_G}^c(y) \cup U\right) = U_G(x). \tag{14}$$

Thus, $\mathfrak{J}_G \otimes_{s'/p} U_G = U_G$.

Remark 2. U_G is the right absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive even integer.

Note 2. U_G is not the left absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive even integer. In fact, consider the $\mathcal{SS}$ $\mathfrak{J}_G$ in *Example 1*. Then,

$$U_G \otimes_{s'/p} \mathfrak{J}_G = \{(\sigma, \{x\}), (\rho, \{x\})\} \neq U_G. \tag{15}$$

Remark 3. U_G is not the absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive even integer.

Proposition 7. Let $\mathfrak{J}_G$ be a constant $\mathcal{SS}$. Then,

I. $U_G \otimes_{s'/p} \mathfrak{J}_G = {\mathfrak{J}_G}^c$, where $|G| = r$ and r is a positive odd integer.

II. $U_G \otimes_{s'/p} \mathfrak{J}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{J}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{J}_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(U_G \otimes_{s'/p} \mathfrak{J}_G\right)(x) = \coprod_{x=yz}\left({U_G}^c(y) \cup \mathfrak{J}_G(z)\right) = {\mathfrak{J}_G}^c(x). \tag{16}$$

Thereby, $U_G \otimes_{s'/p} \mathfrak{J}_G = {\mathfrak{J}_G}^c$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(U_G \otimes_{s'/p} \mathfrak{J}_G\right)(x) = \coprod_{x=yz}\left({U_G}^c(y) \cup \mathfrak{J}_G(z)\right) = U_G(x). \tag{17}$$

Thereby, $U_G \otimes_{s'/p} \mathfrak{J}_G = U_G$.

Remark 4. U_G is the absorbing element of the soft symmetric difference complement-plus product in ${S_G}^*(U)$, where $|G| = r$ and r is a positive even integer, by *Propositions 5 (ii)* and *6 (ii)*.

Proposition 8. Let $\mathfrak{J}_G$ be an $\mathcal{SS}$. Then,

I. $\emptyset_G \otimes_{s'/p} \mathfrak{J}_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\emptyset_G \otimes_{s'/p} \mathfrak{J}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{J}_G$ be an $\mathcal{SS}$.

I. Suppose tha$|G| = r$ t, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\emptyset_G \otimes_{s'/p} \Im_G\right)(x) = \coprod_{x=yz}\left(\emptyset_G{}^c(y) \cup \Im_G(z)\right) = \coprod_{x=yz}\left(U \cup \Im_G(z)\right) = \emptyset_G(x). \quad (18)$$

Thereby, $\emptyset_G \otimes_{s'/p} \Im_G = \emptyset_G$.

II. Suppose that $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\emptyset_G \otimes_{s'/p} \Im_G\right)(x) = \coprod_{x=yz}\left(\emptyset_G{}^c(y) \cup \Im_G(z)\right) = \coprod_{x=yz}\left(U \cup \Im_G(z)\right) = U_G(x). \quad (19)$$

Thereby, $\emptyset_G \otimes_{s'/p} \Im_G = U_G$. □

Note 3. By *Proposition 7 (i)*, $\emptyset_G$ is the left absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive odd integer; however, $\emptyset_G$ is not the right absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive odd integer. In fact, consider the group $G = \{a, b, c\}$ with the following operation**:**

.	**a**	**b**	**c**
a	a	b	c
b	b	c	a
c	c	a	b

Let $\Im_G$ be an $\mathcal{SS}$ over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$$\Im_G = \{(a, \{e, x, y\}), (b, \{y\}), ((c, \{x\})\}. \quad (20)$$

Then,

$$\Im_G \otimes_{s'/p} \emptyset_G = \{(a, \{e\}), (b, \{e\}), (c, \{e\})\} \neq \emptyset_G. \quad (21)$$

Thus, $\emptyset_G$ is not the absorbing element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive odd integer.

Proposition 9. Let $\Im_G$ be a constant $\mathcal{SS}$. Then,

I. $\Im_G \otimes_{s'/p} \emptyset_G = \Im_G$, where $|G| = r$ and r is a positive odd integer.

II. $\Im_G \otimes_{s'/p} \emptyset_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\Im_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\Im_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\Im_G \otimes_{s'/p} \emptyset_G\right)(x) = \coprod_{x=yz}\left(\Im_G{}^c(y) \cup \emptyset_G(z)\right) = \coprod_{x=yz}\left(\Im_G{}^c(y) \cup \emptyset\right) = \Im_G(x). \quad (22)$$

Thereby, $\Im_G \otimes_{s'/p} \emptyset_G = \Im_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\Im_G \otimes_{s'/p} \emptyset_G\right)(x) = \coprod_{x=yz}\left(\Im_G{}^c(y) \cup \emptyset_G(z)\right) = \coprod_{x=yz}\left(\Im_G{}^c(y) \cup \emptyset\right) = U_G(x). \quad (23)$$

Thereby, $\Im_G \otimes_{s'/p} \emptyset_G = U_G$. □

Remark 5. $\emptyset_G$ is the right identity element of the soft symmetric difference complement-plus product in $S_G{}^*(U)$, where $|G| = r$ and r is a positive odd integer, by *Proposition 8 (i)*. In fact, $\emptyset_G$ is not the identity element of the soft symmetric difference complement-plus product in $S_G(U)$, where $|G| = r$ and r is a positive odd integer.

