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Neutrosphic Switch Graphs, Neutrosophic Reversal Switch Graphs and Reversal Neutrosophic Reactive Graphs

Nandini M Chakrapani*

Avinashilingam University; mc.nandini@gmail.com.

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Abstract

This study introduces a broader framework termed Reversal Neutrosophic Switch Graph (RNSG), which facilitates both the activation and deactivation of arrows alongside updating their fuzzy values. The procedure yields the Reversal Neutrosophic Reactive Graph (RNRG) by employing various aggregation functions. Moreover, we propose several processes reliant on aggregate functions such as union, intersection, cartesian product, and extension. Reversal Neutrosophic Fuzzy Switch Graphs (RNFSGs) can effectively model the dynamic components of numerous systems prevalent in engineering, computer science, and related disciplines. The paper further elucidates the relationship between Neutrosophic Graphs and RNSGs, providing a logic for assessing the modelled system's characteristics.

Keywords: Neutrosophic switch graphs, Reversal neutrosophic switch graph, Neutrosophic reactive graphs, Reversal neutrosophic reactive graphs, Aggregations functions.

1 | Introduction

In recent years, Fuzzy Switch Graphs (FSGs) have garnered significant attention due to their ability to model and handle uncertainty in various real-world systems. These graphs have been instrumental in applications ranging from network analysis to decision-making processes. Neutrosophic logic, which extends fuzzy logic by incorporating the degree of indeterminacy, has further enhanced the ability to manage uncertainty. This paper presents an extension of the previous work on FSGs by introducing the concept of Reversal Neutrosophic Switch Graphs (RNSGs) . In the real world, there exist state-based systems displaying fuzzy behaviour.

Corresponding Author: mc.nandini@gmail.com

However, when a state within such a system changes, it necessitates reconfiguration. As we traverse a reactive graph, the relationships between its nodes undergo alterations, a notion initially proposed by Gabbay [1]. This extends beyond static graphs by incorporating high-order edges, also known as high-order arrows or switches. Switch Graphs exemplify certain characteristics, and the operations introduced for Reactive RNSGs in this study facilitate representing system extension into a larger one by highlighting the extension. Each update operation considers the impact of associated aggregations, thus demonstrating the examination of aggregations' impact on each movement. One notable contribution in this domain is the paper "Aggregation-based Operations for Reversal Fuzzy Switch Graphs [2], [3]" which laid the groundwork for aggregating operations within the framework of reversal neturosophic switch graphs and reactive graphs.

2 | Preliminaries

In order to create a self-contained study, we delve into select concepts and discoveries from existing literature within this section. We assume the reader possesses a fundamental grasp of neutrosophic set theory. Additionally, we will link the aggregation functions with the neutrosophic graph to facilitate a more straightforward interpretation.

Definition 1 ([4]). An n-ary function A: [0, 1] n ⟶ [0, 1] is an aggregation function if it satisfies the following conditions:

I. A (0, 0, ..., 0) = 0 and A (1, 1, ..., 1) = 1.

II. $x_i \leq y_i$, for i = 1, ..., n, implies A $(x_1, x_2, ..., x_n) \leq$ A $(y_1, y_2, ..., y_n)$ for all $(x_1, x_2, ..., x_n)$, $(y_1, y_2, ..., y_n) \in [0, 1]^n$ (isotonicity).

The functions: $A_n(\overline{x}) = \frac{1}{n}(x_1 + ... + x_n)$ (arithmetic mean), $A_n(\overline{x}) = x_1 \cdot ... \cdot x_n$ (product), $A_n(\overline{x}) = \sqrt[n]{x_1 x_2 \dots \dots .. x_n}$ (geometric mean), $A_n(\overline{x}) = \max(x_1, x_2,..., x_n)$ (maximum), $A_n(\overline{x}) = \min(x_1, x_2,..., x_n)$ (minimum) and projections, j : $[0, 1]^{n-} \longrightarrow [0, 1]$, s.t. j $(x_1,..., x_j ,..., x_n) = x_j$ are some of aggregation functions.

Definition 2 ([4], [5]). An n-ary aggregation A is idempotent if A (x, x, ..., x) = x, for each x ∈ [0, 1]. If n = 2 (A is binary), A is associative whenever A (A (x_1, x_2), x_3) = A (x_1, A (x_2, x_3)) holds for all x_1, x_2, x_3 in [0, 1] and A is commutative whenever A (x_1, x_2) = A (x_2, x_1) holds for all x_1, x_2 in [0, 1].

Definition 3 ([4], [5]). An aggregation A has a neutral element e ∈ [0, 1] if, for every x ∈ [0, 1], A (e, .., e, x, e, ..., e) = x.

Definition 4 ([2], [3], [6]). A neutrosophic graph is a structure $\langle V, R \rangle$, such that V is a non-empty set of vertices and R is a neutrosophic set R: V × V ⟶ [0, 1]. A neutrosophic graph (NF graph) with underlying set V is defined to be a pair NG = (A, B), where the functions TA, IA, FA: V ⟶ [0, 1] denote the degree of truth membership, degree of indeterminacy membership and the degree of falsity membership of the element vi ∈ V respectively and 0 ≤ TA(x) + IA(x) + FA(x) ≤ 3;

The idea of an enriched graph is introduced by Gabbay [1]. Double arrows been used to express connection between arrows in his work. The double arrows deactivate its target arrows. Gabbay and Marcelino [7] extended the work of Gabbay by introducing Switch Graphs in 2012 [1].

Definition 5 ([8]). A switch graph is an ordered pair $\langle W, R \rangle$ s.t. W is a non-empty set (set of worlds) and R ⊆ A(W) is a set of arrows, called switches, where A(W) = $\bigcup_{i \in N} A_i(w)$ with

$A_0(W) = W \times W$,

$A_{i+1}(W) = A_0(W) \times A_i(W)$.

Santiago et al. [8] provided the fuzzification for switch graphs with some modifications.

Definition 6 ([8]). Let W be a non-empty finite set (set of states or worlds) and the set S=$\bigcup_{n \in N} S^n$ where $S^0 = \emptyset$ and, $S^0 \subseteq W \times W$, $S^{n+1} \subseteq S^0 \times S^n$.

A FSG is a pair M = ⟨W, T: S → [0, 1] ⟩, s.t. t is a membership function for S. The elements $a_i^0 \in S^0$ ($i \in N$) are called zero-order arrows. The elements of S^{n+1}, $n \geq 0$, are called high-order arrows.

3 | Reversal Neutrosophic Switch Graphs

Santiago et al. [8] introduced the notion of FSGs and used then to model some biological applications (Circadian rhythm in cyanobacteria and autoregulative protein). This work applied aggregation functions in order to combine fuzzy values of graph edges. The FSGs, however, are not able to model reactive systems which requires the activation and deactivation of transitions. In order to overcome that, this section presents the notion of Reversal Fuzzy Switch Graphs (RNSGs) [9].

Definition 7 (Neutrosophic Switch Graph (NSG)). A NSG is a four tuple (W, T, I, F) where T: S→ [0,1], I: S→[0,1], F: S→[0,1] are called truth membership degree, indeterminacy membership degree, and falsity membership degree of x on S, respectively, satisfy the condition $0 \leq T(x) + I(x) + F(x) \leq 3$, for all $x \in S$. The elements $a_i^0 \in S^0$ ($i \in N$) are called zero-order arrows. The elements of S^{n+1}, $n \geq 0$, are called high-order arrows.

Note: *Fig. 1.b* shows a NSG N with $S^0 = \{(u, v), (v, w), (w, u),(v, z)\}$, $S^1 = \{((v, w),(w, u)) , ((v, z),(v, w))\}$ and $S^2 = \{((u, v), ((v, w),(w, u)))\}$. Associated to S^0, S^1 and S^2, we have the following membership values:

Truth membership values:

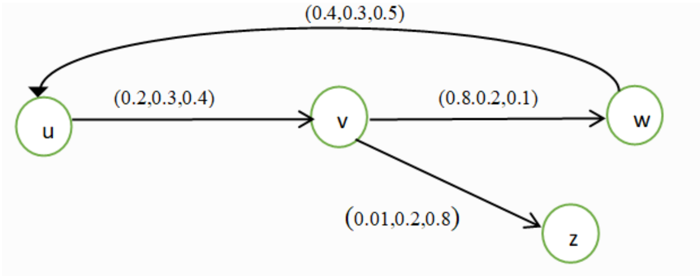
$$T(u,v) = 0.2, T(v,w) = 0.8, T(v,z) = 0.01, T(w,u) = 0.4, T((v,w),(w,u)) = 0.7, T((v,z),(v,w)) = 0.1, T((u,v),((v,w),(w,u))) = 0.1.$$

Indeterminacy membership values:

