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Quadripartitioned Single Valued Neutrosophic Refined Resolvable and Irresolvable Spaces

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Abstract

In this paper, the concepts of Quadripartitioned Single Valued Neutrosophic Refined resolvable,Quadripartitioned Single Valued Neutrosophic Refined irresolvable,Quadripartitioned Single Valued Neutrosophic Refined open hereditarily irresolvable and maximally Quadripartitioned Single Valued Neutrosophic Refined irresolvable spaces are introduced.Also we examine several properties of the Quadripartitioned Single Valued Neutrosophic Refined open hereditarily irresolvable spaces besides giving characterization of these spaces by means of somewhat Quadripartitioned Single Valued Neutrosophic Refined continuous functions and somewhat Quadripartitioned Single Valued Neutrosophic Refined open functions.

Keywords: Quadripartitioned Single Valued Neutrosophic Refined resolvable,Quadripartitioned Single Valued Neutrosophic Refined open hereditarily irresolvable,somewhat Quadripartitioned Single Valued Neutrosophic Refined continuous functions and somewhat Quadripartitioned Single Valued Neutrosophic Refined open functions.

1 | Introduction

The fuzzy set was proposed by Zadeh[13] in 1965. The intuitionistic fuzzy set, an extension of fuzzy sets, was first introduced by K. Atanassov [1]in 1986. Smarandache [8]developed neurothosophic sets, an extension of intuitionistic fuzzy sets and fuzzy sets. Neutosophic set theory deals with the uncertainty issue. Wang [11]suggested single-valued neutrosphic sets, which are an extension of intuitionistic fuzzy sets, fuzzy sets, and the classical set. The four components of the quadripartitioned single valued neutrosophic sets developed by Chatterjee[7] are the membership functions for truth, contradiction, unknown, and falsity. E. Hewit [6]

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proposed the ideas of resolvability and irresolvability in topological spaces. A. Gelikin[] proposed the idea of open hereditarily irresolvable spaces in classical topology. G. Thangaraj and Balasubramanian [10] established the idea of fuzzy resolvable and fuzzy irresolvable spaces (2009). The concepts of resolvability and irresolvability in intuitionistic fuzzy topological spaces were introduced by Dhavaseelan et al [4]. M. Caldas et al.[3]proposed the idea of neutrosophic fuzzy resolvable spaces and irresolvable spaces.

In this paper, the concept of quadripartitioned single valued neutrosophic refined resolvable, quadripartitioned single valued neutrosophic refined irresolvable, quadripartitioned single valued neutrosophic refined open hereditarily irresolvable, somewhat quadripartitioned single valued neutrosophic refined continuous and open fuctions and discussed some of its properties.

2 | Preliminary Concepts

[10, 13] Let E be a universe.A neutrosophic refined set (NRS) A on E can be defined as follows: A={⟨x,$(T_A^1(x), T_A^2(x), ...T_A^P(x))$,$(I_A^1(x), I_A^2(x), ..., I_A^P(x))$,$(F_A^1(x), F_A^2(x), ..., F_A^P(x))$⟩:x∈E} where$T_A^1(x), T_A^2(x), ...T_A^P(x)$:E→[0,1],$I$ such that 0≤T_A^i+I_A^i+F_A^i≤3 (i=1,2,...P) and for any x∈E.$(T_A^1(x), T_A^2(x), ...T_A^P(x))$,$(I_A^1(x), I_A^2(x), ...I_A^P(x))$ and $(F_A^1(x), F_A^2(x), ...F_A^P(x))$ are the truth membership sequence,indeterminacy membership sequence and falsity membership sequence of the element x respectively. P is also known as dimension of NRS A. [7] Let X be a nonempty set.A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth membership function $T_A(x)$,a contradiction membership function $D_A(x)$,an unknown membership function $Y_A(x)$ and a falsity membership function $F_A(x)$ such that for each x∈X. $T_A(x)$,$D_A(x)$,$Y_A(x)$,$F_A(x)$∈[0,1] and 0≤ $T_A(x)$ +$D_A(x)$+$Y_A(x)$+$F_A(x)$≤ 4. [2] Let X be a universe.A Quadripartitioned single valued neutrosophic refined set(QSVNRS)A on X can be defined as follows
A={⟨x,$(T_A^1(x), T_A^2(x), ...T_A^P(x))$,$(D_A^1(x), D_A^2(x), ..., D_A^P(x))$,
$(Y_A^1(x), Y_A^2(x), ..., Y_A^P(x))$, $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$⟩:x∈X}
where$T_A^1(x), T_A^2(x), ...T_A^P(x)$:X→[0,1],$D_A^1(x), D_A^2(x), ...D_A^P(x)$:X→[0,1],$Y_A^1(x), Y_A^2(x), ...Y_A^P(x)$:X→[0,1]
and $F_A^1(x), F_A^2(x), ...F_A^P(x)$:X→[0,1] such that 0≤$T_A^i$+$D_A^i$+$Y_A^i$+$F_A^i$≤4 (i=1,2,...P) and for any x∈E.
$(T_A^1(x), T_A^2(x), ...T_A^P(x))$,$(D_A^1(x), D_A^2(x), ...D_A^P(x))$,$(Y_A^1(x), Y_A^2(x), ...Y_A^P(x))$ and $(F_A^1(x), F_A^2(x), ...F_A^P(x))$ are the truth membership sequence,a contradiction membership sequence,an unknown membership sequence and falsity membership sequence of the element x respectively. Also P is called the dimension of QSVNRS(A). [2] The complement of a quadripartitioned single valued neutrosophic refined set A on X is denotedby $A^{\tilde{c}}$ and is defined as
$A^{\tilde{c}}$={⟨x,$(F_A^1(x), F_A^2(x), ...F_A^P(x))$,$(Y_A^1(x), Y_A^2(x), ..., Y_A^P(x))$,$(D_A^1(x), D_A^2(x), ..., D_A^P(x))$,
i.e.,$T_A^i(x)$=$F_A^i(x)$, $D_A^i(x)$=$Y_A^i(x)$,$Y_A^i(x)$=$D_A^i(x)$,$F_A^i(x)$=$T_A^i(x)$ for all x∈X and i=1,2....P. [2] Let A,B∈QSVNRS(X).Then,
1.The union of A and B is denoted by A$\tilde{\cup}$B = C_1 and is defined by