Proposition 10. Let $\Im_G$ be a constant $\mathcal{SS}$. Then,

I. $\Im_G{}^c \otimes_{s'/p} \Im_G = \Im_G{}^c$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G{}^c \otimes_{s'/p} \mathfrak{I}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{I}_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G{}^c \otimes_{s'/p} \mathfrak{I}_G\right)(x) = \coprod_{x=yz}\left((\mathfrak{I}_G{}^c)^c(y) \cup \mathfrak{I}_G(z)\right) = \coprod_{x=yz}\left(\mathfrak{I}_G(y) \cup \mathfrak{I}_G(z)\right) = \mathfrak{I}_G{}^c(x). \quad (24)$$

Thereby, $\mathfrak{I}_G{}^c \otimes_{s'/p} \mathfrak{I}_G = \mathfrak{I}_G{}^c$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G{}^c \otimes_{s'/p} \mathfrak{I}_G\right)(x) = \coprod_{x=yz}\left((\mathfrak{I}_G{}^c)^c(y) \cup \mathfrak{I}_G(z)\right) = \coprod_{x=yz}\left(\mathfrak{I}_G(y) \cup \mathfrak{I}_G(z)\right) = U_G(x). \quad (25)$$

Thereby, $\mathfrak{I}_G{}^c \otimes_{s'/p} \mathfrak{I}_G = U_G$.

Proposition 11. Let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c = \mathfrak{I}_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{I}_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \mathfrak{I}_G{}^c(z)\right) = \mathfrak{I}_G(x). \quad (26)$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c = \mathfrak{I}_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \mathfrak{I}_G{}^c(z)\right) = U_G(x). \quad (27)$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \mathfrak{I}_G{}^c = U_G$.

Proposition 12. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\mathfrak{I}_G \tilde{\subseteq}_A \wp_G$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s and $\mathfrak{I}_G \tilde{\subseteq}_A \wp_G$. Then, $\mathfrak{I}_G(y) \subseteq \wp_G(z)$, for each $y, z \in G$.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \wp_G(z)\right) = \emptyset_G(x). \quad (28)$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \emptyset_G$. Here note that $\mathfrak{I}_G{}^c(y) \cup \wp_G(z) = \left(\mathfrak{I}_G(y) \setminus \wp_G(z)\right)'$, for all $y, z \in G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left(\mathfrak{I}_G{}^c(y) \cup \wp_G(z)\right) = U_G(x). \quad (29)$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$.

Proposition 13. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\mathfrak{I}_G \tilde{\subseteq}_S \wp_G$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: the proof is similar to the proof of *Proposition 11*.

Proposition 14. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\wp_G \tilde{\subseteq}_S (\mathfrak{I}_G)^c$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \mathfrak{I}_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s and $\wp_G \widetilde{\subseteq}_S (\mathfrak{I}_G)^c$. Then, for all $x \in G$, $\mathfrak{I}_G(x) = A$, $\wp_G(x) = B$, where A and B are two fixed sets and $B \subseteq A'$.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right) = \mathfrak{I}_G(x). \quad \textbf{(30)}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = \mathfrak{I}_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right) = U_G(x). \quad \textbf{(31)}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$.

Proposition 15. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $(\mathfrak{I}_G)^c \widetilde{\subseteq}_S \wp_G$. Then,

I. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = {\wp_G}^c$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s and $(\mathfrak{I}_G)^c \widetilde{\subseteq}_S \wp_G$. Then, for all $x \in G$, $\mathfrak{I}_G(x) = A$, $\wp_G(x) = B$, where A and B are two fixed sets and $A' \subseteq B$.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right) = {\wp_G}^c(x). \quad \textbf{(32)}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = {\wp_G}^c$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)(x) = \coprod_{x=yz}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right) = U_G(x). \quad \textbf{(33)}$$

Thereby, $\mathfrak{I}_G \otimes_{s'/p} \wp_G = U_G$.

Proposition 16. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, $\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)^c = \mathfrak{I}_G \otimes_{s/d} \wp_G$.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, for all $x \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)^c(x) = \left(\coprod_{x=yz}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right)\right)' = \underset{x=yz}{\triangle}\left({\mathfrak{I}_G}^c(y) \cup \wp_G(z)\right)' = \underset{x=yz}{\triangle}\left(({\mathfrak{I}_G}^c)^c(y) \cap {\wp_G}^c(z)\right) = \underset{x=yz}{\triangle}\left(\mathfrak{I}_G(y) \cap {\wp_G}^c(z)\right) = \underset{x=yz}{\triangle}\left(\mathfrak{I}_G(y) \setminus \wp_G(z)\right) = \left(\mathfrak{I}_G \otimes_{s/d} \wp_G\right)(x). \quad \textbf{(34)}$$

Thereby, $\left(\mathfrak{I}_G \otimes_{s'/p} \wp_G\right)^c = \mathfrak{I}_G \otimes_{s/d} \wp_G$.

4 | Conclusion

This paper introduces the soft symmetric difference complement–plus product, a novel binary operation on soft sets defined over group-structured parameter domains. It is shown to be compatible with extended notions of soft equality and subsethood and satisfies core algebraic properties such as closure, associativity, commutativity, and idempotency. The operation also interacts coherently with identity and absorbing elements, as well as null and absolute soft sets. These results support the development of a generalized soft group theory and significantly extend the algebraic scope of soft set theory, making it applicable in areas like uncertainty modeling, abstract algebra, and multi-criteria decision-making.

Author Contributions

Z. A.: investigation, visualization, conceptualization, writing-review, validation

A. S.: supervision, visualization, conceptualization, validation, review.

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The author confirms consent for the publication of this work.

Ethics Approval and Consent to Participate

This study does not involve any research conducted on human participants or animals.

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All the data are available in this paper.

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The authors stated that there are no conflicts of interest regarding the publication of this article.

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