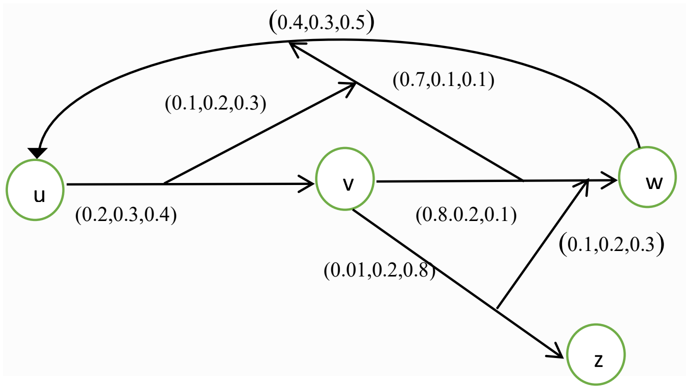
$$I(u,v) = 0.3, I(v,w) = 0.2, (v,z) = 0.2, I(w,u) = 0.3, I\big((v,w),(w,u)\big) = 0, I((v,z),(v,w)) = 0.2, I((u,v),((v,w),(w,u))) = 0.2.$$

Falsity membership values:

F(u, v) =0.4, F (v, w) = 0.1, F(v, z) = 0.8, F(w ,u) = 0.5, F((v, w), (w, u))= 0.1, F((v, z),(v ,w)) = 0.3, F((u, v), ((v, w),(w, u))) = 0.5, and T(a) = 0, I(a) =0 ,F(a) =0, for all $a \in S^j$ and $j > 2$.



a.



b.

Fig. 1. a. neutrosophic graph, and b. NSG.

Definition 8 (Reconfiguration of neutrosophic switch graph). Given an aggregation function A: $[0, 1]^3 \rightarrow [0, 1]$ and a NSG $N = \langle W, T: S \rightarrow [0,1],\ I: S \rightarrow [0,1],\ F: S \rightarrow [0,1] \rangle$, the structure $N_A^{a_i^0} = \langle W, T_{a_i^0}^A: S \rightarrow [0,1], I_{a_i^0}^A: S \rightarrow [0,1], F_{a_i^0}^A: S \rightarrow [0,1] \rangle$ such that

$$T_{a_i^0}^A(a) = \begin{cases} T(a), \text{if } (a_i^0, a) \notin S \\ A(T(a_i^0), T(a_i^0, a), T(a)), \text{ othervise} \end{cases},$$

$$I_{a_i^0}^A(a) = \begin{cases} I(a), \text{if } (a_i^0, a) \notin S \\ A(I(a_i^0), I(a_i^0, a), I(a)), \text{ othervise} \end{cases},$$

$$F_{a_i^0}^A(a) = \begin{cases} F(a), \text{if } (a_i^0, a) \notin S \\ A(F(a_i^0), F(a_i^0, a), F(a)), \text{ othervise} \end{cases},$$

is called the reconfiguration of N, based on A, after crossing a_i^0.

Definition 9. An aggregation A: $[0, 1]^n \rightarrow [0, 1]$ is shift-invariant if, for all $\lambda \in [-1, 1]$ and for all $(x_1,..., x_n) \in [0, 1]^n$, $A(x_1 + \lambda,..., x_n + \lambda) = A(x_1,..., x_n) + \lambda$ whenever $(x_1 + \lambda,..., x_n + \lambda) \in [0, 1]$ n and $A(x_1,..., x_n) + \lambda \in [0, 1]$. Santiago et al. [8] extend the notion of FSGs for Fuzzy Reactive Graphs. In what follows, given a NSG, $N = N\ \langle W, T: S \rightarrow [0,1],\ I: S \rightarrow [0,1],\ F: S \rightarrow [0,1] \rangle$, we define the set $S\rightarrow = \{a_i^0 \in S0; [a_i^0, a] \in S$, with $a \in S\}$.

Remark 1. Any reconfiguration of a NSG, N is an NSG.

Example 1. *Fig. 2* shows the reconfiguration, $N_{arith}^{(u,v)}$, of the *Fig. 1.(b)* obtained after crossing (u, v) by using the arithmetic mean. In this case,

$A_T(0.2, 0.1, 0.7) = \frac{0.2 + 0.1 + 0.7}{3} = \frac{1}{3}$, $A_I(0.3,0.2, 0.1) = \frac{0.3 + 0.2 + 0.1}{3} = 0.2$, $A_F(0.4, 0.3, 0.1) = \frac{0.4 + 0.3 + 0.1}{3} = \frac{0.8}{3}$.

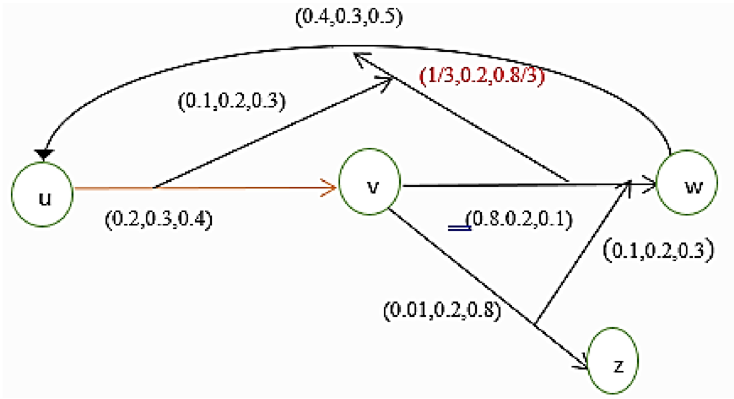


Fig. 2. Reconfiguration of the NSG in Fig. 1.b. after crossing the arrow (u,v)(u,v)(u,v) using the arithmetic mean aggregation function.

Definition 10. Let $N = \langle W, T: S \to [0,1],\ I: S \to [0,1],\ F: S \to [0,1] \rangle$ be a NFSG, $A_N = \{A_1,..., A_k : [0, 1]^3 \to [0, 1]\}$ a set of aggregation functions and a function $Ag_N : S_\to \to A_N$. The pair $N_R = \langle N, AgN \rangle$ is called a Neutrosophic Reactive Graph (NRG).

Definition 11. Let $N = \langle W, T: S \to [0,1],\ I: S \to [0,1],\ F: S \to [0,1] \rangle$ be a NFSG, $A_M = \{A_1,..., A_k : [0,1]^3 \to [0, 1]\}$ a set of aggregation functions and a function $A_{g_N}: S_\to \to A_N$. The pair $N_R = \langle N, A_{g_N} \rangle$ is called a NRG. Given $a_i^0 \in S^0$, the reconfiguration of N_R after crossing a_i^0 is the NRG $N_R^{a_i^0} = \langle N_{Ag}^{a_i^0},\ A_g \rangle$ s.t.

$$T_{a_i^0}^{Ag}(a) = \begin{cases} A_g(a_i^0)(T(a_i^0), T(a_i^0, a), T(a)), & \text{if } (a_i^0, a) \in S \\ T(a), \text{othervise} \end{cases},$$

$$I_{a_i^0}^{Ag}(a) = \begin{cases} A_g(a_i^0)(I(a_i^0), I(a_i^0, a), I(a)), & \text{if } (a_i^0, a) \in S \\ I(a), \text{othervise} \end{cases},$$

$$F_{a_i^0}^{Ag}(a) = \begin{cases} A_g(a_i^0)(F(a_i^0), F(a_i^0, a), F(a)), & \text{if } (a_i^0, a) \in S \\ F(a), \text{othervise} \end{cases}.$$

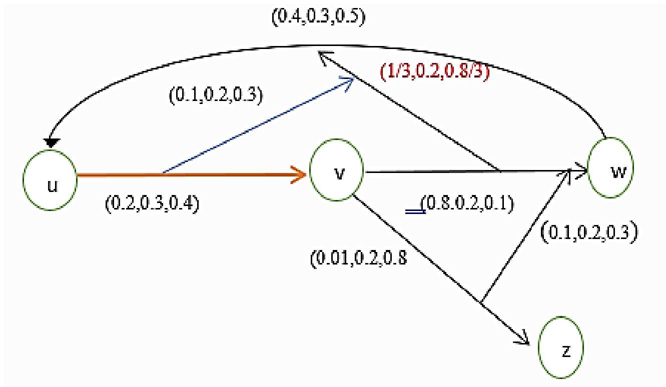
Example 2. Let N be the NSG in *Fig. 1.(b)*, consider $S^0 = \{a_1^0 = (u, v), a_2^0 = (v, w), a_3^0 = (v, z), a_4^0 = (w, u)\}$ and $A_T = \{arith, max\}$. Making $Ag(a_1^0) = arith$ and $A_g(a_2^0) = Ag(a_3^0) = max$,

$A_I = \{arith, max\}$. Making $Ag(a_1^0) = arith$ and $Ag(a_2^0) = Ag(a_3^0) = max$,

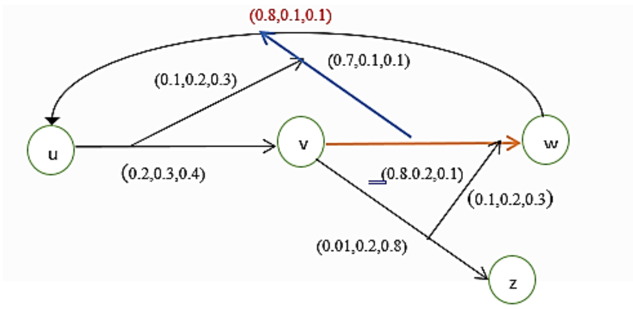
$A_F = \{arith, max\}$. Making $Ag(a_1^0) = arith$ and $Ag(a_2^0) = Ag(a_3^0) = min$,

the structure N_R is a NRG. *Fig. 3* contains $N_R^{a_1^0}$ and $N_R^{a_2^0}$, i.e. the reconfiguration of N_R after crossing a_1^0 and a_2^0, respectively.