C_1={⟨x,$(T_{C_1}^1(x), T_{C_1}^2(x), ...T_{C_1}^P(x))$,$(D_{C_1}^1(x), D_{C_1}^2(x), ..., D_{C_1}^P(x))$,$(Y_{C_1}^1(x), Y_{C_1}^2(x), ..., Y_{C_1}^P(x))$,
where $T_{C_1}^i(x)$=max$\{T_A^i(x), T_A^i(x)\}$,$D_{C_1}^i(x)$=max$\{D_A^i(x), D_A^i(x)\}$,$Y_{C_1}^i(x)$=min$\{Y_A^i(x), Y_A^i(x)\}$,
$F_{C_1}^i(x)$=min$\{F_A^i(x), F_A^i(x)\}$ for all x∈X and i=1,2...P.
2.The intersection of A and B is denoted by A$\tilde{\cap}$B = E and is defined by
E={⟨x,$(T_E^1(x), T_E^2(x), ...T_E^P(x))$,$(D_E^1(x), D_E^2(x), ..., D_E^P(x))$,$(Y_E^1(x), Y_E^2(x), ..., Y_E^P(x))$,
where $T_E^i(x)$=min$\{T_A^i(x), T_A^i(x)\}$,$D_E^i(x)$=min$\{D_A^i(x), D_A^i(x)\}$,$Y_E^i(x)$=max$\{Y_A^i(x), Y_A^i(x)\}$,
$F_E^i(x)$=max$\{F_A^i(x), F_A^i(x)\}$ for all x∈X and i=1,2...P. [2] A quadripartitioned single valued neutrosophic refined topology(QSVNRT) on a nonempty set X in a family of QSVNRS in X which satisfy the following conditions.
1. $\tilde{\phi}$, $\tilde{X}$ $\in \tau$.
2. $H_1 \tilde{\cap}$ $H_2 \in \tau$ for any H_1, $H_2 \in \tau$.
3. $\tilde{\cup}$ $H_i \in \tau$ for every{ H_i : i ∈ I}$\tilde{\subseteq}$ τ.
Here the pair (X,τ) is called a quadripartitioned single valued neutrosophic refined topological space(QSVNRTS).All the elements of τ are called quadripartitioned single valued neutrosophic refined open set (QNROS) in X. A QSVNRS K is quadripartitioned single valued neutrosophic refined closed set (QNRCS) if and only if its complement of K is QNROS. [2] Let (X,τ) be a QSVNRTS and A={⟨x,$(T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x))$⟩:x∈X} for i = 1,2....P be QSVNRS in X.Then quadripartitioned single valued neutrosophic refined closure (QNR(cl(A)) and quadripartitioned single valued neutrosophic refined interior (QNRint(A)) of A are defined by

QNRcl(A) = $\tilde{\cap}$\{K:K is a QNRCS in X and A $\tilde{\subseteq}$ K\}
QNRint(A) = $\tilde{\cup}$\{L:L is a QNROS in X and L $\tilde{\subseteq}$ K\}

uadripartitioned Single valued Neutrosophic Refined Resolvable and Irresolvable Spaces

3 | Quadripartitioned Single valued Neutrosophic Refined Resolvable and Irresolvable Spaces

A Quadripartitioned single valued neutrosophic refined set (QSVNRS) U in Quadripartitioned single valued neutrosophic refined topological space (QSVNRTS) (X,τ) is called a quadripartitioned single valued neutrosophic refined dense if there exists no quadripartitioned single valued neutrosophic refined closed set V in (X,τ) such that U $\tilde{\subset}$ V $\tilde{\subset}\tilde{X}$. Let QSVNRTS (X,τ) is called quadripartitioned single valued neutrosophic refined resolvable if there exists a quadripartitioned single valued neutrosophic refined dense set U in (X,τ) such that QNRcl($A^{\tilde{c}}$) = $\tilde{X}$.Otherwise (X,τ) is called quadripartitioned single valued neutrosophic refined irresolvable. Let X = \{e,f\}.Define the quadripartitioned single valued neutrosophic refined sets U,V and W as follows.

