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The Randic Index in Neutrosophic Graphs and its Applications

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Abstract

In this paper introduces and investigates the Randic index for unclear graphs and neutrosophic subgraphs. It effective models in uncertainty and imprecision in edge and vertex relations. This study examines the Randic index's features for neutrosophic graphs and the impact of isomorphism between graphs. Furthermore, the notion is extended to directed neutrosophic graphs, offering a novel approach to analysing directed structures under uncertainty. The Randic index can help determine the best site for a waste management plant, as demonstrated in a real-world example. This approach emphasises the Randic index's practical importance in urban planning and infrastructure development, demonstrating its larger impact across several disciplines.

Keywords: Randic index, Neutrosophic graph, Randic index in neutrosophic graph.

1 | Introduction

Graph theory may help to address a wide range of real-world issues in computer applications, systems analysis, computer networks, transportation, operations research and economics. A graph is essentially a relational model that is used to describe real-world difficulties requiring relationships between entities. The graph's vertices and edges represent the items and their relationships, respectively. The information available in many optimization situations is inaccurate or inexact for a number of reasons, including information loss, a lack of supporting evidence, flawed statistical data, and inadequate information. In general, knowledge about a real-life scenario may be ambiguous.

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Cantor proposed the fundamental concept of classical graph theory. In a classical graph, each vertex or edge can have one of two outcomes: it is either part of the graph or not. Consequently, standard graphs cannot describe uncertain optimization situations. Real-world problems are typically unexpected, making it difficult to mimic them using conventional graphs. Zadeh [23] proposed fuzzy set theory. According to Smarandache's theory, neutrosophic sets are a useful mathematical tool for dealing with confusing, inconsistent, and incomplete data in the real world. Neutrosophic sets in the real standard or nonstandard unit interval [0, 1] are defined separately by a truth-membership function (t), indeterminacy-membership function (i) and falsity-membership function (f). Wang created the concept of single-valued neutrosophic sets, a subclass of neutrosophic sets, to help with the use of NS in practical applications. Satham Hussain [15] introduced the concepts of neutrosophic vague graphs and strong neutrosophic vague graphs to help motivate the prior study.

Nagoor Gani [9, 10] defines regular and irregular fuzzy graphs and discusses the features of neighborly regular and irregular fuzzy graphs. Then, Liangsong Huang [7] introduced a research of regular and irregular neutrosophic graphs, the two types of degree, d_m-regular, td_m-regular and m-highly irregular neutrosophic graphs, and certain features are presented.

Suriya [21] defines complex neutrosophic vague soft graphs, strong complex neutrosophic vague soft graphs, and constant complex neutrosophic vague soft graphs. Furthermore, after careful examination, we discovered some remarkable characteristics of the complement and self-complement of the complex neutrosophic vague soft graph.

Yahya [22] the Randic index of vague graphs and subgraphs are introduced along with their features. This study examines the top and lower bounds of the Randic index of vague graphs using isomorphic features. Randic indices for directed ambiguous graphs are introduced. Finally, an application of Randic index in building was given.

Vague graphs and neutrosophic graphs are sophisticated mathematical constructs that describe uncertainty in real-world networks. A vague graph is based on vague set theory, with each vertex or edge represented by a pair of truth-membership and false-membership degrees, whose sum cannot exceed one. The remaining implicitly denotes doubt or reluctance. Neutosophic graphs, on the other hand, take this notion a step further by incorporating three independent components: truth (T), indeterminacy (I) and falsity (F), with no restrictions on their total. This additional degree of flexibility enables neutrosophic graphs to better describe incomplete, inconsistent or contradictory information than vague graphs. While vague graphs are appropriate for moderate levels of uncertainty, where hesitation can be estimated using known truth and falsity values, neutrosophic graphs provide a richer and more flexible framework for highly uncertain or indeterminate environments, making them especially useful in fields such as artificial intelligence, decision-making systems and medical diagnosis.

We present the Randic index and neutrosophic subgraph features. This study examines a truth-membership function, indeterminacy-membership function and falsity-membership function of the Randic index in neutrosophic graphs using various isomorphic features. Randic indexes for neutrosophic graphs and neutrosophic digraphs are introduced then theorems as followed by example also explend. An application of Randic index in building is shown.

2 | Preliminaries

This section provides a basic description and example to support the key findings.

Definition 1. *Where $\mathfrak{P}_A : X \to [0,1]$ and $\mathfrak{R}_A : X \to [0,1]$ are true membership and falsehood membership functions, respectively, such that a vague set A on a non-empty set X is a pair ($\mathfrak{P}_A, \mathfrak{R}_A$).*

$$0 \leq \mathfrak{P}_A(x) + \mathfrak{R}_A(x) \leq 1$$

for every $x \in X$.
Consider two non-empty sets, X and Y. The criterion that a vague relation B of X to Y meets is that $R = (\mathfrak{P}_B, \mathfrak{R}_B)$, where $\mathfrak{P}_B : X \times Y \to [0,1]$ and $\mathfrak{R}_B : X \times Y \to [0,1]$ is a vague set B on $X \times Y$

$$0 \leq \mathfrak{P}_B(x,y) + \mathfrak{R}_B(x,y) \leq 1$$

for any $x, y \in X$.

Definition 2. *Consider the graph $G^* = (V, E)$. When $J = (\mathfrak{P}_J, \mathfrak{R}_J)$ is a vague set on V and $K = (\mathfrak{P}_K, \mathfrak{R}_K)$ is a vague set on $E \subseteq V \times V$, such that for each $xy \in E$,*

$$\mathfrak{P}_K(xy) \leq \min\{\mathfrak{P}_J(x), \mathfrak{P}_J(y)\} \text{ and } \mathfrak{R}_K(xy) \leq \min\{\mathfrak{R}_J(x), \mathfrak{R}_J(y)\}.$$

Definition 3. *A neutrosophic set A contained neutrosophic set B, i.e., $A \subseteq B$ if for all $x \in X$, $\mathfrak{P}_A(x) \leq \mathfrak{P}_B(x), \mathfrak{Q}_A(x) \geq \mathfrak{Q}_B(x)$ and $\mathfrak{R}_A(x) \geq \mathfrak{R}_B(x)$.*

Definition 4. *Let X be a space of point (objects) and let X represent the generic elements in X. There is only one neutrosophic set A in X and its truth membership function defines it as the $\mathfrak{P}_A(x)$, a membership function for indeterminacy $\mathfrak{Q}_A(x)$ and the membership function for falsehood $\mathfrak{R}_A(x)$.*
For each point x in X, $\mathfrak{P}_A(x), \mathfrak{Q}_A(x), \mathfrak{R}_A(x) \in [0, 1]$
$A = \{x, \mathfrak{P}_A(x), \mathfrak{Q}_A(x), \mathfrak{R}_A(x)\}$ and $0 \leq \mathfrak{P}_A(x) + \mathfrak{Q}_A(x) + \mathfrak{R}_A(x) \leq 3$.