Gabbay et al. [7] categorize the high-order arrows within switch graphs into two distinct types: connecting arrows and disconnecting arrows. As their labels imply, when a connecting/disconnecting arrow's source arrow is traversed, its target arrow becomes activated/deactivated. It's worth noting that high-order arrows within NSGs and NRGs do not possess this characteristic.



a.



b.

Fig. 3. a. reconfiguration of n_r after crossing a_1^0 $N_R^{a_1^0}$, and b. reconfiguration of n_r after crossing a_2^0 $N_R^{a_2^0}$.

Definition 12 (Reversal Neutrosophic Switch Graphs). Let W be a non-empty finite set of states or worlds, the following family of sets defined recursively:

$$\begin{Bmatrix} S^0 \subseteq W \times W, \\ S^{n+1} \subseteq S^0 \times S^n \times \{\bullet, \circ\} \end{Bmatrix},$$

s.t. $S^0 = \emptyset$ and for any $n \geq 1$, a_i^0 (, a, •) ∉ S^n or (a_i^0, a, ∘) ∉ S^n. Consider $S = \bigcup_{n \in N} S^n$, a RNSG is a 4- tuple $N = \langle W, T: S \rightarrow [0,1] \times \{ON, OFF\}, I: S \rightarrow [0,1] \times \{ON, OFF\}, F: S \rightarrow [0,1] \times \{ON, OFF\} \rangle$.

Arrows marked with • in their third component are termed as connecting arrows, while those with ∘ are referred to as disconnecting arrows.

In [1], the scenario where both a connecting and a disconnecting arrow operate simultaneously on the same arrow is avoided by selecting one type of high-order arrow as predominant. However, *Definition 12* adopts a different approach. It stipulates that within a RNSG, there cannot be a connecting and a disconnecting arrow, originating from the same source arrow, acting on the same target arrow. Visually, active arrows are depicted with solid lines, whereas inactive arrows are represented with dashed lines. Connecting arrows alter the state of the targeted arrow from active to inactive, while disconnecting arrows change it from inactive to active, distinguished by black and white arrowheads, respectively.

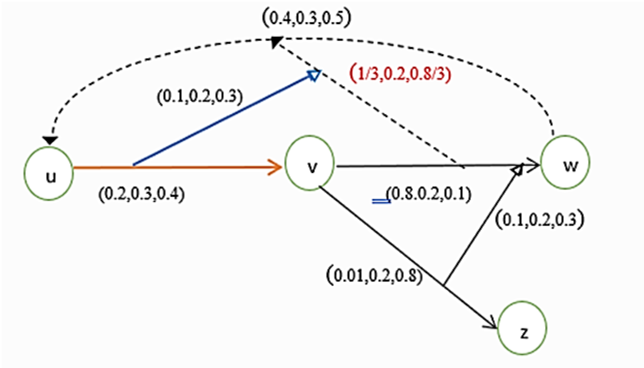


Fig. 4. An example of a RNSG with connecting and disconnecting high-order arrows.

Example 3. *Fig. 4* shows a RNSG.

Definition 13. Given N and an aggregation function A : $[0, 1]^3 \to [0, 1]$ and a RNSG with the structure

$N_A^{a_i^0}=\langle W,\ T_{a_i^0}^A: S \to [0,1] \times \{ON, OFF\},\ I_{a_i^0}^A: S \to [0,1] \times \{ON, OFF\}, F_{a_i^0}^A: S \to [0,1] \times \{ON, OFF\}\rangle$ based on A after crossing an active zero-order arrow a_i^0 s.t

$$T_{a_i^0}^A(a) = \begin{Bmatrix} \left(A(T_1(a_i^0), T_1(\llbracket a_i^0, a, \bullet \rrbracket), T_1(a), ON\right), & \text{if } \llbracket a_i^0, a, \bullet \rrbracket \in S^* \\ \left(A(T_1(a_i^0), T_1(\llbracket a_i^0, a, \circ \rrbracket), T_1(a), OFF\right), & \text{if } \llbracket a_i^0, a, \circ \rrbracket \in S^* \\ T(a), \text{othervise} & \end{Bmatrix},$$

$$I_{a_i^0}^A(a) = \begin{Bmatrix} \left(A(I_1(a_i^0), I_1(\llbracket a_i^0, a, \bullet \rrbracket), I_1(a), ON\right), & \text{if } \llbracket a_i^0, a, \bullet \rrbracket \in S^* \\ \left(A(I_1(a_i^0), I_1(\llbracket a_i^0, a, \circ \rrbracket), I_1(a), OFF\right), & \text{if } \llbracket a_i^0, a, \circ \rrbracket \in S^* \\ I(a), \text{othervise} & \end{Bmatrix},$$

$$F_{a_i^0}^A(a) = \begin{Bmatrix} \left(A(F(a_i^0), F_1(\llbracket a_i^0, a, \bullet \rrbracket), F_1(a), ON\right), & \text{if } \llbracket a_i^0, a, \bullet \rrbracket \in S^* \\ \left(A(F_1(a_i^0), F_1(\llbracket a_i^0, a, \circ \rrbracket), F_1(a), OFF\right), & \text{if } \llbracket a_i^0, a, \circ \rrbracket \in S^* \\ I(a), \text{othervise} & \end{Bmatrix}.$$

$N_A^{a_i^0}$ is called reconfiguration of N, based on A, after crossing a^0.

Remark 2. The restructuring of an RNSG, denoted as N, results in another RNSG. Additionally, within the set S, arrows can possess a null fuzzy value. Visually, these arrows will only be depicted if there exists the potential for altering this value through a higher-order arrow.

Notation 1: to enhance the clarity of graph presentations and movements, we introduce the following conventions: Red arrows will indicate crossings over the graph. Blue arrows, of higher order, will depict actions on the graph configuration after a crossing. The initial crossed arrow will feature a single arrowhead, followed by a double arrowhead for the second crossing, and a triple arrowhead for the third, and so forth. In cases of repeated movements on the same arrow, the arrowheads will indicate the order of the last movement. For example, if the movement is made three times on the same arrow, graphically we will see only a red triple-headed arrow in the graph.

Example 4. *Fig. 5* shows the reconfiguration, $N_{[uv]}^{arith}$, of RNSG *N* in *Fig. 4*.

Note that the previously active arrow $\llbracket [vw], [wu], \bullet \rrbracket$ becomes inactive due the action of the disconnection arrow $\llbracket [uv], \llbracket [vw], [wu], \bullet \rrbracket, \circ \rrbracket$.

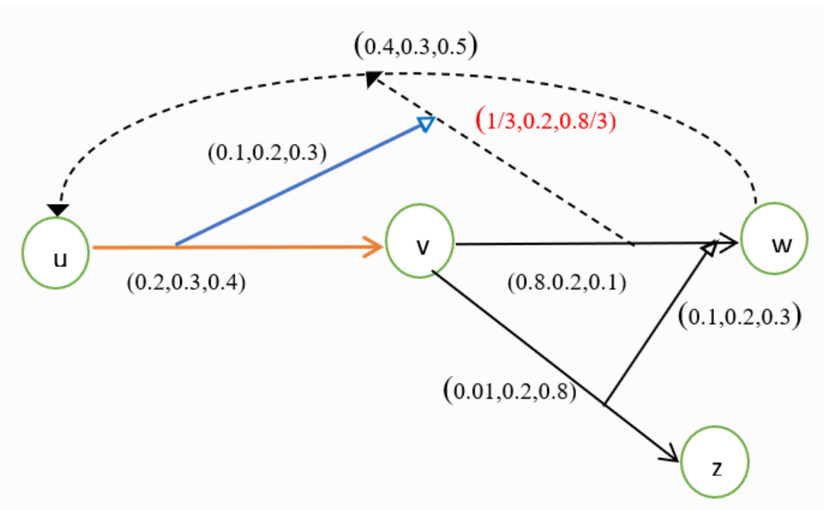


Fig. 5. Reconfiguration of the RNSG in Fig. 4 after crossing an active zero-order arrow and deactivation of a previously active arrow by a disconnecting arrow.