U=$\Big\{$⟨ e,\{0.2,0.3,0.2,0.1\},\{0.4,0.3,0.1,0.2\},\{0.5,0.1,0.1,0.6\}⟩,
⟨ f,\{0.3,0.1,0.5,0.4\},\{0.4,0.5,0.6,0.1\}, \{0.3,0.4,0.1,0.2\}⟩$\Big\}$
V=$\Big\{$⟨ e,\{0.3,0.1,0.6,0.2\},\{0.1,0.6,0.2,0.5\},\{0.1,0.5,0.1,0.4\}⟩,
⟨ f,\{0.5,0.1,0.2,0.3\},\{0.2,0.3,0.5,0.4\}, \{0.3,0.4,0.1,0.2\}⟩$\Big\}$
W=$\Big\{$⟨ e,\{0.1,0.2,0.3,0.4\},\{0.4,0.6,0.1,0.3\},\{0.5,0.6,0.1,0.5\}⟩,
⟨ f,\{0.4,0.5,0.2,0.1\},\{0.4,0.5,0.1,0.4\}, \{0.5,0.1,0.2,0.3\}⟩$\Big\}$

It is clear that τ = \{$\tilde{\phi}$, $\tilde{X}$, U\} is a quadripartitioned single valued neutrosophic refined topology on X. Then (X,τ) is a quadripartitioned single valued neutrosophic refined topological space. Now QNRint(V) = $\tilde{\phi}$, QNRint(W) = $\tilde{\phi}$, QNRcl(V) = $\tilde{X}$, QNRcl(W) = $\tilde{X}$.Hence there exists a quadripartitioned single valued neutrosophic refined dense set V and W in (X,τ) such that QNRcl($V^{\tilde{c}}$) = $\tilde{X}$ and QNRcl($W^{\tilde{c}}$) = $\tilde{X}$.Therefore the QSVNR topological space (X,τ) is QSVNR resolvable space. Let X = \{e,f\}.Define the quadripartitioned single valued neutrosophic refined sets U,V and W as follows.

U=$\Big\{$⟨ e,\{0.1,0.2,0.5,0.2\},\{0.3,0.2,0.4,0.6\},\{0.4,0.3,0.2,0.7\}⟩,
⟨ f,\{0.2,0.1,0.5,0.6\},\{0.2,0.3,0.6,0.4\}, \{0.3,0.2,0.5,0.6\}⟩$\Big\}$
V=$\Big\{$⟨ e,\{0.2,0.4,0.3,0.1\},\{0.5,0.4,0.3,0.5\},\{0.7,0.5,0.1,0.6\}⟩,
⟨ f,\{0.3,0.4,0.4,0.3\},\{0.4,0.6,0.1,0.3\}, \{0.5,0.3,0.1,0.2\}⟩$\Big\}$
W=$\Big\{$⟨ e,\{0.2,0.3,0.2,0.1\},\{0.4,0.6,0.1,0.3\},\{0.5,0.4,0.1,0.4\}⟩,
⟨ f,\{0.3,0.2,0.4,0.2\},\{0.6,0.7,0.3,0.4\}, \{0.5,0.6,0.2,0.3\}⟩$\Big\}$

It is clear that τ = \{$\tilde{\phi}$, $\tilde{X}$, U\} is a quadripartitioned single valued neutrosophic refined topology on X. Then (X,τ) is a quadripartitioned single valued neutrosophic refined topological space. Now QNRint(V) = U, QNRint(W) = U, QNRcl(V) = $\tilde{X}$, QNRcl(W) = $\tilde{X}$. Thus V and W are quadripartitioned single valued neutrosophic refined dense set in (X,τ) such that QNRcl($V^{\tilde{c}}$) = $U^{\tilde{c}}$ and QNRcl($W^{\tilde{c}}$) = $U^{\tilde{c}}$.Hence the QSVNR topological space (X,τ) is called a QSVNR irresolvable. Let (X,τ) be a QSVNRTS. (X,τ) is a quadripartitioned single valued neutrosophic refined resolvable space iff (X,τ) has a pair of quadripartitioned single valued neutrosophic refined dense set U_1 and U_2 such that $U_1 \tilde{\subseteq} U_2^{\tilde{c}}$.