Definition 5. *The Randic index of a neutrosophic graph $G = (S, K)$ is shown by $\mathfrak{RI}(G)$ and explained as follows:*

$$\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{P}}(G), \mathfrak{RI}_{\mathfrak{Q}}(G), \mathfrak{RI}_{\mathfrak{R}}(G))$$

where,

$$\mathfrak{RI}_{\mathfrak{P}}(G) = \sum_{i \neq j, c_i c_j \in E} \left(\mathfrak{P}_S(c_i)\mathfrak{P}_S(c_j)\mathbb{D}_{\mathfrak{P}}(c_i)\mathbb{D}_{\mathfrak{P}}(c_j)\right)^{-\frac{1}{2}}$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) = \sum_{i \neq j, c_i c_j \in E} \left(\mathfrak{R}_S(c_i)\mathfrak{R}_S(c_j)\mathbb{D}_{\mathfrak{R}}(c_i)\mathbb{D}_{\mathfrak{R}}(c_j)\right)^{-\frac{1}{2}}$$

where true and false parts of the node c_i degrees are $\mathbb{D}_{\mathfrak{P}}$ and $\mathbb{D}_{\mathfrak{R}}$, respectively.

3 | The Randic index in Neutrosophic Graphs

Definition 6. *The Randic index of a neutrosophic graph $G = (S, K)$ is shown by $\mathfrak{RI}(G)$ and explained as follows:*

$$\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{P}}(G), \mathfrak{RI}_{\mathfrak{Q}}(G), \mathfrak{RI}_{\mathfrak{R}}(G))$$

where,

$$\mathfrak{RI}_{\mathfrak{P}}(G) = \sum \frac{1}{\sqrt{(\mathfrak{P}_S(c_i)\mathfrak{P}_S(c_j)\mathbb{D}_{\mathfrak{P}}(c_i)\mathbb{D}_{\mathfrak{P}}(c_j))}}$$

$$\mathfrak{RI}_{\mathfrak{Q}}(G) = \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(c_i)\mathfrak{Q}_S(c_j)\mathbb{D}_{\mathfrak{Q}}(c_i)\mathbb{D}_{\mathfrak{Q}}(c_j))}}$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) = \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{R}_S(c_i)\mathfrak{R}_S(c_j)\mathbb{D}_{\mathfrak{R}}(c_i)\mathbb{D}_{\mathfrak{R}}(c_j))}}$$

that the true, indeterminacy and false parts of the node c_i degrees are $\mathbb{D}_{\mathfrak{P}}$, $\mathbb{D}_{\mathfrak{Q}}$ and $\mathbb{D}_{\mathfrak{R}}$, respectively.

Example 1. *Assume the neutrosophic graphs G as figure 1. Here, $\mathbb{D}(v) = (0.4, 1.5, 1.4)$, $\mathbb{D}(w) = (0.5, 1.5, 1.5)$, $\mathbb{D}(x) = (0.5, 2.4, 2.4)$, $\mathbb{D}(y) = (0.4, 2.5, 2.3)$ and $\mathbb{D}(z) = (0.3, 1.8, 1.5)$.*

$$\sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{P}_S(c_i)\mathfrak{P}_S(c_j)\mathbb{D}_{\mathfrak{P}}(c_i)\mathbb{D}_{\mathfrak{P}}(c_j))}} = 6.455 + 8.165 + 11.180 + 8.839 + 11.785 + 10.541$$
$$= 56.965$$

$$\sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(c_i)\mathfrak{Q}_S(c_j)\mathbb{D}_{\mathfrak{Q}}(c_i)\mathbb{D}_{\mathfrak{Q}}(c_j))}} = 1.260 + 0.813 + 0.657 + 1.017 + 0.831 + 0.813$$

$$= 4.578$$

$$\sum_{i\neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{R}_S(c_i)\mathfrak{R}_S(c_j)\mathbb{D}_{\mathfrak{R}}(c_i)\mathbb{D}_{\mathfrak{R}}(c_j))}} = 1.260 + 0.813 + 0.657 + 1.017 + 0.831 + 0.813$$

$$= 5.397$$

$$\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{P}}(G), \mathfrak{RI}_{\mathfrak{Q}}(G), \mathfrak{RI}_{\mathfrak{R}}(G)) = (56.965, 4.578, 5.391).$$

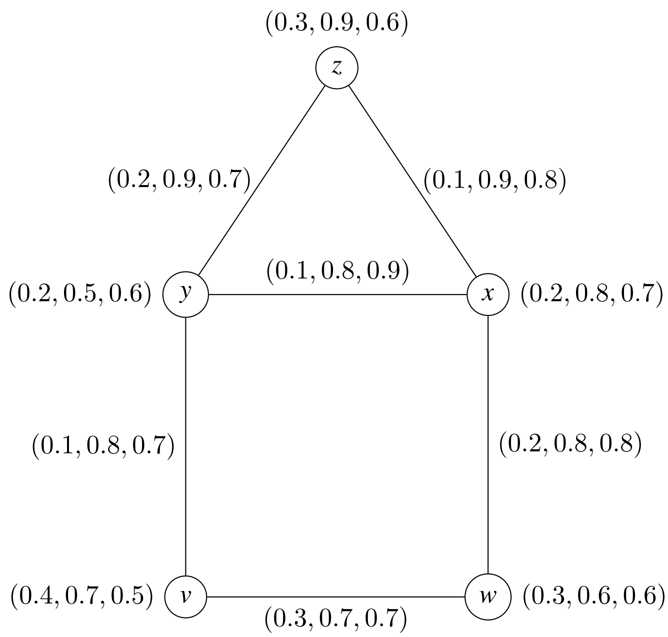

Fig. 1. Neutrosophic graph G.

Theorem 1. *Assume $G = (S, K)$ be a connected neutrosophic Graph and $G' = (S', K')$ such that $V' = V - \{x_n)\}, x_n \in V$ with $|V| = n$. Thus, $\mathfrak{RI}_{\mathfrak{P}}(G) \geq \mathfrak{RI}_{\mathfrak{P}}(G')$, $\mathfrak{RI}_{\mathfrak{Q}}(G) \geq \mathfrak{RI}_{\mathfrak{Q}}(G')$ and $\mathfrak{RI}_{\mathfrak{R}}(G) \geq \mathfrak{RI}_{\mathfrak{R}}(G')$.*

Proof: *Since G contains n notes i.e.,$V = \{x_1, x_2, ..., x_n\}$, $V = \{x_1, x_2, ..., x_{n-1}\}$. Hence V' must be a subset of V as so G' is a neutrosophic subgraph of G. Hence,*

$$\mathfrak{P}_S(x_i) = \mathfrak{P}_{S'}(x_i), \mathfrak{Q}_S(x_i) = \mathfrak{Q}_{S'}(x_i), \mathfrak{R}_S(x_i) = \mathfrak{R}_{S'}(x_i)$$

$$\mathfrak{P}_K(x_i x_j) = \mathfrak{P}_{K'}(x_i x_j), \mathfrak{Q}_K(x_i x_j) = \mathfrak{Q}_{K'}(x_i x_j), \mathfrak{R}_K(x_i x_j) = \mathfrak{R}_{K'}(x_i x_j)$$

for all $x_i \in V'$ and for all $x_i x_j \in E'$.