Definition 14 (Reversal Neutrosophic Reactive Graphs). Given N, the set of ternary aggregation functions $A = \{A_1,...,A_k : [0, 1]^3 \longrightarrow [0, 1]\}$ and a function A_g: $S \longrightarrow \longrightarrow A$. The pair $N_R = \langle N, Ag \rangle$ is called RNFRG. If $a_i^0 \in S0$ is an active arrow, the reconfiguration of NR after crossing a_i^0 is the NFRG $N_R^{a_i^0} = \langle N^{a_i^0}, A_g \rangle$ is the RNRG where $N^{a_i^0} = \langle W, T^{Ag}_{a_i^0}, F^{Ag}_{a_i^0}, I^{Ag}_{a_i^0} \rangle$ is the RNRG s.t.

$$T^{Ag}_{a_i^0}(a) = \begin{cases} \left(A_g(T_1(a_i^0)), T_1([\![a_i^0, a, \bullet]\!]), T_1(a), ON\right), & \text{if } [\![a_i^0, a, \bullet]\!] \in S^* \\ \left(A_g(T_1(a_i^0)), T_1([\![a_i^0, a, \circ]\!]), T_1(a), OFF\right), & \text{if } [\![a_i^0, a, \circ]\!] \in S^* \\ T(a), \text{othervise} \end{cases},$$

$$I^{Ag}_{a_i^0}(a) = \begin{cases} \left(A_g(I_1(a_i^0)), I_1([\![a_i^0, a, \bullet]\!]), I_1(a), ON\right), & \text{if } [\![a_i^0, a, \bullet]\!] \in S^* \\ \left(A_g(I_1(a_i^0)), I_1([\![a_i^0, a, \circ]\!]), I_1(a), OFF\right), & \text{if } [\![a_i^0, a, \circ]\!] \in S^* \\ I(a), \text{othervise} \end{cases},$$

$$F^{Ag}_{a_i^0}(a) = \begin{cases} \left(A_g(F(a_i^0)), F_1([\![a_i^0, a, \bullet]\!]), F_1(a), ON\right), & \text{if } [\![a_i^0, a, \bullet]\!] \in S^* \\ \left(A_g(F_1(a_i^0)), F_1([\![a_i^0, a, \circ]\!]), F_1(a), OFF\right), & \text{if } [\![a_i^0, a, \circ]\!] \in S^* \\ I(a), \text{othervise} \end{cases}.$$

Remark 3. RNRGs generalizes RNSGs as they allow relating to different zero-order arrows, source of high-order arrows, different aggregations.

Example 5. Let be the RNSG N in *Fig. 6.*(a), we have $S^0 = \{a_1^0 = [uv], a_2^0 = [vw], a_3^0 = [vz], a_4^0 = [wu]\}$ and consider A = {arith, max}. Making $A_g(a_1^0)$ = arith and $A_g(a_2^0) = A_g(a_3^0)$ = max, the *Fig. 6.(b)* contains the result of crossing a_1^0 and then a_2^0, which we denote by $N_R^{a_1^0 a_2^0}$. In addition, the *Fig. 6.*(c) shows the reconfiguration of N after crossing just the arrow a_2^0 i.e., by $N_R^{a_2^0}$.

4 | Operations on Reversal Neutrosophic Switch Graphs

In what follow we present some operations in order to build a reactive system by combining two pre-existing reactive systems. The operations that follow will be built from arbitrary binary or ternary aggregations. In the following we consider:

I. the RNSGs N = $\langle W, T: S \to [0,1] \times \{ON, OFF\}, I: S \to [0,1] \times \{ON, OFF\}, F: S \to [0,1] \times \{ON, OFF\} \rangle$ P = $\langle V, T: S' \to [0,1] \times \{ON, OFF\}, I: S' \to [0,1] \times \{ON, OFF\}, F: S' \to [0,1] \times \{ON, OFF\} \rangle$.

II. The RNRGs $N_R = \langle N, Ag_N \rangle$ and $P_R = \langle P, Ag_P \rangle$ s.t., $A_N = \{A_{(1,N)}, A_{(2,N)}, \ldots, A_{(n,N)} : [0, 1]^3 \longrightarrow [0, 1]\}$ and $A_P = \{A_{(1,P)}, A_{(2,P)}, \ldots, A_{(m, P)}: [0, 1]^3 \longrightarrow [0, 1]\}$, are sets of aggregation functions and $A_{gN}: S\longrightarrow \longrightarrow A_N$, $A_{gP}: S' \longrightarrow \longrightarrow A_P$, are functions.

Given membership functions:

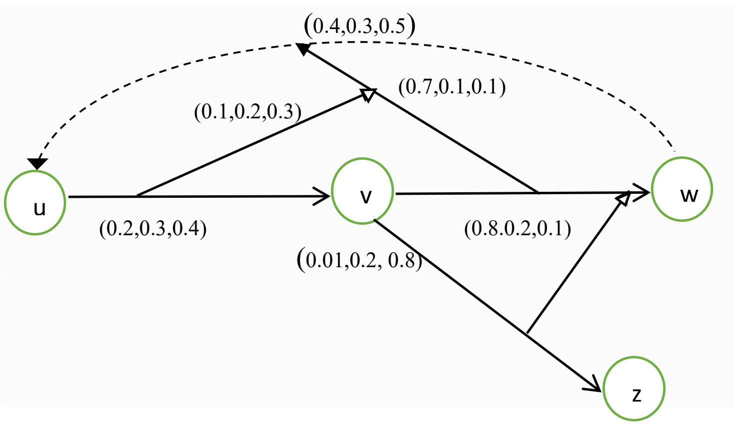
$G_T: X \longrightarrow [0, 1] \times \{ON, OFF\}$ and $H_T: Y \longrightarrow [0, 1] \times \{ON, OFF\}$,

$G_I: X \longrightarrow [0, 1] \times \{ON, OFF\}$ and $H_I: Y \longrightarrow [0, 1] \times \{ON, OFF\}$,

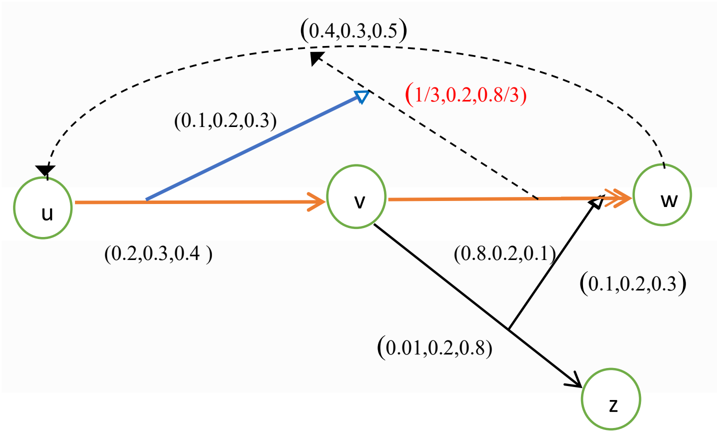
$G_F: X \longrightarrow [0, 1] \times \{ON, OFF\}$ and $H_F: Y \longrightarrow [0, 1] \times \{ON, OFF\}$,

We consider the sets:

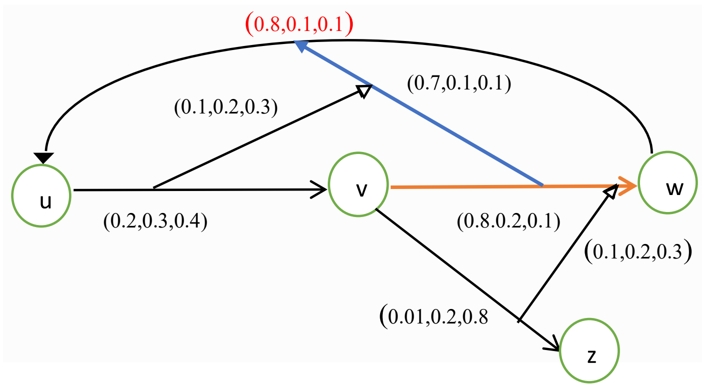
I. $(G_T \cap H_T)ON = \{a \in X \cap Y, G_{T2}(a) = H_{T2}(a) = ON\}$.
II. $(G_T \cap H_T)OFF = \{a \in X \cap Y, G_{T2}(a) = H_{T2}(a) = OFF\}$.
III. $(G_T \cap H_T)\neq = \{a \in X \cap Y, G_{T2}(a) \neq H_{T2}(a)\}$.
IV. $(G_I \cap H_I)ON = \{a \in X \cap Y, G_{I2}(a) = H_{I2}(a) = ON\}$.
V. $(G_I \cap H_I)OFF = \{a \in X \cap Y, G_{I2}(a) = H_{I2}(a) = OFF\}$.
VI. $(G_I \cap H_I)\neq = \{a \in X \cap Y, G_{I2}(a) \neq H_{I2}(a)\}$.
VII. $(G_F \cap H_F)ON = \{a \in X \cap Y, G_{F2}(a) = H_{F2}(a) = ON\}$.
VIII. $(G_F \cap H_F)OFF = \{a \in X \cap Y, G_{F2}(a) = H_{F2}(a) = OFF\}$.
IX. $(G_F \cap H_F)\neq = \{a \in X \cap Y, G_{F2}(a) \neq H_{F2}(a)\}$.



a.



b.



c.