Proof: Let (X,τ) be a QSVNRTS and (X,τ) is a QSVNR resolvable space.Suppose that for all QSVNR dense sets U_i and U_j, we have $U_i \tilde{\nsubseteq} U_j^{\tilde{c}}$.Then $U_i \tilde{\supset} U_j^{\tilde{c}}$.Then QNRcl($U_i$) $\tilde{\supset}$ QNRcl($U_j^{\tilde{c}}$) which implies that $\tilde{X} \tilde{\supset}$ QNRcl($U_j^{\tilde{c}}$). Then QNRcl($U_j^{\tilde{c}}$) $\neq \tilde{X}$. Also $U_j \tilde{\supset} U_i^{\tilde{c}}$, then QNRcl($U_j$) $\tilde{\supset}$ QNRcl($U_i^{\tilde{c}}$) which implies that $\tilde{X} \tilde{\supset}$ QNRcl($U_i^{\tilde{c}}$). Then QNRcl($U_i^{\tilde{c}}$) $\neq \tilde{X}$.Hence QNRcl($U_i^{\tilde{c}}$) = $\tilde{X}$, but QNRcl(U_i) $\neq \tilde{X}$ for all QSVNRS U_i in (X,τ) which is a contradiction.Hence (X,τ) has a pair of QSVNR dense set U_1 and U_2 such that
$U_1 \tilde{\subseteq} U_2^{\tilde{c}}$.
Conversely, suppose that the QSVNRTS (X,τ) has a pair of QSVNR dense set U_1 and U_2 such that
$U_1 \tilde{\subseteq} U_2^{\tilde{c}}$. Suppose that (X,$\tau$) is a QSVNR irresolvable space.Then for all QSVNR dense sets U_1 and U_2 in

(X,τ), we have QNRcl($U_1^{\tilde{c}}$) $\neq \tilde{X}$.Then QNRcl($U_2^{\tilde{c}}$) $\neq \tilde{X}$ implies that there exists a QNRCS K in (X,τ) such that $U_2^{\tilde{c}} \tilde{\subset}$ K $\tilde{\subset}\tilde{X}$.Then $U_1 \tilde{\subset} U_2^{\tilde{c}} \tilde{\subset}$ K $\tilde{\subset}\tilde{X}$ implies that $U_1 \tilde{\subset}$ K $\tilde{\subset}\tilde{X}$. which is a contradiction. Hence (X,τ) is a quadripartitioned single valued neutrosophic refined resolvable space. □

If (X,τ) is a quadripartitioned single valued neutrosophic refined irresolvable if and only if QNRint(U) $\neq \tilde{\phi}$ for all QSVNR dense set U in (X,τ).

Proof: Since (X,τ) is a QSVNR irresolvable space for all QSVNR dense set U in (X,τ), QNRcl($U^{\tilde{c}}$) $\neq \tilde{X}$. Then (QNRint(U))$^{\tilde{c}} \neq \tilde{X}$ which implies QNRint(U) $\neq \tilde{\phi}$.
Conversely QNRint(U) $\neq \tilde{\phi}$ for all QSVNR dense set U in (X,τ). Suppose that (X,τ) is QSVNR resolvable space. Then there exists a QSVNR dense set U in (X,τ) such that QNRcl($U^{\tilde{c}}$) = $\tilde{X}$.This implies that (QNRint(U))$^{\tilde{c}}$ = $\tilde{X}$, implies QNRint(U) = $\tilde{\phi}$. Which is a contradiction. Hence (X,τ) is a QSVNR irresolvable space. □

A quadripartitioned single valued neutrosophic refined topological space (X,τ) is called a quadripartitioned single valued neutropsphic refined submaximal space if for each QSVNRS U in (X,τ), QNRcl(U) = $\tilde{X}$. If the QSVNRTS (X,τ) is QSVNR submaximal, then (X,τ) is QSVNR irresolvable.

Proof: Let (X,τ) be a QSVNR submaximal space. Assume that (X,τ) is a QSVNR resolvable space. Let U be a QSVNR dense set in (X,τ). Then QNRcl($U^{\tilde{c}}$) = $\tilde{X}$. Hence (QNRint(U))$^{\tilde{c}}$ = $\tilde{X}$, implies QNRint(U) = $\tilde{\phi}$. Then U $\notin \tau$.Which is a contradiction to QSVNR submaximal space of (X,τ). Hence (X,τ) is a QSVNR irresolvable space.The converse of the Proposition 3.8. is not true. See Example 3.4. □

A quadripartitioned single valued neutroosphic refined topological space (X,τ) is called a maximal quadripartitioned single valued neutropsphic refined irresolvable space if (X,τ) is QSVNR irresolvable and every QSVNR dense set U of (X,τ) is QSVNR open. Let X = {e,f}.Define the quadripartitioned single valued neutrosophic refined sets V,W, V$\tilde{\cup}$W, V$\tilde{\cap}$W as follows