$$\mathbb{D}_{\mathfrak{P}}(x_i) = \sum \mathfrak{P}_K(x_i x_j), \mathbb{D}_{\mathfrak{Q}}(x_i) = \sum_{x_i x_j \in E} \mathfrak{Q}_K(x_i x_j), \mathbb{D}_{\mathfrak{R}}(x_i) = \sum_{x_i x_j \in E} \mathfrak{R}_K(x_i x_j)$$

i.e. $\mathbb{D}_{\mathfrak{P}}(x_i), \mathbb{D}_{\mathfrak{Q}}(x_i), \mathbb{D}_{\mathfrak{R}}(x_i)$ are sume of truth, indetetminacy and falsity membership values for edges occurring in x_i in G, respectively. Hence, $\mathbb{D}_{\mathfrak{R}}(x_i)\mathbb{D}_{\mathfrak{R}}(x_j)$ is positive real number. Similarly, we can show that $\mathbb{D}'_{\mathfrak{R}}(x_i)\mathbb{D}'_{\mathfrak{R}}(x_j)$ is also positive real number. Thus,

$$\frac{1}{\sqrt{(\mathfrak{P}_S(c_i)\mathfrak{P}_S(c_j)\mathbb{D}_{\mathfrak{P}}(c_i)\mathbb{D}_{\mathfrak{P}}(c_j))}} \geq 0, \qquad \frac{1}{\sqrt{(\mathfrak{P}_{S'}(c_i)\mathfrak{P}_{S'}(c_j)\mathbb{D}'_{\mathfrak{P}}(c_i)\mathbb{D}'_{\mathfrak{P}}(c_j))}} \geq 0,$$

$$\frac{1}{\sqrt{(\mathfrak{Q}_S(c_i)\mathfrak{Q}_S(c_j)\mathbb{D}_{\mathfrak{Q}}(c_i)\mathbb{D}_{\mathfrak{Q}}(c_j))}} \geq 0, \qquad \frac{1}{\sqrt{(\mathfrak{Q}_{S'}(c_i)\mathfrak{Q}_{S'}(c_j)\mathbb{D}'_{\mathfrak{Q}}(c_i)\mathbb{D}'_{\mathfrak{Q}}(c_j))}} \geq 0,$$

$$\frac{1}{\sqrt{(\mathfrak{R}_S(c_i)\mathfrak{R}_S(c_j)\mathbb{D}_{\mathfrak{R}}(c_i)\mathbb{D}_{\mathfrak{R}}(c_j))}} \geq 0, \qquad \frac{1}{\sqrt{(\mathfrak{R}_{S'}(c_i)\mathfrak{R}_{S'}(c_j)\mathbb{D}'_{\mathfrak{R}}(c_i)\mathbb{D}'_{\mathfrak{R}}(c_j))}} \geq 0.$$

So,

$$\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{P}_S(c_i)\mathfrak{P}_S(c_j)\mathbb{D}_{\mathfrak{P}}(c_i)\mathbb{D}_{\mathfrak{P}}(c_j))}}\geq\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{P}_{S'}(c_i)\mathfrak{P}_{S'}(c_j)\mathbb{D}'_{\mathfrak{P}}(c_i)\mathbb{D}'_{\mathfrak{P}}(c_j))}}$$
$$\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{Q}_S(c_i)\mathfrak{Q}_S(c_j)\mathbb{D}_{\mathfrak{Q}}(c_i)\mathbb{D}_{\mathfrak{Q}}(c_j))}}\geq\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{Q}_{S'}(c_i)\mathfrak{Q}_{S'}(c_j)\mathbb{D}'_{\mathfrak{Q}}(c_i)\mathbb{D}'_{\mathfrak{Q}}(c_j))}}$$
$$\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{R}_S(c_i)\mathfrak{R}_S(c_j)\mathbb{D}_{\mathfrak{R}}(c_i)\mathbb{D}_{\mathfrak{R}}(c_j))}}\geq\sum_{1\leq i\neq j\geq a, c_ic_j\in E}\frac{1}{\sqrt{(\mathfrak{R}_{S'}(c_i)\mathfrak{R}_{S'}(c_j)\mathbb{D}'_{\mathfrak{R}}(c_i)\mathbb{D}'_{\mathfrak{R}}(c_j))}}$$

Therefore, $\mathfrak{RI}_{\mathfrak{P}}(G)\geq\mathfrak{RI}_{\mathfrak{P}}(G')$, $\mathfrak{RI}_{\mathfrak{Q}}(G)\geq\mathfrak{RI}_{\mathfrak{Q}}(G')$ *and* $\mathfrak{RI}_{\mathfrak{R}}(G)\geq\mathfrak{RI}_{\mathfrak{R}}(G')$.

Example 2. *Assume the neutrosophic subgraph* $G'=(S',K')$ *of figure 2 and neutrosophic graph G of figure 1. Clearly,*

$$\mathfrak{P}_S(x_i)\geq\mathfrak{P}_{S'}(x_i),\mathfrak{Q}_S(x_i)\leq\mathfrak{Q}_{S'}(x_i),\mathfrak{R}_S(x_i)\leq\mathfrak{R}_{S'}(x_i),\forall x_i\in V'$$
$$\mathfrak{P}_K(x_ix_j)\geq\mathfrak{P}_{K'}(x_ix_j),\mathfrak{Q}_K(x_ix_j)\leq\mathfrak{Q}_{K'}(x_ix_j),\mathfrak{R}_K(x_ix_j)\leq\mathfrak{R}_{K'}(x_ix_j),\forall x_ix_j\in E'.$$

Hence, G' *is a neutrosophic subgraph of neutrosophic graph G.*

$$\mathfrak{RI}_{\mathfrak{P}}(G')=6.455+10.541+20.412+12.500=49.908$$
$$\mathfrak{RI}_{\mathfrak{Q}}(G')=1.029+0.932+0.988+1.091=4.04$$
$$\mathfrak{RI}_{\mathfrak{R}}(G')=1.260+0.966+0.936+1.220=4.382$$

Therefore, $\mathfrak{RI}_{\mathfrak{P}}(G)\geq\mathfrak{RI}_{\mathfrak{P}}(G')$, $\mathfrak{RI}_{\mathfrak{Q}}(G)\geq\mathfrak{RI}_{\mathfrak{Q}}(G')$ *and* $\mathfrak{RI}_{\mathfrak{R}}(G)\geq\mathfrak{RI}_{\mathfrak{R}}(G')$.

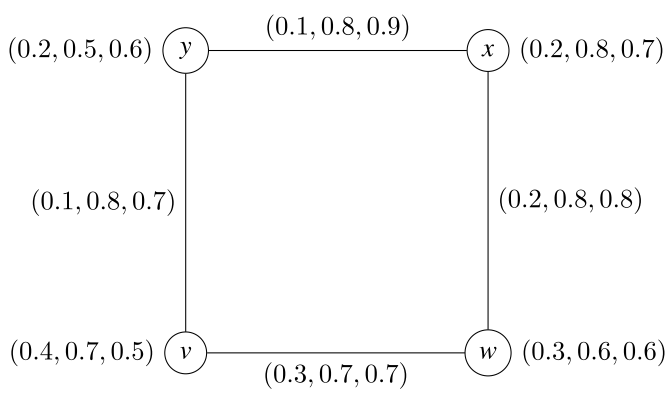

Fig. 2. Neutrosophic subgraph G'of the neutrosophic graph G of fig. 1.