Fig. 6. RNSG and its configurations.

4.1 | Δ-union and Δ-intersection of Reversal Neutrosophic Switch Graphs

Given an aggregation $\Delta: [0,1]^2 \longrightarrow [0,1]$, let be the following operations:

Definition 15. The structure $N \cup_\Delta P = \langle W \cup V, (T_1 \cup_\Delta T_2): S \cup_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\}, (I_1 \cup_\Delta I_2): S \cup_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\}, (F_1 \cup_\Delta F_2): S \cup_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\} \rangle$ s.t.

$$T_1 \cup_\Delta T_2 = \left\{ \begin{array}{l} T_1(a), \text{case } S - S' \\ T_2(a), \text{case } S' - S \\ \left(\Delta\left(T_1(a), T_2(a)\right), ON\right), \text{case } a \in (S \cap S')_{ON} \cup (S \cap S')_{\neq} \\ \left(\Delta\left(T_1(a), T_2(a)\right), OFF\right), \text{case } a \in (S \cap S')_{OFF} \end{array} \right\},$$

with N and P RNSGs, s.t, for any $n \geq 1$, $[\![a_i^0, a, \bullet]\!] \notin S^n \cup (S')^n$ or $[\![a_i^0, a, \circ]\!] \notin S^n \cup (S')^n$, is called Δ-union of N and P.

Definition 16. The structure $N \cap_\Delta P = \langle W \cap V, (T_1 \cap_\Delta T_2): S \cap_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\}, (I_1 \cap_\Delta I_2): S \cap_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\}, (F_1 \cap_\Delta F_2): S \cap_\Delta S' \longrightarrow [0,1] \times \{ON, OFF\} \rangle$ s.t.

$$T_1 \cap_\Delta T_2 = \begin{cases} \mathrm{T}_1(\mathrm{a}), \text{case } \mathrm{S}-\mathrm{S}' \\ \mathrm{T}_2(\mathrm{a}), \text{case } \mathrm{S}'-\mathrm{S} \\ (\Delta(T_1(a), T_2(a)), ON), \mathit{case}\ a \in (S \cap S')_{ON} \\ (\Delta(T_1(a), T_2(a)), OFF), \mathit{case}\ a \in (S \cap S')_{OFF} \cup (S \cap S')_{\neq} \end{cases},$$

is called Δ-intersection of N and P with N and P, RNSGs.

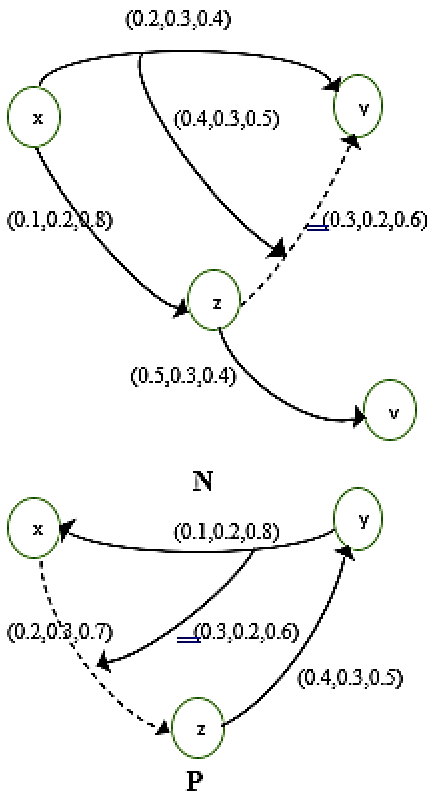


Fig. 7. RNSGs n and p.

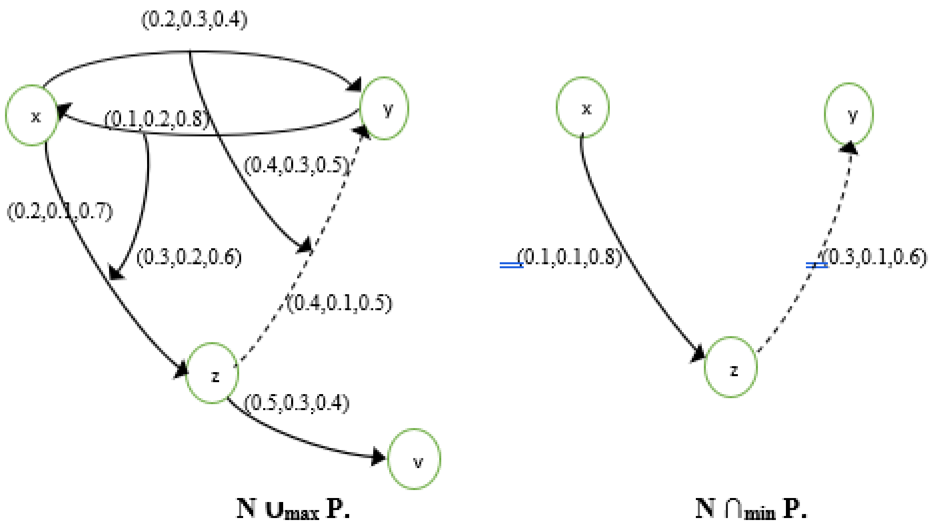


Fig. 8. Max-union and min-intersection of n and p.

Remark 4. The Δ-union and Δ-intersection represent RNSGs. Moreover, specifically addresses RNSGs N and P, where the Δ-union doesn't result in connection and disconnection arrows sharing the same source and target arrows. This condition ensures that the union adheres to *Definition 15*. The same requirement does not occur since, in order to $[\![a_i^0, a, \circ]\!]$ and $[\![a_i^0, a, \bullet]\!]$ be in the Δ-intersection, both must be in N and P, meaning that they are not RNSGs. It is not possible to have the case $W \cap V = \emptyset$.

Example 6. *Fig. 7* shows two RNSGs N and P, whereas the *Fig. 8* shows $N \cup_{max} P$ and $N \cap_{min} P$. In what follows we relate some properties of aggregations with properties of Δ- union and Δ -intersection.

Proposition 1. Given three RNSGs, N, P and $Q = \langle K, T: S'' \to [0,1] \times \{ON, OFF\}, I: S'' \to [0,1] \times \{ON, OFF\}, F: S'' \to [0,1] \times \{ON, OFF\}\rangle$. The following properties are satisfied:

I. $N \cup \Delta P = P \cup \Delta N$ and $N \cap \Delta P = P \cap \Delta N$ if is commutative.

II. $N \cup \Delta (P \cup \Delta Q) = (N \cup \Delta P) \cup \Delta Q$ and $N \cap \Delta (P \cap \Delta Q) = (N \cap \Delta P) \cap \Delta Q$ if Δ is commutative and associative.

III. $N \cup \Delta N = N$ and $N \cap \Delta N = N$ if Δ is idempotent.

Proof:

I. This proof is obvious.
II. $N \cup\Delta (P \cup\Delta Q) = \langle V \cup (W \cup K), [T_1 \cup (T_2 \cup T_3)]: S \cup (S' \cup S'') \to [0,1] \times \{ON, OFF\}, [I_1 \cup (I_2 \cup I_3)]: S \cup (S' \cup S'')' \to [0,1] \times \{ON, OFF\}, [F_1 \cup (F_2 \cup F_3)]: S \cup (S' \cup S'') \to [0,1] \times \{ON, OFF\}\rangle$. We know that $V \cup (W \cup K) = (V \cup W) \cup K$. So, we just need show that $T_1 \cup (T_2 \cup T_3) = (T_1 \cup T_2) \cup T_3$, $I_1 \cup (I_2 \cup I_3) = (I_1 \cup I_2) \cup I_3$ and $F_1 \cup (F_2 \cup F_3) = (F_1 \cup F_2) \cup F_3$.