V=$\Big\{\langle$ e,{0.1,0.3,0.5,0.4},{0.5,0.1,0.3,0.2},{0.1,0.2,0.5,0.4}$\rangle$,
$\langle$ f,{0.3,0.2,0.1,0.5},{0.1,0.5,0.1,0.4}, {0.1,0.3,0.4,0.6}$\rangle\Big\}$

W=$\Big\{\langle$ e,{0.3,0.1,0.2,0.5},{0.4,0.2,0.3,0.5},{0.2,0.1,0.5,0.4}$\rangle$,
$\langle$ f,{0.3,0.1,0.2,0.4},{0.2,0.1,0.6,0.5}, {0.1,0.3,0.4,0.6}$\rangle\Big\}$

V$\tilde{\cup}$W=$\Big\{\langle$ e,{0.3,0.3,0.2,0.4},{0.5,0.2,0.3,0.2},{0.2,0.2,0.5,0.4}$\rangle$,
$\langle$ f,{0.3,0.2,0.1,0.4},{0.2,0.5,0.1,0.4}, {0.1,0.3,0.4,0.2}$\rangle\Big\}$

V$\tilde{\cap}$W=$\Big\{\langle$ e,{0.1,0.1,0.5,0.5},{0.4,0.1,0.3,0.5},{0.1,0.1,0.5,0.4}$\rangle$,
$\langle$ f,{0.3,0.1,0.2,0.5},{0.1,0.1,0.6,0.5},{0.1,0.3,0.4,0.6}$\rangle\Big\}$

It is clear that τ = {$\tilde{\phi}$, $\tilde{X}$, V, W, V$\tilde{\cup}$W, V$\tilde{\cap}$W } is a quadripartitioned single valued neutrosophic refined topology on X. Then (X,τ) is a quadripartitioned single valued neutrosophic refined topological space. Now QNRint($V^{\tilde{c}}$) = $\tilde{\phi}$, QNRint($W^{\tilde{c}}$) = W, QNRint(V$\tilde{\cup}$W)$^{\tilde{c}}$ = $\tilde{\phi}$, QNRint(V$\tilde{\cap}$W)$^{\tilde{c}}$ = W and QNRcl(V) = $\tilde{X}$, QNRcl(W) = $W^{\tilde{c}}$, QNRcl(V$\tilde{\cup}$W) = $\tilde{X}$, QNRcl(V$\tilde{\cap}$W) = $W^{\tilde{c}}$, QNRcl(V$\tilde{\cup}$W)$^{\tilde{c}}$ = (V$\tilde{\cup}$W)$^{\tilde{c}}$, QNRcl($V^{\tilde{c}}$) = $V^{\tilde{c}}$. Hence (X,τ) is a QSVNR irresolvable and every QSVNR dense set of (X,τ) is quadripartitioned single valued neutrosophic refined open. Therefore (X,τ) is a maximally QSVNR irresolvable space. uadripartitioned Single valued Neutrsophic Refined Open Hereditarily Irresolvable

4 | Quadripartitioned Single valued Neutrsophic Refined Open Hereditarily Irresolvable

A quadripartitioned single valued neutroosphic refined topological space (X,τ) is said to be QSVNR open hereditarily irresolvable if QNRint(QNRcl(U)) $\neq \tilde{\phi}$ and QNRint(U) $\neq \tilde{\phi}$, for any QSVNRS U in (X,τ). Let X = {e,f}.Define the quadripartitioned single valued neutrosophic refined sets U_1, U_2 and U_3 as follows

U_1=$\Big\{\langle$ e,{0.1,0.3,0.4,0.2},{0.3,0.1,0.2,0.5},{0.2,0.4,0.5,0.3}$\rangle$,