Theorem 2. *Assume* $G=(S,K)$ *is a neutrosophic graphs of the graph* $G*=(V,E)$. *Then,* $\mathfrak{RI}_{\mathfrak{P}}(G)\geq\frac{b}{(a-1)}$, $\mathfrak{RI}_{\mathfrak{Q}}(G)\geq\frac{b}{(a-1)}$ *and* $\mathfrak{RI}_{\mathfrak{R}}(G)\geq\frac{b}{(a-1)}$, *where,* $a=|V|$ *and* $b=|E|$.

Proof: *Since G is a neutrosophic graph,* $0\leq\mathfrak{P}_S(b_i)\leq 1$, $0\leq\mathfrak{P}_S(b_j)\leq 1$, $0\leq\mathfrak{Q}_S(b_i)\leq 1$, $0\leq\mathfrak{Q}_S(b_j)\leq 1$, $0\leq\mathfrak{R}_S(b_i)\leq 1$, $0\leq\mathfrak{R}_S(b_j)\leq 1\ \forall\, b_i,b_j\in E$ *. So,*

$$\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\leq 1,\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\leq 1,\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\leq 1\qquad (i)$$

Since $|V|=a$, *each vertex is connected to at most* $(a-1)$ *vertices. But,* $\mathbb{D}_{\mathfrak{P}}(b_i)=\sum_{b_ib_j\in E}\mathfrak{P}(b_ib_j)$, $\mathbb{D}_{\mathfrak{Q}}(b_i))=\sum_{b_ib_j\in E}\mathfrak{Q}(b_ib_j)$, $\mathbb{D}_{\mathfrak{R}}(b_i))=\sum_{b_ib_j\in E}\mathfrak{R}(b_ib_j)$, *so* $\mathbb{D}_{\mathfrak{P}}(b_i)\leq a-1$, $\mathbb{D}_{\mathfrak{Q}}(b_i)\leq a-1$ *and* $\mathbb{D}_{\mathfrak{R}}(b_i)\leq a-1$, $\forall\, b_i\in V$. *This implies that*

$$\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j)\leq(a-1)^2,\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j)\leq(a-1)^2,\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j)\leq(a-1)^2\qquad (ii)$$

So, from (i) and (ii), we get:

$$\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j) \leq (a-1)^2,$$
$$\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j) \leq (a-1)^2,$$
$$\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j) \leq (a-1)^2.$$

Hence,

$$\frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j)}} \geq \frac{1}{(a-1)},$$
$$\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j)}} \geq \frac{1}{(a-1)},$$
$$\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j)}} \geq \frac{1}{(a-1)}.$$

Therefore,

$$\sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j)}} \geq \frac{b}{(a-1)},$$
$$\sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j)}} \geq \frac{b}{(a-1)},$$
$$\sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j)}} \geq \frac{b}{(a-1)}.$$

Therefore, $\mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) \geq \frac{b}{(a-1)}$, $\mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) \geq \frac{b}{(a-1)}$ *and* $\mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) \geq \frac{b}{(a-1)}$.

Example 3. *Assume the neutrosophic graph* G *in Example 1. Here,* $a = |V| = 5$, $b = |E| = 6$, $\mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) = 56.965, \mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) = 4.578, \mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) = 5.391$ *and* $\frac{b}{(a-1)} = \frac{6}{5} = 1.2$.
Therefore, $\mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) > \frac{b}{(a-1)}$, $\mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) > \frac{b}{(a-1)}$ *and* $\mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) > \frac{b}{(a-1)}$.

Theorem 3. *Assume* $G = (S, K)$ *be a strong neutrosophic graph so that* S *is a constant function and* $|V| = a$. *Then,* $\mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) \leq \frac{a}{2v_1^2}$, $\mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) \leq \frac{a}{2v_2^2}$ *and* $\mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) \leq \frac{a}{2v_3^2}$, *where,* $(v_1, v_2, v_3) = (\mathfrak{P}_S(b_i), \mathfrak{Q}_S(b_i), \mathfrak{R}_S(b_i))$, $b_i \in V$.

Proof: *Since a strong neutrosophic graph is generally a neutrosophic subgraph of the corresponding complete neutrosophic graph, using theorem 2, it is simple to prove that* $\mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) \leq \frac{a}{2v_1^2}$, $\mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) \leq \frac{a}{2v_2^2}$ *and* $\mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) \leq \frac{a}{2v_3^2}$.

Theorem 4. *Assume* G *be a complete neutrosophic graph so that* S *is a constant function. Then* $\mathfrak{R}\mathfrak{I}(G) = \left(\frac{a}{2v_1^2}, \frac{a}{2v_2^2}, \frac{a}{2v_3^2}\right)$, *where* $a = |V|$ *and* $(v_1, v_2, v_3) = (\mathfrak{P}_S(b_i), \mathfrak{Q}_S(b_i), \mathfrak{R}_S(b_i))$, $b_i \in V$.

Proof: *Since* S *is a constant and* $\mathfrak{P}_S(b_i) = v_1$, $\mathfrak{Q}_S(b_i) = v_2$, $\mathfrak{R}_S(b_i) = v_3$, $b_i \in V$. *Since* G *is complete neutrosophic graph,* $\mathfrak{P}_K(b_ib_j) = \{\mathfrak{P}_S(b_i) \wedge \mathfrak{P}_S(b_j)\}$, $\mathfrak{Q}_K(b_ib_j) = \{\mathfrak{Q}_S(b_i) \wedge \mathfrak{Q}_S(b_j)\}$ *and* $\mathfrak{R}_K(b_ib_j) = \{\mathfrak{R}_S(b_i) \wedge \mathfrak{R}_S(b_j)\}$, $\forall b_ib_j \in E$.

Since G *is complete and* $|V| = a$, *there are* $\frac{a(a-1)}{2}$ *pairs of vertices and* $\frac{a(a-1)}{2}$ *edges. Additionally, each vertices in* G *is adjacent to* $(a-1)$ *vertices. Then,*

$$\mathbb{D}_{\mathfrak{P}}(b_i) = \sum_{b_ib_j\in E} \mathfrak{P}_K(b_ib_j) = v_1 \cdot v_1 \cdots v_1(a-1 \ \ times) = (a-1)v_1$$

$$\mathbb{D}_{\mathfrak{Q}}(b_i) = \sum_{b_ib_j\in E} \mathfrak{Q}_K(b_ib_j) = v_2 \cdot v_2 \cdots v_2(a-1 \ \ times) = (a-1)v_2$$

$$\mathbb{D}_{\mathfrak{R}}(b_i) = \sum_{b_ib_j\in E} \mathfrak{R}_K(b_ib_j) = v_3 \cdot v_3 \cdots v_3(a-1 \ \ times) = (a-1)v_3$$

Therefore,

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{P}}(G) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j)}}\\ &= \frac{a(a-1)}{2}\cdot\frac{1}{\sqrt{(v_1v_1(a-1)v_1(a-1)v_1)}}\\ &= \frac{a}{2v_1^2}\end{aligned}$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{Q}}(G) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j)}}\\ &= \frac{a(a-1)}{2}\cdot\frac{1}{\sqrt{(v_2v_2(a-1)v_2(a-1)v_2)}}\\ &= \frac{a}{2v_2^2}\end{aligned}$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{R}}(G) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j)}}\\ &= \frac{a(a-1)}{2}\cdot\frac{1}{\sqrt{(v_3v_3(a-1)v_3(a-1)v_3)}}\\ &= \frac{a}{2v_3^2}\end{aligned}$$

Hence, $\mathfrak{RI}(G) = \left(\frac{a}{2v_1^2}, \frac{a}{2v_2^2}, \frac{a}{2v_3^2}\right)$.