Case 1. $a \in S - (S' \cup_\Delta S'')$, then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = S(a) = [T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = S(a) = [I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a)$

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = S(a) = [F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a)$

Case 2. $a \in S' - (S \cup S'')$, then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = S'(a) = [T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = S'(a) = [I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a)$

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = S'(a) = [F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a)$

Case 3. $A \in S'' - (S \cap S')$, then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = S''(a) = [T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = S''(a) = [I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a)$

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = S''(a) = [F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a)$

Case 4. $A \in [(S \cap S') - S'']$, then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a)$

$= \Delta\ (T_1(a), (T_2 \cup_\Delta T_3)(a)) = \Delta\ (T_1(a), T_2(a))$

$= (T_1 \cap_\Delta T_2)(a) = ((T_1 \cup_\Delta T_2) \cup_\Delta T_3)(a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \Delta\ (I_1(a), (I_2 \cup_\Delta F_3)(a))$

$= ((I_1 \cup_\Delta I_2) \cup_\Delta I_3)(a)$

And $[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \Delta\ (F_1(a), (F_2 \cup_\Delta F_3)(a))$

$= ((F_1 \cup_\Delta F_2) \cup_\Delta F_3)(a)$

Then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), ON \rangle$

$= [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S \cap S') - S'']_{ON} \cup [(S \cap S') - S'']_{\neq}$ or

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), OFF \rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S \cap S') - S'']_{OFF}$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), ON \rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S \cap S') - S'']_{ON} \cup [(S \cap S') - S'']_{\neq}$ or

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), OFF \rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S \cap v) - S'']_{OFF}$ and

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), ON \rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S \cap S') - S'']_{ON} \cup [(S \cap S') - S'']_{\neq}$ or

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), OFF \rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S \cap S') - S'']_{OFF}$

Case $a \in [(S \cap S'') - S']$,

then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \Delta\ (T_1(a), (T_2 \cup_\Delta T_3)(a))$

$= \Delta\ (T_1(a), T_3(a))$

$= (T_1 \cap_\Delta T_3)(a)$

$= ((T_1 \cup_\Delta T_2) \cup_\Delta T_3)(a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \Delta\ (I_1(a), (I_2 \cup_\Delta I_3)(a))$

$= ((I_1 \cup_\Delta I_2) \cup_\Delta I_3)(a)$.

And $[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \Delta\ (F_1(a), (F_2 \cup_\Delta F_3)(a))$

$= ((F_1 \cup_\Delta F_2) \cup_\Delta F_3)(a)$

Then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), ON \rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S \cap S'') - S']_{ON} \cup [(S \cap S'') - S']_{\neq}$ or

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), OFF \rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S \cap S'') - S']_{OFF}$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), ON\rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S \cap S'') - S']_{ON} \cup [(S \cap S'') - S']_{\neq}$ or

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), OFF\rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S \cap S'') - S']_{OFF}$ and

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), ON\rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S \cap S'') - S']_{ON} \cup [(S \cap S'') - S']_{\neq}$ or

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), OFF\rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S \cap S'') - S']_{OFF}$

Case $a \in [(S' \cap S'') - S]$, then

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \Delta\ (T_1(a), (T_2 \cup_\Delta T_3)(a))$

$= \Delta\ (T_2(a), T_3(a))$

$= (T_2 \cap_\Delta T_3)(a)$

$= ((T_1 \cup_\Delta T_2) \cup_\Delta T_3)(a)$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \Delta\ (I_1(a), (I_2 \cup_\Delta I_3)(a))$

$= ((I_1 \cup_\Delta I_2) \cup_\Delta I_3)(a)$.

And $[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \Delta\ (F_1(a), (F_2 \cup_\Delta F_3)(a))$

$= ((F_1 \cup_\Delta F_2) \cup_\Delta F_3)(a)$

Then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), ON\rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S' \cap S'') - S]_{ON} \cup [(S' \cap S'') - S]_{\neq}$ or

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), OFF\rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [(S' \cap S'') - S]_{OFF}$

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), ON\rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S' \cap S'') - S]_{ON} \cup [(S' \cap S'') - S]_{\neq}$ or

$[I_1 \cup_\Delta (I_2 \cup_\Delta I_3)](a) = \langle [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a), OFF\rangle = [(I_1 \cup_\Delta I_2) \cup_\Delta I_3](a)$

If $a \in [(S' \cap S'') - S]_{OFF}$ and

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), ON\rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S' \cap_\Delta S'') - S]_{ON} \cup [(S' \cap_\Delta S'') - S]_{\neq}$ or

$[F_1 \cup_\Delta (F_2 \cup_\Delta F_3)](a) = \langle [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a), OFF\rangle = [(F_1 \cup_\Delta F_2) \cup_\Delta F_3](a)$

If $a \in [(S' \cap S'') - S]_{OFF}$

Case $a \in (S \cap S' \cap S'')$

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \Delta\ (T_1(a), (T_2 \cup_\Delta T_3)(a))$

$= \Delta\ (T_1(a), \Delta(T_2(a), T_3(a)))$

$= \Delta\ (\Delta(T_1(a), T_2(a), T_3(a))$ (Associative property)

$= \Delta\ ((T_1 \cup_\Delta T_2)(a), T_3(a))$

$= ((T_1 \cup_\Delta T_2) \cup_\Delta T_3)(a)$

Then $[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), ON\rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in [S \cap S' \cap S'']_{ON} \cup [S \cap S' \cap S'']_{\neq}$ or

$[T_1 \cup_\Delta (T_2 \cup_\Delta T_3)](a) = \langle [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a), OFF \rangle = [(T_1 \cup_\Delta T_2) \cup_\Delta T_3](a)$

If $a \in (S \cap S' \cap S'')_{OFF}$

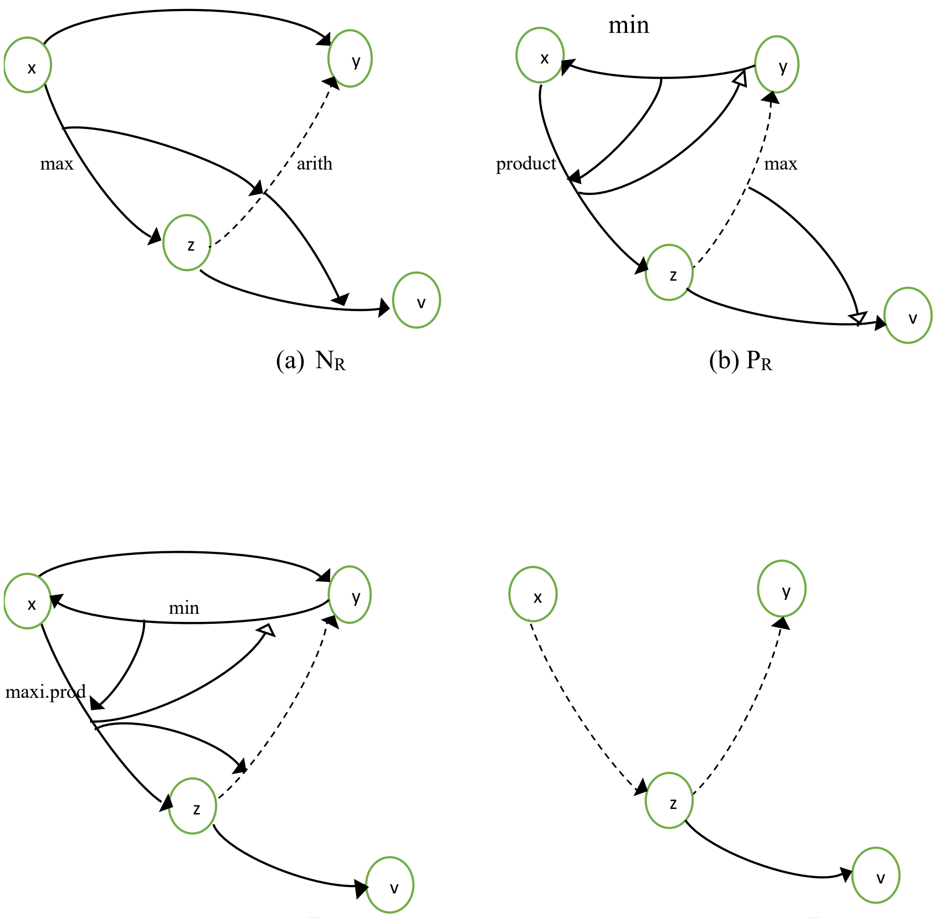
Similarly, we can prove $N \cap_\Delta (P \cap_\Delta Q) = (N \cap_\Delta P) \cap_\Delta Q$

Definition 17. Let be the RSNRGs N_R and P_R. The structure $N_R \cup_\Delta^\nabla P_R = \langle N \cup_\Delta P, Ag^\nabla_{N\cup_\Delta P} \rangle$ s.t the function $Ag^\nabla_{N\cup_\Delta P} : (S_\to \cup S'_\to) \to A_N \cup A_P \cup \{\pi_1 : [0,1]^3 \to [0,1]\}$, defined by

$$Ag^\nabla_{N\cup_\Delta P}(a_i^0) = \left\{ \begin{array}{c} Ag_N(a_i^0) \text{ if } a_i^0 \in S_\to - S'_\to \\ Ag_P(a_i^0) \text{ if } a_i^0 \in S'_\to - S_\to \\ \pi_1(\nabla\left(\left(Ag_N(a_i^0), Ag_P(a_i^0)\right), Ag_N(a_i^0), Ag_P(a_i^0)\right), \text{if } a_i^0 \in (S \cap S')^U_\to \end{array} \right\},$$

is called Δ^∇ - union of N_R and P_R.