⟨ f,{0.2,0.1,0.4,0.3},{0.3,0.2,0.4,0.6}, {0.1,0.3,0.4,0.5}⟩}

U_2={⟨ e,{0.3,0.5,0.2,0.1},{0.5,0.3,0.1,0.4},{0.4,0.5,0.3,0.2}⟩,

⟨ f,{0.5,0.3,0.2,0.1},{0.6,0.5,0.1,0.3}, {0.3,0.5,0.2,0.4}⟩}

U_3={⟨ e,{0.1,0.4,0.3,0.1},{0.4,0.1,0.1,0.5},{0.2,0.5,0.4,0.3}⟩,

⟨ f,{0.3,0.1,0.2,0.2},{0.4,0.3,0.3,0.5}, {0.1,0.3,0.4,0.2}⟩}

It is obvious that $\tau = \{\tilde{\phi}, \tilde{X}, U_1, U_2\}$ is a quadripartitioned single valued neutrosophic refined topology on X. Then (X,τ) is a quadripartitioned single valued neutrosophic refined topological space.Now QNRcl(U_1) = $(U_1)^{\tilde{c}}$; QNRcl(U_2) = $\tilde{X}$ and QNRint(U_2) = U_1. Also QNRint(QNRcl(U_1)) = QNRint($(U_1)^{\tilde{c}}$) = $(U_1)^{\tilde{c}} \neq \tilde{\phi}$ and QNRint(U_1) = $U_1 \neq \tilde{\phi}$, QNRint(QNRcl(U_2) = QNRint($\tilde{X}$) = $\tilde{X} \neq \tilde{\phi}$ and QNRint(U_2) = $U_2 \neq \tilde{\phi}$, QNRint(QNRcl(U_3)) = QNRint($(U_1)^{\tilde{c}}$ = $(U_1)^{\tilde{c}} \neq \tilde{\phi}$ and QNRint(U_3) = $U_1 \neq \tilde{\phi}$ and QNRint(QNRcl($(U_3)^{\tilde{c}}$) = QNRint$(U_1)^{\tilde{c}}$ = $(U_1)^{\tilde{c}} \neq \tilde{\phi}$ and QNRint$(U_1)^{\tilde{c}}$ = $U_1 \neq \tilde{\phi}$. Hence if QNRint(QNRcl(U)) $\neq \tilde{\phi}$, then QNRint(U) $\neq \tilde{\phi}$ for any nonzero QSVNRS U in (X,τ).Thus (X,τ) is a quadripartitioned single valued neutrosophicrefined open hereditarily irresolvable space. Let (X,τ) be a QSVNRTS.If (X,τ) is QSVNR open hereditarily irresolvable then (X,τ) is QSVNR irresolvable.

Proof: Let U be a QSVNR dense set in (X,τ).Then QNRcl(U) = $\tilde{X}$, which implies that QNRint(QNRcl(U)) = $\tilde{X} \neq \tilde{\phi}$.Since (X,$\tau$) is QSVNR open hereditarily irresolvable, we have QNRint(U) $\neq \tilde{\phi}$. Therefore by Proposition 3.6. QNRint(U) $\neq \tilde{\phi}$ for all QSVNR dense set in (X,τ), implies that (X,τ) is QSVNR irresolvable.The converse of the above proposition need not be true by the following example. □

Let X = {e,f}.Define the quadripartitioned single valued neutrosophic refined sets U, V and W as follows

U={⟨ e,{0.3,0.2,0.5,0.4},{0.4,0.1,0.3,0.5},{9.2,0.3,0.5,0.4}⟩,

⟨ f,{0.1,0.2,0.3,0.5},{0.3,0.1,0.5,0.6}, {0.2,0.3,0.5,0.4}⟩}

V={⟨ e,{0.4,0.3,0.4,0.3},{0.5,0.2,0.1,0.4},{0.3,0.4,0.2,0.1}⟩,

⟨ f,{0.2,0.3,0.1,0.4},{0.4,0.2,0.4,0.5}, {0.3,0.4,0.3,0.2}⟩}

W={⟨ e,{0.4,0.6,0.1,0.1},{0.6,0.4,0.1,0.4},{0.5,0.6,0.2,0.1}⟩,

⟨ f,{0.6,0.4,0.1,0.1},{0.7,0.6,0.1,0.2}, {0.4,0.6,0.1,0.3}⟩}

It is obvious that $\tau = \{\tilde{\phi}, \tilde{X}$, U, V } is a quadripartitioned single valued neutrosophic refined topology on X. Then (X,τ) is a quadripartitioned single valued neutrosophic refined topological space.Now W and $\tilde{X}$ are QSVNR dense sets in (X,τ).Then QNRint(W) = U $\neq \tilde{\phi}$ and QNRint($\tilde{X}$) $\neq \tilde{\phi}$. Hence (X,τ) is a QSVNR irresolvable. But QNRint(QNRcl($W^{\tilde{c}}$)) = QNRint($U^{\tilde{c}}$) = U $\neq \tilde{\phi}$ and QNRint($W^{\tilde{c}}$) = $\tilde{\phi}$.Therefore (X,τ) is not a QSVNR open hereditarily irresolvable space. Let (X,τ) be a QSVNR open hereditarily irresolvable. Then QNRint(U) $\tilde{\nsubseteq}$ (QNRint(V))$^{\tilde{c}}$ for any two QSVNR dense sets U and V in (X,τ).

Proof: Let U and V be any QSVNR dense sets in (X,τ). Then QNRcl(U) = $\tilde{X}$ and QNRcl(V) = $\tilde{X}$ implies that QNRint(QNRcl(U)) $\neq \tilde{\phi}$ and QNRint(QNRcl(V)) $\neq \tilde{\phi}$. Since (X,τ) is QSVNR open hereditarily irresolvable, QNRint(U) $\neq \tilde{\phi}$ and QNRint(V) $\neq \tilde{\phi}$.Hence by proposition 3.5, U $\tilde{\nsubseteq}$ $V^{\tilde{c}}$. Therefore QNRint(U) $\tilde{\subseteq}$ U $\tilde{\nsubseteq}$ $V^{\tilde{c}}$ $\tilde{\subseteq}$ (QNRint(V))$^{\tilde{c}}$. Hence we have QNRint(U) $\tilde{\subseteq}$ (QNRint(V))$^{\tilde{c}}$ for any two QSVNR dense sets U and V in (X,τ). □

Let (X,τ) be a QSVNRTS.If(X,τ) is QSVNR open hereditarily irresolvable,then QNRint(U) = $\tilde{\phi}$ for any nonzero QSVNR dense set U in (X,τ) implies that QNRint(QNRcl(U)) = $\tilde{\phi}$.