Proposition 1. *If two neutrosophic graphs G and G' are isomorphic to each other, then,* $\mathfrak{RI}(G) = \mathfrak{RI}(G')$

Proof: *Assume $G = (S, K)$ and $G' = (S', K')$ be two isomorphic neutrosophic graphs. Then , there is a bijection $l : G \to G'$ so that*

$$\begin{aligned}\mathfrak{P}_S(b_i) &= \mathfrak{P}_{S'}(l(b_i)) = \mathfrak{P}_S(b_i'),\\ \mathfrak{Q}_S(b_i) &= \mathfrak{Q}_{S'}(l(b_i)) = \mathfrak{Q}_S(b_i'),\\ \mathfrak{R}_S(b_i) &= \mathfrak{R}_{S'}(l(b_i)) = \mathfrak{R}_S(b_i'),\end{aligned}$$

$\forall\ b_i \in V$ *and*

$$\begin{aligned}\mathfrak{P}_K(b_ib_j) &= \mathfrak{P}_{K'}(l(b_i)l(b_j)) = \mathfrak{P}_K(b_i'b_j'),\\ \mathfrak{Q}_K(b_ib_j) &= \mathfrak{Q}_{K'}(l(b_i)l(b_j)) = \mathfrak{Q}_K(b_i'b_j'),\\ \mathfrak{R}_K(b_ib_j) &= \mathfrak{R}_{K'}(l(b_i)l(b_j)) = \mathfrak{R}_K(b_i'b_j'),\end{aligned}$$

$\forall\ b_ib_j \in E.$
So, for each $b_i \in V$ there exists a vertex $b_i' \in V'$ so that $\mathbb{D}(b_i) = \mathbb{D}(b_i')$.
Thus, we have the proof.

Example 4. *Assume the neutrosophic graphs G and G' of figure 3. Here, $\mathfrak{P}_{S'}(l(b_i)) = \mathfrak{P}_S(a_i)$, $\mathfrak{Q}_{S'}(l(b_i)) = \mathfrak{Q}_S(a_i)$, $\mathfrak{R}_{S'}(l(b_i)) = \mathfrak{R}_S(a_i)$, $\mathfrak{P}_{K'}(l(b_i)l(b_j)) = \mathfrak{P}_K(a_ia_j)$, $\mathfrak{Q}_{K'}(l(b_i)l(b_j)) = \mathfrak{Q}_K(a_ia_j)$, $\mathfrak{R}_{K'}(l(b_i)l(b_j)) = \mathfrak{R}_K(a_ia_j)$, $\forall 1 \geq i, j \leq 3$. Here,*

$$\mathfrak{RI}_{\mathfrak{P}}(G) = 25.000 + 35.355 + 35.355 = 95.710 = \mathfrak{RI}_{\mathfrak{P}}(G')$$
$$\mathfrak{RI}_{\mathfrak{Q}}(G) = 1.207 + 1.304 + 1.102 = 3.613 = \mathfrak{RI}_{\mathfrak{Q}}(G')$$
$$\mathfrak{RI}_{\mathfrak{R}}(G) = 1.181 + 1.056 + 1.398 = 3.635 = \mathfrak{RI}_{\mathfrak{R}}(G')$$

Thus, $\mathfrak{RI}(G) = \mathfrak{RI}(G')$.

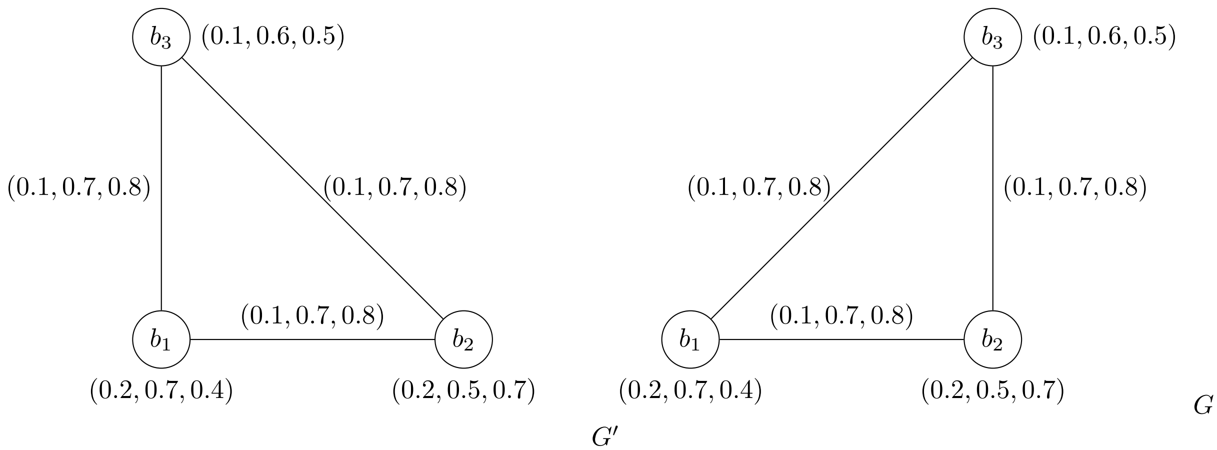

Fig. 3. Two isomorphic neutrosophic graph G and G' .

Theorem 5. *Assume $G = (S, K)$ be a neutrosophic so that*

$$\mu_1 = \min\{\mathbb{D}_{\mathfrak{P}}(b_i)\}, \mu_2 = \min\{\mathbb{D}_{\mathfrak{Q}}(b_i)\}, \mu_3 = \min\{\mathbb{D}_{\mathfrak{R}}(b_i)\}$$

$$\eta_1 = \max\{\mathbb{D}_{\mathfrak{P}}(b_i)\}, \eta_2 = \max\{\mathbb{D}_{\mathfrak{Q}}(b_i)\}, \eta_3 = \max\{\mathbb{D}_{\mathfrak{R}}(b_i)\}$$