The structure $N_R \cap_\Delta^\nabla P_R = \langle N \cap_\Delta P, Ag^\nabla_{N\cap_\Delta P} \rangle$ s.t the function $Ag^\nabla_{N\cap_\Delta P} : (S_\to \cap S'_\to) \to \{\pi_1 : [0,1]^3 \to [0,1]\}$, defined by $Ag^\nabla_{N\cap_\Delta P}(a_i^0) = \pi_1(\nabla\left(\left(Ag_N(a_i^0), Ag_P(a_i^0)\right), Ag_N(a_i^0), Ag_P(a_i^0)\right), \text{if } a_i^0 \in (S \cap S')^I_\to$, is called Δ^∇ - intersection of N_R and P_R.



(a) N_R (b) P_R

(c) $N_R \cup_\Delta^\nabla P_R$. **(d) $N_R \cap_\Delta^\nabla P_R$.**

Fig. 9. Δ^∇ -union and Δ^∇ -intersection of N_R and P_R.

The next proposition guarantees that the reconfiguration of an Δ^{∇}-union of RFRGs for arrows contained in only one of the graphs is equal to Δ^{∇}-union of its reconfigured components. *Fig. 10* shows the case that the edge [xy] has been crossed. For this example, consider Ag_{NE} ([xy]) = product and Δ = max. The demonstration of the following Proposition will be done by using the previous patterns for connecting and disconnecting arrows. For σ = ∘ and σ = •, we use OFF and ON, respectively. Succinctly, we will use OFF\ON for ∘ \•.

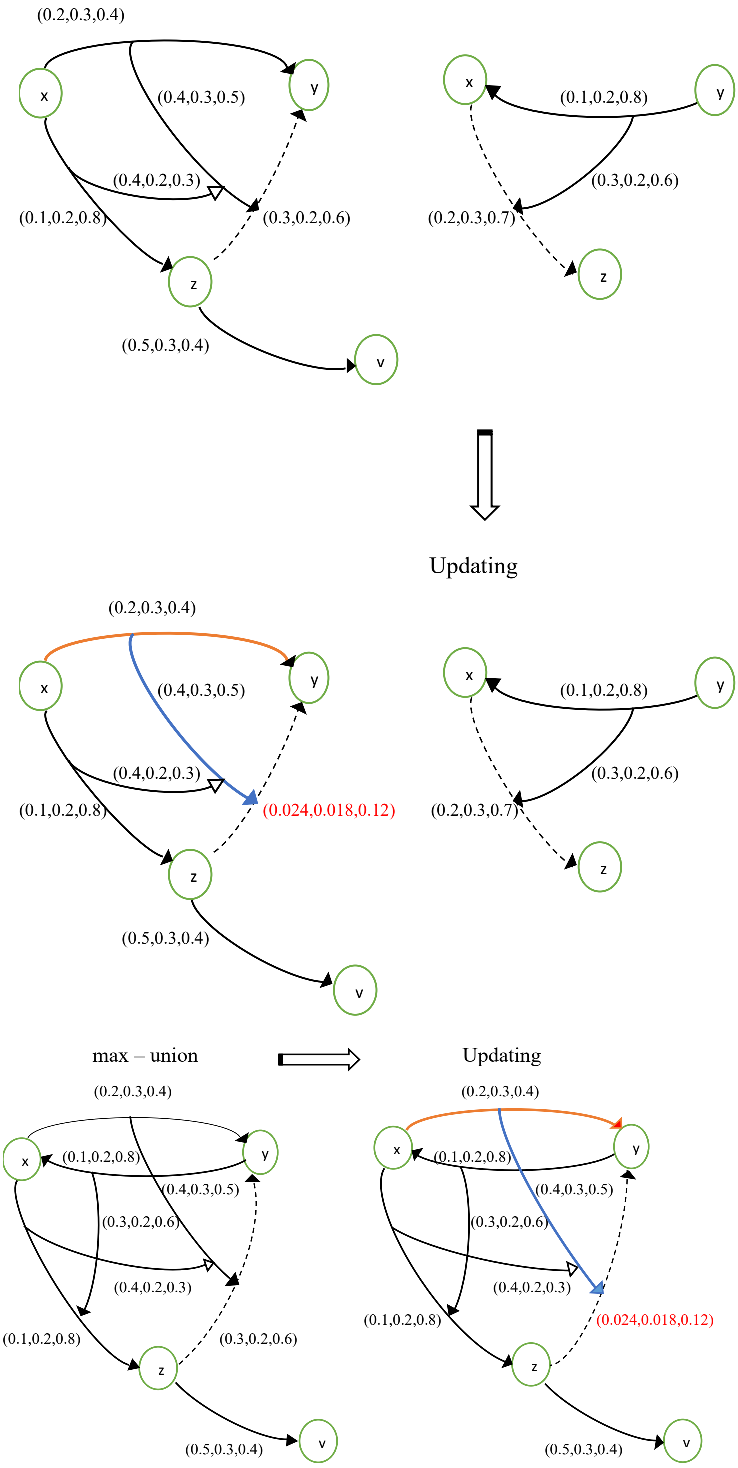


Fig. 10. Reconfiguration of the Δ-union of two RNRGs after crossing the arrow [xy] and the effect of aggregation functions on connecting and disconnecting arrows.

Proposition 2. Given $N_R \cup_{\Delta}^{\nabla} P_R$ and let an active arrow $a_i^0 \in S^0 - (S')^0$ ($(S')^0 - S^0$) and $a \in S - S'$ ($S' - S$), then:

$$(T_1 \cup_\Delta T_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = \left\{\begin{array}{c} T_1(a), \text{if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ T_2(a), \text{if } a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ \left(Ag(a_i^0)(T_1(a_i^0), T_1([\![a_i^0, a, \sigma]\!]), T_1(a)), OFF\backslash ON\right), \text{ if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \\ \left(Ag(a_i^0)(T_2(a_i^0), T_2([\![a_i^0, a, \sigma]\!]), T_2(a)), OFF\backslash ON\right), \text{ if} a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \end{array}\right\},$$

$$(I_1 \cup_\Delta I_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = \left\{\begin{array}{c} I_1(a), \text{if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ I_2(a), \text{if } a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ \left(Ag(a_i^0)(I_1(a_i^0), I_1([\![a_i^0, a, \sigma]\!]), I_1(a)), OFF\backslash ON\right), \text{ if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \\ \left(Ag(a_i^0)(I_2(a_i^0), I_2([\![a_i^0, a, \sigma]\!]), I_2(a)), OFF\backslash ON\right), \text{ if} a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \end{array}\right\},$$

$$(F_1 \cup_\Delta F_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = \left\{\begin{array}{c} F_1(a), \text{if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ F_2(a), \text{if } a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \text{ and } [\![a_i^0, a, \sigma]\!] \notin (S \cup S')^* \\ \left(Ag(a_i^0)(F_1(a_i^0), F_1([\![a_i^0, a, \sigma]\!]), F_1(a)), OFF\backslash ON\right), \text{ if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \\ \left(Ag(a_i^0)(F_2(a_i^0), F_2([\![a_i^0, a, \sigma]\!]), F_2(a)), OFF\backslash ON\right), \text{ if} a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \\ \text{and } [\![a_i^0, a, \sigma]\!] \in (S \cup S')^* \end{array}\right\}.$$

for $\sigma = \circ/\bullet$.

Proof:

Case 1. $[\![a_i^0, a, \sigma]\!] \notin (S \cup S')^*$, $(T_1 \cup_\Delta T_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = (T_1 \cup_\Delta T_2)(a)$, $(I_1 \cup_\Delta I_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = (I_1 \cup_\Delta I_2)(a)$ and $(F_1 \cup_\Delta F_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = (F_1 \cup_\Delta F_2)(a)$