Proof: Let U be a QSVNR dense set in (X,τ) such that QNRint(U) = $\tilde{\phi}$. We claim that QNRint(QNRcl(U)) = $\tilde{\phi}$. Suppose that QNRint(QNRcl(U)) = $\tilde{\phi}$.Since (X,τ) is QSVNR open hereditarily irresolvable, we have QNRint(U) $\neq \tilde{\phi}$. Which is a contradiction to QNRint(U) = $\tilde{\phi}$. Hence QNRint(QNRcl(U)) = $\tilde{\phi}$. □

Let (X,τ) be a QSVNRTS. If (X,τ) is QSVNR open hereditarily irresolvable,then QNRcl(U) = $\tilde{X}$ for any nonzero QSVNR dense set U in (X,τ) implies that QNRcl(QNRint(U)) = $\tilde{\phi}$.

Proof: Let U be a QSVNRS in (X,τ) such that QNRcl(U) = $\tilde{X}$. Then we have $(\text{QNRcl(U)})^{\tilde{c}} = \tilde{\phi}$, which implies that QNRint($U^{\tilde{c}}$) = $\tilde{\phi}$. Since (X,τ) is QSVNR open hereditarily irresolvable by proposition 4.6. We have that QNRint(QNRcl($U^{\tilde{c}}$)) = $\tilde{\phi}$. Therefore $(\text{QNRcl(QNRint(U))})^{\tilde{c}} = \tilde{\phi}$ implies QNRcl(QNRint(U)) = $\tilde{X}$. □

5 | Somewhat QSVNR Continuous and Somewhat QSVNR open

Let (X, τ) and (Y, δ) be any two QSVNRTS. A function g: (X, τ) $\rightarrow$ (Y, δ) is called somewhat QSVNR continuous if U $\in \delta$ and g^{-1}(U) $\neq \tilde{\phi}$, then there exists a V $\in \tau$ such that V $\neq \tilde{\phi}$ and V $\tilde{\subseteq}$ g^{-1}(U). Let (X, τ) and (Y, δ) be any two QSVNRTS. A function g: (X, τ) $\rightarrow$ (Y, δ) is called somewhat QSVNR open if U $\in \delta$ and U $\neq \tilde{\phi}$, then there exists a V $\in \delta$ such that V $\neq \tilde{\phi}$ and V $\tilde{\subseteq}$ g(U). Let (X, τ) and (Y, δ) be any two QSVNRTS. If the function g: (X, τ) $\rightarrow$ (Y, δ) is called somewhat QSVNR continuous and one-to-one. If QNRint(U) = $\tilde{\phi}$ for any nonzero QSVNRS U in (X, τ) then QNRint(g(U)) = $\tilde{\phi}$ in (Y, δ).

Proof: Let U be a nonzero QSVNRS in (X, τ) such that QNRint(U) = $\tilde{\phi}$. To prove that QNRint(g(U)) = $\tilde{\phi}$. Suppose that QNRint(g(U)) $\neq \tilde{\phi}$ in (Y, δ). Then there exists a nonzero QSVNRS V in (Y, δ) such that V $\tilde{\subseteq}$ g(U). Then g^{-1}(V) $\tilde{\subseteq}$ g^{-1}(g(U)). Since g is somewhat continuous, there exists a W $\in \tau$ such that W $\neq \tilde{\phi}$ and S $\tilde{\subseteq}$ g^{-1}(V). Hence S $\tilde{\subseteq}$ g^{-1}(V) $\tilde{\subseteq}$ U, implies that QNRint(U) $\neq \tilde{\phi}$. Which is a contradiction. Hence QNRint(g(U)) = $\tilde{\phi}$ in (Y, δ). □

Let (X, τ) and (Y, δ) be any two QSVNRTS. If the function g: (X, τ) $\rightarrow$ (Y, δ) is called somewhat QSVNR continuous, one-to-one, QNRint((QNRcl(U)) = $\tilde{\phi}$ for any nonzero QSVNRS U in (X, τ) then QNRint(QNRcl(g(U))) = $\tilde{\phi}$ in (Y, δ).