$\forall b_i \in V$.
Then, $\frac{f_1}{\eta_1} \leq \mathfrak{R}\mathfrak{I}_{\mathfrak{P}}(G) \leq \frac{f_1}{\mu_1}$, $\frac{f_2}{\eta_2} \leq \mathfrak{R}\mathfrak{I}_{\mathfrak{Q}}(G) \leq \frac{f_2}{\mu_2}$ *and* $\frac{f_1}{\eta_3} \leq \mathfrak{R}\mathfrak{I}_{\mathfrak{R}}(G) \leq \frac{f_3}{\mu_3}$, *where*

$$f_1 = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}}, \qquad f_2 = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}},$$

$$f_3 = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}}.$$

Proof: *Since G is a neutrosophic graph and $b_i \in V$,*

$$\mathbb{D}_{\mathfrak{P}}(b_i) = \sum_{b_i b_j \in E} \mathfrak{P}_K(b_i b_j), \mathbb{D}_{\mathfrak{Q}}(b_i) = \sum_{b_i b_j \in E} \mathfrak{Q}_K(b_i b_j), \mathbb{D}_{\mathfrak{R}}(b_i) = \sum_{b_i b_j \in E} \mathfrak{R}_K(b_i b_j).$$

Again,

$$\mu_1 = \min\{\mathbb{D}_{\mathfrak{P}}(b_i)\}, \mu_2 = \min\{\mathbb{D}_{\mathfrak{Q}}(b_i)\}, \mu_3 = \min\{\mathbb{D}_{\mathfrak{R}}(b_i)\}$$

$$\eta_1 = \max\{\mathbb{D}_{\mathfrak{P}}(b_i)\}, \eta_2 = \max\{\mathbb{D}_{\mathfrak{Q}}(b_i)\}, \eta_3 = \max\{\mathbb{D}_{\mathfrak{R}}(b_i)\}$$

$\forall b_i \in V$, *so we have*

$$\mu_1 \leq \mathbb{D}_{\mathfrak{P}}(b_i) \leq \eta_1, \mu_2 \leq \mathbb{D}_{\mathfrak{Q}}(b_i) \leq \eta_2, \mu_3 \leq \mathbb{D}_{\mathfrak{R}}(b_i) \leq \eta_3,$$

$\forall b_i \in V$. *Then*

$$\mu_1^2 \leq \mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j) \leq \eta_1^2, \mu_2^2 \leq \mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j) \leq \eta_2^2, \mu_3^2 \leq \mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j) \leq \eta_3^2,$$

$\forall b_i, b_j \in V$. *Thus,*

$$\mathfrak{P}(b_i)\mathfrak{P}(b_j)\mu_1^2 \leq \mathfrak{P}(b_i)\mathfrak{P}(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j) \leq \mathfrak{P}(b_i)\mathfrak{P}(b_j)\eta_1^2,$$
$$\mathfrak{Q}(b_i)\mathfrak{Q}(b_j)\mu_2^2 \leq \mathfrak{Q}(b_i)\mathfrak{Q}(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j) \leq \mathfrak{Q}(b_i)\mathfrak{Q}(b_j)\eta_2^2,$$
$$\mathfrak{R}(b_i)\mathfrak{R}(b_j)\mu_3^2 \leq \mathfrak{R}(b_i)\mathfrak{R}(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j) \leq \mathfrak{R}(b_i)\mathfrak{R}(b_j)\eta_3^2,$$

$\forall b_i, b_j \in V$. *So,*

$$\sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\eta_1^2}} \leq \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j)}}$$
$$\leq \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mu_1^2}}$$

$$\sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\eta_2^2}} \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j)}} \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mu_2^2}}$$

$$\sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\eta_3^2}} \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j)}} \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mu_3^2}}$$

Hence,

$$\sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}}\cdot\frac{1}{\eta_1} \leq \mathfrak{RI}_{\mathfrak{P}}(G) \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}}\cdot\frac{1}{\mu_1}$$
$$\sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}}\cdot\frac{1}{\eta_2} \leq \mathfrak{RI}_{\mathfrak{Q}}(G) \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}}\cdot\frac{1}{\mu_2}$$
$$\sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}}\cdot\frac{1}{\eta_3} \leq \mathfrak{RI}_{\mathfrak{R}}(G) \leq \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}}\cdot\frac{1}{\mu_3}$$

Therefore, $\frac{f_1}{\eta_1} \leq \mathfrak{RI}_{\mathfrak{P}}(G) \leq \frac{f_1}{\mu_1}$, $\frac{f_2}{\eta_2} \leq \mathfrak{RI}_{\mathfrak{Q}}(G) \leq \frac{f_2}{\mu_2}$ *and* $\frac{f_1}{\eta_3} \leq \mathfrak{RI}_{\mathfrak{R}}(G) \leq \frac{f_3}{\mu_3}$.

Example 5. *Assume the neutrosophic graph G of Example 4. Here,* $V = \{b_1, b_2, b_3\}$, $E = \{b_1b_2, b_2b_3, b_1b_3\}$

$$\big(\mathbb{D}_{\mathfrak{P}}(b_1), \mathbb{D}_{\mathfrak{Q}}(b_1), \mathbb{D}_{\mathfrak{R}}(b_1)\big) = \big(\mathbb{D}_{\mathfrak{P}}(b_2), \mathbb{D}_{\mathfrak{Q}}(b_2), \mathbb{D}_{\mathfrak{R}}(b_2)\big) = \big(\mathbb{D}_{\mathfrak{P}}(b_3), \mathbb{D}_{\mathfrak{Q}}(b_3), \mathbb{D}_{\mathfrak{R}}(b_3)\big) = (0.2, 1.4, 1.6)$$

So,

$$\mu_1 = \eta_1 = 0.2, \mu_2 = \eta_2 = 1.4, \mu_3 = \eta_3 = 1.6$$
$$\mathfrak{RI}_{\mathfrak{P}}(G) = 95.710, \mathfrak{RI}_{\mathfrak{Q}}(G) = 3.613, \mathfrak{RI}_{\mathfrak{R}}(G) = 3.635$$

$$f_1 = \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}} = \frac{1}{\sqrt{0.2\times 0.2}} + \frac{1}{\sqrt{0.2\times 0.1}} + \frac{1}{\sqrt{0.1\times 0.2}} = 19.142$$

$$f_1 = \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}} = \frac{1}{\sqrt{0.7\times 0.5}} + \frac{1}{\sqrt{0.5\times 0.6}} + \frac{1}{\sqrt{0.6\times 0.7}} = 5.059$$

$$f_1 = \sum_{i\neq j, b_ib_j\in E}\frac{1}{\sqrt{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}} = \frac{1}{\sqrt{0.4\times 0.7}} + \frac{1}{\sqrt{0.7\times 0.5}} + \frac{1}{\sqrt{0.5\times 0.4}} = 5.816$$

So, $\frac{f_1}{\mu_1} = \frac{f_1}{\eta_1} = 95.71$, $\frac{f_2}{\mu_2} = \frac{f_2}{\eta_2} = 3.613$, $\frac{f_3}{\mu_3} = \frac{f_3}{\eta_3} = 3.635$.
Therefore,

$$\mathfrak{RI}_{\mathfrak{P}}(G) = \frac{f_1}{\mu_1} = \frac{f_1}{\eta_1} = 95.71$$

$$\mathfrak{RI}_{\mathfrak{Q}}(G) = \frac{f_2}{\mu_2} = \frac{f_2}{\eta_2} = 3.613$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) = \frac{f_3}{\mu_3} = \frac{f_3}{\eta_3} = 3.635.$$

Theorem 6. *Assume $G = (S, K)$ be a neutrosophic graph so that $|V| = a$. Then,*

$$\mathfrak{RI}_{\mathfrak{P}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}$$

$$\mathfrak{RI}_{\mathfrak{Q}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}.$$