Case 2. $[\![a_i^0, a, \sigma]\!] \in (S \cup S')^*$

$$(T_1 \cup_\Delta T_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = \left(Ag_{N \cup_\Delta P}^{\nabla}(a_i^0)\left((T_1 \cup_\Delta T_2)(a_i^0), (T_1 \cup_\Delta T_2)([\![a_i^0, a\ \sigma]\!]), (T_1 \cup_\Delta T_2)(a)\right), OFF\backslash ON\right)$$

$$\stackrel{\text{def}}{=} \left\{\begin{array}{c} \left(Ag_{N \cup_\Delta P}^{\nabla}(a_i^0)\,(a_i^0)(T_1(a_i^0), T_1([\![a_i^0, a, \sigma]\!]), T_1(a)), OFF\backslash ON\right), \text{if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \\ \left(Ag_{N \cup_\Delta P}^{\nabla}(a_i^0)\,(a_i^0)(T_2(a_i^0), T_2([\![a_i^0, a, \sigma]\!]), T_2(a)), OFF\backslash ON\right), \text{if} a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \end{array}\right\}$$

$$\stackrel{\text{def}}{=} \left\{\begin{array}{c} \left(Ag(a_i^0)(T_1(a_i^0), T_1([\![a_i^0, a, \sigma]\!]), T_1(a)), OFF\backslash ON\right), \text{if } a_i^0 \in (S^0 - (S')^0)^*, a \in S - S' \\ \left(Ag(a_i^0)(T_2(a_i^0), T_2([\![a_i^0, a, \sigma]\!]), T_2(a)), OFF\backslash ON\right), \text{if} a_i^0 \in ((S')^0 - S^0)^*, a \in S' - S \end{array}\right\}$$

$$(I_1 \cup_\Delta I_2)_{a_i^0}^{Ag_{N \cup_\Delta P}}(a) = \left(Ag_{N \cup_\Delta P}^{\nabla}(a_i^0)\left((I_1 \cup_\Delta I_2)(a_i^0), (I_1 \cup_\Delta I_2)([\![a_i^0, a, \sigma]\!]), (I_1 \cup_\Delta I_2)(a)\right), OFF\backslash ON\right)$$

$$\stackrel{\text{def}}{\equiv}\begin{Bmatrix}\left(Ag^{\nabla}_{N\cup_\Delta P}\,(a_i^0)\,(a_i^0)\left(I_1(a_i^0)\,,I_1([\![a_i^0\,,a\,,\sigma]\!]),I_1(a)\right),OFF\backslash ON\right), \text{if } a_i^0\in(S^0-(S')^0)^*, a\in\ S\ -\ S' \\ \left(Ag^{\nabla}_{N\cup_\Delta P}\,(a_i^0)\,(a_i^0)\left(I_2(a_i^0)\,,I_2([\![a_i^0\,,a\,,\sigma]\!]),I_2(a)\right),OFF\backslash ON\right), \text{if} a_i^0\ \in((S')^0-S^0)^*, a\in S'-S\end{Bmatrix}$$

$$\stackrel{\text{def}}{\equiv}\begin{Bmatrix}\left(Ag(a_i^0)\left(I_1(a_i^0)\,,I_1([\![a_i^0\,,a\,,\sigma]\!]),I_1(a)\right),OFF\backslash ON\right), \text{if } a_i^0\in(S^0-(S')^0)^*, a\in\ S\ -\ S' \\ \left(Ag(a_i^0)\left(I_2(a_i^0)\,,I_2([\![a_i^0\,,a\,,\sigma]\!]),I_2(a)\right),OFF\backslash ON\right), \text{if} a_i^0\ \in((S')^0-S^0)^*, a\in S'-S\end{Bmatrix}$$

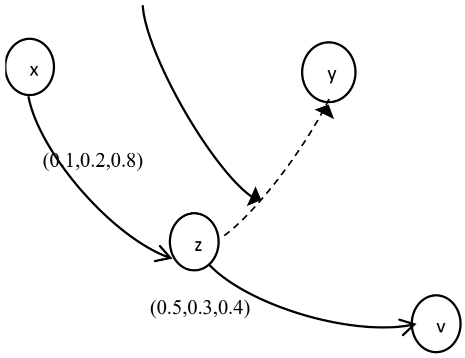
$$(F_1\cup_\Delta F_2)_{a_i^0}^{Ag\ N\cup_\Delta P}\,(a)=\left(Ag^{\nabla}_{N\cup_\Delta P}(a_i^0)\left((F_1\cup_\Delta F_2)(a_i^0)\,,(F_1\cup_\Delta F_2)([\![a_i^0\,,a\,,\ \sigma]\!]),(F_1\cup_\Delta F_2)(a)\right),OFF\backslash ON\right)$$

$$\stackrel{\text{def}}{\equiv}\begin{Bmatrix}\left(Ag^{\nabla}_{N\cup_\Delta P}\,(a_i^0)\,(a_i^0)\left(F_1(a_i^0)\,,F_1([\![a_i^0\,,a\,,\sigma]\!]),F_1(a)\right),OFF\backslash ON\right), \text{if } a_i^0\in(S^0-(S')^0)^*, a\in\ S\ -\ S' \\ \left(Ag^{\nabla}_{N\cup_\Delta P}\,(a_i^0)\,(a_i^0)\left(F_2(a_i^0)\,,F_2([\![a_i^0\,,a\,,\sigma]\!]),F_2(a)\right),OFF\backslash ON\right), \text{if} a_i^0\ \in((S')^0-S^0)^*, a\in S'-S\end{Bmatrix}$$

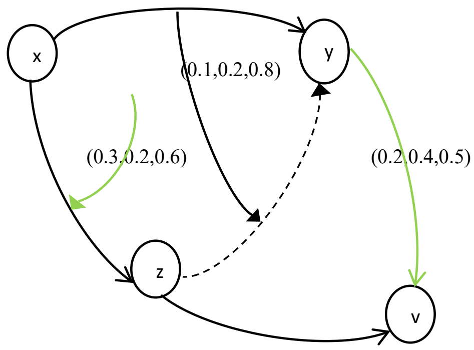
$$\stackrel{\text{def}}{\equiv}\begin{Bmatrix}\left(Ag(a_i^0)\left(F_1(a_i^0)\,,F_1([\![a_i^0\,,a\,,\sigma]\!]),F_1(a)\right),OFF\backslash ON\right), \text{if } a_i^0\in(S^0-(S')^0)^*, a\in\ S\ -\ S' \\ \left(Ag(a_i^0)\left(F_2(a_i^0)\,,F_2([\![a_i^0\,,a\,,\sigma]\!]),F_2(a)\right),OFF\backslash ON\right), \text{if} a_i^0\ \in((S')^0-S^0)^*, a\in S'-S\end{Bmatrix}.$$

for $\sigma = \circ/\bullet$.

The reconfiguration of the Δ-union of two NRSGs, for arrows in intersection, can be not equivalent to the Δ- union of their reconfigurations due the action of Δ. For these arrows, after the Δ-union, neutrosophic fuzzy values different from the original will be assigned and this fact promotes the non-validity of the above proposition. The *Proposition 2* it will also not be valid for arrows from the Δ-intersection between two graphs. Consider the min-intersection of the *Fig. 8(b)* which cannot even be updated.



a.



b.

Fig. 11. Extension of a RNSG; a. Original RNSG before extension, b. Extended RNSG with additional arrows and updated neutrosophic membership values.

4 | Extensions of Reversal Neutrosophic Switch Graphs and Reversal Neutrosophic Reactive Graphs

This section introduces an operation which enables to reuse a model in order to build a more complex one. An extension of a system N = $\langle W, T: S \rightarrow [0,1] \times \{\circ, \bullet\}, I: S \rightarrow [0,1] \times \{ON, OFF\}, F: S \rightarrow [0,1] \times \{ON, OFF\} \rangle$ involves a non-empty finite set of new arrows (S_E), a truth membership function ($T_E : S_E \rightarrow [0, 1] \times \{ON, OFF\}$), indeterminacy membership function $I_E: S \rightarrow [0,1] \times \{ON, OFF\}$, falsity membership function $F_E: S \rightarrow [0,1] \times \{ON, OFF\}$ and a finite set (possibly empty) of new nodes (W_E) to be connected to the original graph. For example, considering the RNSG N in *Fig. 11(a)*, the *Fig. 11(b)* shows its extension with $W_E = \emptyset$, $S_E = \{[yx], [yv], [yx], [xz], \bullet\}$, A membership function s.t. $T_E([yx]) = (0.1, ON)$, $I_E([yx]) = (0.2, ON)$, $F_E([yx]) = (0.8, ON)$,$T_E([yv]) = (0.2, ON)$, $I_E([yv]) = (0.4, ON)$, $F_E([yv]) = (0.5, ON)$, and $T_E([yx], [xz], \bullet) = (0.3, ON)$, $I_E([yx], [xz], \bullet) = (0.2, ON)$, $F_E([yx], [xz], \bullet) = (0.6, ON)$.

5 | Conclusion

Graph theory has widely used to model and study applications in different areas like Engineering, science and Biology. Fuzzy graphs can handle uncertain or vague situations occur in real life problems. neutrosophic fuzzy graphs have shown more advantages in solving such situations. Reversal Switch Graphs (RSGs) are structures designed to model reactive systems which provide the activation and deactivation of resources. In this paper I introduced neutrosophic FSGs and Reversal Neutrosophic Fuzzy Switch Graphs (RNFSG). Also some properties of RNFSGs have discussed. They are strongly based on the idea that the evolution of some systems results from the aggregation of previous information. In this study, three aggregation-based operations (union, intersection, and extension) are provided along with some revisions to the current theory. This research will extend to some real-life applications in Engineering.

Conflicts of Interest

The authors assert that they do not possess any identifiable conflicting financial interests or personal connections that may have seemed to impact the findings presented in this paper.

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