Proof: Let U be a nonzero QSVNRS in (X, τ) such that QNRint(QNRcl(U)) = $\tilde{\phi}$. We claim that QNRint(g(U)) = $\tilde{\phi}$. Suppose that QNRint(QNRcl(g(U))) $\neq \tilde{\phi}$ in (Y, δ). Then QNRcl(g(U)) $\neq \tilde{\phi}$ and $(\text{QNRcl(g(U))})^{\tilde{c}} \neq \tilde{\phi}$. Now $(\text{QNRcl(g(U))})^{\tilde{c}} \neq \tilde{\phi} \in \delta$. Since g is somewhat QSVNR continuous, there exists a V $\in \tau$, such that V $\neq \tilde{\phi}$ and V $\tilde{\subseteq}$ $g^{-1}(\text{QNRcl(g(U))})^{\tilde{c}}$. That is V $\tilde{\subseteq}$ $(g^{-1}(\text{QNRcl(g(U))}))^{\tilde{c}}$, which implies that g^{-1}(QNRcl(g(U)) $\tilde{\subseteq}$ $V^{\tilde{c}}$. Since g is one-to-one, thus U $\tilde{\subseteq}$ g^{-1}(g(U)) $\tilde{\subseteq}$ g^{-1}(QNRcl(g(U))) $\tilde{\subseteq}$ $V^{\tilde{c}}$ which implies U $\tilde{\subseteq}$ $V^{\tilde{c}}$. Therefore V $\tilde{\subseteq}$ $U^{\tilde{c}}$. This implies that QNRint($U^{\tilde{c}}$) $\neq \tilde{\phi}$. Let QNRint($U^{\tilde{c}}$) = W $\neq \tilde{\phi}$. Then we have QNRcl(QNRint($U^{\tilde{c}}$)) = QNRcl(W) $\neq$ $\tilde{X}$ which implies that QNRint(QNRcl(U)) $\neq \tilde{\phi}$. Which is a contradiction. Hence QNRint(QNRcl(g(U))) = $\tilde{\phi}$ in (Y, δ). □

Let (X, τ) and (Y, δ) be any two QSVNRTS.If the function g: (X, τ) $\rightarrow$ (Y, δ) is called somewhat QSVNR open and QNRint(U) = $\tilde{\phi}$ for any nonzero QSVNRS U in (Y, δ), then QNRint(g^{-1}(U)) = $\tilde{\phi}$ in (X, τ).

Proof: Let U be a nonzero QSVNR in (Y, δ) such that QNRint(U) = $\tilde{\phi}$. We claim that QNRint(g^{-1}(U)) = $\tilde{\phi}$ in (X, τ). Suppose that QNRint(g^{-1}(U)) $\neq \tilde{\phi}$ in (X, τ). Then there exists a nonzero QSVNR open set V in (X, τ) such that V $\tilde{\subseteq}$ g^{-1}(U). Thus,we have g(V) $\tilde{\subseteq}$ g(g^{-1}(U)) $\tilde{\subseteq}$ U.This implies that g(V) $\tilde{\subseteq}$ U. Since g is somewhat QSVNR open, there exists a W $\in \delta$ such that W $\neq \tilde{\phi}$ and W $\tilde{\subseteq}$ g(V). Therefore W $\tilde{\subseteq}$ g(V) $\tilde{\subseteq}$ U which implies W $\tilde{\subseteq}$ U. Hence QNRint(U) $\neq \tilde{\phi}$ which is a contradiction. Hence QNRint(g^{-1}(U)) = $\tilde{\phi}$ in (X, τ). □

Let (X, τ) and (Y, δ) be any two QSVNRTS. Let (X, τ) be a QSVNR open hereditarily irresolvable space. If the function g: (X, τ) $\rightarrow$ (Y, δ) is somewhat QSVNR open, somewhat QSVNR continuous, one-to-one and onto function, then (Y, δ) is a QSVNR open hereditarily space.

Proof: Let U be a nonzero QSVNRS in (Y, δ) such that QNRint(U) = $\tilde{\phi}$. Now QNRint(U) = $\tilde{\phi}$ and g is somewhat QSVNR open implies that by proposition 5.5, QNRint(g^{-1}(U)) = $\tilde{\phi}$ in (X, τ). Since (X, τ) is a QSVNR open hereditarily irresolvable space, we have QNRint(QNRcl(g^{-1}(U)) = $\tilde{\phi}$ in (X, τ) by proposition 4.6. Since QNRint(QNRcl(g^{-1}(U)) = $\tilde{\phi}$ and g is somewhat QSVNR continuous by proposition 5.4, we have that QNRint(QNRcl(g^{-1}(U)) = $\tilde{\phi}$. Since g is onto, thus QNRintQNRcl(U) = $\tilde{\phi}$. Hence by proposition 4.6, (Y, δ) is a QSVNR open hereditarily irresolvable space. □

6 | Conclusion

In this paper we defined Quadripartitioned single valued neutrosophic refined resolvable and irresolvable spaces and studied some of its properties.Further we defined continuous and open functions in Quadripartitioned single valued neutrosophic refined topological spaces and also quadripartitioned single valued neutrosophic refined open hereditarily space.

Conflict of Interest

"The authors declare no conflict of interest."

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