Proof: *Since $|V| = a$, consider, every vertex of G is connected with at most $a-1$ other vertices. Therefore,*

$$\sum_{b_i b_j \in E} \mathfrak{P}_K(b_i b_j) \leq (a-1)\mathfrak{P}_S(b_i)$$

$$\sum_{b_i b_j \in E} \mathfrak{Q}_K(b_i b_j) \leq (a-1)\mathfrak{Q}_S(b_i)$$

$$\sum_{b_i b_j \in E} \mathfrak{R}_K(b_i b_j) \leq (a-1)\mathfrak{R}_S(b_i).$$

So,

$$\mathbb{D}_{\mathfrak{P}}(b_i) \leq (a-1)\mathfrak{P}_S(b_i), \mathbb{D}_{\mathfrak{Q}}(b_i) \leq (a-1)\mathfrak{Q}_S(b_i), \mathbb{D}_{\mathfrak{R}}(b_i) \leq (a-1)\mathfrak{R}_S(b_i)$$

$\forall b_i \in V$. *Hence,*

$$\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j) \leq \mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)(a-1)\mathfrak{P}_S(b_i)(a-1)\mathfrak{P}_S(b_j)$$

$$\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j) \leq \mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)(a-1)\mathfrak{Q}_S(b_i)(a-1)\mathfrak{Q}_S(b_j)$$

$$\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j) \leq \mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)(a-1)\mathfrak{R}_S(b_i)(a-1)\mathfrak{R}_S(b_j)$$

$\forall b_i b_j \in E$. *So,*

$$\sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{P}_S(b_i)\mathfrak{P}_S b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j))}} \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}$$

$$\sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(b_i)\mathfrak{Q}_S b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j))}} \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}$$

$$\sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{R}_S(b_i)\mathfrak{R}_S b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j))}} \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}$$

Thus,

$$\mathfrak{RI}_{\mathfrak{P}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{P}_S(b_i)\mathfrak{P}_S(b_j)}$$

$$\mathfrak{RI}_{\mathfrak{Q}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{Q}_S(b_i)\mathfrak{Q}_S(b_j)}$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{R}_S(b_i)\mathfrak{R}_S(b_j)}.$$

Definition 7. *The degree of a vertex b_i in a neutrosophic digraph $\overrightarrow{G} = (S, \overrightarrow{K})$ is $\mathbb{D}(b_i) = (\mathbb{D}_{\mathfrak{P}}(b_i), \mathbb{D}_{\mathfrak{Q}}(b_i), \mathbb{D}_{\mathfrak{R}}(b_i))$, where*

$$\mathbb{D}_{\mathfrak{P}}(b_i) = \sum_{i \neq j} (\mathfrak{P}_K(\overrightarrow{b_i b_j}) + \mathfrak{P}_K(\overrightarrow{b_j b_i}))$$

$$\mathbb{D}_{\mathfrak{Q}}(b_i) = \sum_{i \neq j} (\mathfrak{Q}_K(\overrightarrow{b_i b_j}) + \mathfrak{Q}_K(\overrightarrow{b_j b_i}))$$

$$\mathbb{D}_{\mathfrak{R}}(b_i) = \sum_{i \neq j} (\mathfrak{R}_K(\overrightarrow{b_i b_j}) + \mathfrak{R}_K(\overrightarrow{b_j b_i})).$$

Example 6. *Assume the neutrosophic digraph $\overrightarrow{G}$ of figure 4. Here, $\big(\mathfrak{P}_S(b_i), \mathfrak{Q}_S(b_i), \mathfrak{R}_S(b_i)\big)$, $i = 1, 2, 3$.*

$$\mathbb{D}_{\mathfrak{P}}(b_1) = \mathbb{D}_{\mathfrak{P}}(b_2) = \mathbb{D}_{\mathfrak{P}}(b_3) = 0.6$$

$$\mathbb{D}_{\mathfrak{Q}}(b_1) = \mathbb{D}_{\mathfrak{Q}}(b_2) = \mathbb{D}_{\mathfrak{Q}}(b_3) = 3.0$$

$$\mathbb{D}_{\mathfrak{R}}(b_1) = \mathbb{D}_{\mathfrak{R}}(b_2) = \mathbb{D}_{\mathfrak{R}}(b_3) = 2.6$$

$$\mathbb{D}(b_1) = \mathbb{D}(b_2) = \mathbb{D}(b_3) = (0.6, 3.0, 2.6).$$

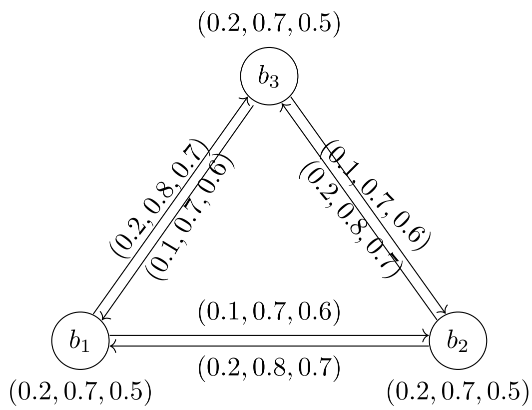

Fig. 4. A neutrosophic digraph →−G

Definition 8. *The Randic index of a neutrosophic digraph $\overrightarrow{G} = (S, \overrightarrow{K})$ is denoted by $\mathfrak{RI}(\overrightarrow{G})$ and is denoted as:*

$$\mathfrak{RI}(\overrightarrow{G}) = (\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G}))$$

$$= \begin{cases} \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{P}_S(b_i)\mathfrak{P}_S b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j))}}, \\ \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(b_i)\mathfrak{Q}_S b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j))}}, \\ \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{R}_S(b_i)\mathfrak{R}_S b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j))}}. \end{cases}$$

Example 7. *Consider the neutrosophic digraph $\overrightarrow{G}$ of example 6. We have:*

$$\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}) = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{P}_S(b_i)\mathfrak{P}_S b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j))}}$$

$$= 3 \cdot \frac{1}{\sqrt{(0.2 \times 0.2 \times 0.6 \times 0.6)}}$$

$$= 25$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(b_i)\mathfrak{Q}_S b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j))}}\\ &= 3\cdot\frac{1}{\sqrt{(0.7\times 0.7\times 3.0\times 3.0)}}\\ &= 1.429\end{aligned}$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G}) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{(\mathfrak{R}_S(b_i)\mathfrak{R}_S b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j))}}\\ &= 3\cdot\frac{1}{\sqrt{(0.5\times 0.5\times 2.6\times 2.6)}}\\ &= 2.308\end{aligned}$$

Hence, $\mathfrak{RI}(\overrightarrow{G}) = (\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G})) = (25, 1.429, 2.308)$.

Theorem 7. *Assume* $\overrightarrow{G} = (S, \overrightarrow{K})$ *be a neutrosophic digraph with* $|V| = b$ *so that* $S = (\mathfrak{P}_S, \mathfrak{Q}_S, \mathfrak{R}_S,)$ *is constant. If* $\mathbb{D} = (x_1, x_2, x_3)$, $\forall b_i \in V$, *then* $\mathfrak{RI}(\overrightarrow{G}) = b(\frac{1}{v_1x_1}, \frac{1}{v_2x_2}, \frac{1}{v_3x_3})$, *where* $(v_1, v_2, v_3) = (\mathfrak{P}_S(b_i), \mathfrak{Q}_S(b_i), \mathfrak{R}_S(b_i))$, $\forall b_i \in V$.
Proof: *Let the neutrosophic digraph* $\overrightarrow{G} = (S, \overrightarrow{K})$ *so that* $S = (\mathfrak{P}_S, \mathfrak{Q}_S, \mathfrak{R}_S,)$ *is constant,* $\mathbb{D} = (x_1, x_2, x_3)$ *and* $(v_1, v_2, v_3) = (\mathfrak{P}_S(b_i), \mathfrak{Q}_S(b_i), \mathfrak{R}_S(b_i))$, $\forall b_i \in V$.

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{(\mathfrak{P}_S(b_i)\mathfrak{P}_S b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j))}}\\ &= \frac{b}{\sqrt{v_1v_1x_1x_1}}\\ &= \frac{b}{v_1x_1}\end{aligned}$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(b_i)\mathfrak{Q}_S b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j))}}\\ &= \frac{b}{\sqrt{v_2v_2x_2x_2}}\\ &= \frac{b}{v_2x_2}\end{aligned}$$

$$\begin{aligned}\mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G}) &= \sum_{i\neq j, b_ib_j\in E} \frac{1}{\sqrt{(\mathfrak{R}_S(b_i)\mathfrak{R}_S b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j))}}\\ &= \frac{b}{\sqrt{v_3v_3x_3x_3}}\\ &= \frac{b}{v_3x_3}\end{aligned}$$

Therefore, $\mathfrak{RI}(\overrightarrow{G}) = (\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}), \mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G})) = b\big(\frac{1}{v_1x_1}, \frac{1}{v_2x_2}, \frac{1}{v_3x_3}\big)$.

Example 8. *Let the neutrosophic digraph $\overrightarrow{G}$ of example 6. Here, $|V| = b = 3$, degree of every vertex =(x_1, x_2, x_3)= (0.6,3.0,2.6), membership value of every vertex =$(v_1, v_2, v_3) = (0.2, 0.7, 0.5)$ and $\mathfrak{RI}(\overrightarrow{G}) = (25.000, 1.129, 2.308)$. Now,*

$$\frac{b}{v_1 x_1} = \frac{3}{0.2 \times 0.6} = 25.000$$
$$\frac{b}{v_2 x_2} = \frac{3}{0.7 \times 3.0} = 1.429$$
$$\frac{b}{v_3 x_3} = \frac{3}{0.5 \times 2.6} = 2.308$$

Therefore, theorem 6 is verified.

Theorem 8. *Assume $G = (S, K)$ be a regular and totally regular neutrosophic graph so that there is an edge between each pair of vertices and $|V| = a$. Then, $\mathfrak{RI}(G) = \frac{a(a-1)}{2}\big(\frac{1}{v_1x_1}, \frac{1}{v_2x_2}, \frac{1}{v_3x_3}\big)$, where $x_1 = \mathbb{D}_{\mathfrak{P}}(b_i)$, $x_2 = \mathbb{D}_{\mathfrak{Q}}(b_i)$, $x_3 = \mathbb{D}_{\mathfrak{R}}(b_i)$, $v_1 = \mathfrak{P}(b_i)$, $v_2 = \mathfrak{Q}(b_i)$, $v_3 = \mathfrak{R}(b_i)$, $\forall b_i \in V$.*

Proof: *Since G is regular, $\mathbb{D}_{\mathfrak{P}}(b_i) = x_1$, $\mathbb{D}_{\mathfrak{Q}}(b_i) = x_2$, $\mathbb{D}_{\mathfrak{R}}(b_i) = x_3$, $\forall b_i \in V$. Again, G is totally regular neutrosophic graph, so let $t\mathbb{D}_{\mathfrak{P}}(b_i) = h_1$, $t\mathbb{D}_{\mathfrak{Q}}(b_i) = h_2$, $t\mathbb{D}_{\mathfrak{R}}(b_i) = h_3$, $\forall b_i \in V$. We no that, $t\mathbb{D}_{\mathfrak{P}}(b_i) = \mathbb{D}_{\mathfrak{P}}(b_i) + \mathfrak{P}_S(b_i)$, $t\mathbb{D}_{\mathfrak{Q}}(b_i) = \mathbb{D}_{\mathfrak{Q}}(b_i) + \mathfrak{Q}_S(b_i)$, $t\mathbb{D}_{\mathfrak{R}}(b_i) = \mathbb{D}_{\mathfrak{R}}(b_i) + \mathfrak{R}_S(b_i)$, $\forall b_i \in V$.*

Thus,

$$v_1 = \mathfrak{P}_S(b_i) = t\mathbb{D}_{\mathfrak{P}}(b_i) - \mathbb{D}_{\mathfrak{P}}(b_i) = h_1 - x_1$$
$$v_2 = \mathfrak{Q}_S(b_i) = t\mathbb{D}_{\mathfrak{Q}}(b_i) - \mathbb{D}_{\mathfrak{Q}}(b_i) = h_2 - x_2$$
$$v_3 = \mathfrak{R}_S(b_i) = t\mathbb{D}_{\mathfrak{R}}(b_i) - \mathbb{D}_{\mathfrak{R}}(b_i) = h_3 - x_3$$

Hence, S is constant function and $v_1 = \mathfrak{P}_S(b_i)$, $v_2 = \mathfrak{Q}_S(b_i)$, $v_3 = \mathfrak{R}_S(b_i)$, $\forall b_i \in V$. Since $|V| = a$ and there is an edge between each pair of vertices in G, so there are $\frac{a(a-1)}{2}$ number of edges in G. Now

$$\mathfrak{RI}_{\mathfrak{P}}(\overrightarrow{G}) = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{P}_S(b_i)\mathfrak{P}_S b_j)\mathbb{D}_{\mathfrak{P}}(b_i)\mathbb{D}_{\mathfrak{P}}(b_j))}}$$
$$= \frac{a(a-1)}{2} \cdot \frac{1}{\sqrt{v_1 v_1 x_1 x_1}}$$
$$= \frac{a(a-1)}{2 v_1 x_1}$$

$$\mathfrak{RI}_{\mathfrak{Q}}(\overrightarrow{G}) = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(b_i)\mathfrak{Q}_S b_j)\mathbb{D}_{\mathfrak{Q}}(b_i)\mathbb{D}_{\mathfrak{Q}}(b_j))}}$$
$$= \frac{a(a-1)}{2} \cdot \frac{1}{\sqrt{v_2 v_2 x_2 x_2}}$$
$$= \frac{a(a-1)}{2 v_2 x_2}$$

$$\mathfrak{RI}_{\mathfrak{R}}(\overrightarrow{G}) = \sum_{i \neq j, b_i b_j \in E} \frac{1}{\sqrt{(\mathfrak{R}_S(b_i)\mathfrak{R}_S b_j)\mathbb{D}_{\mathfrak{R}}(b_i)\mathbb{D}_{\mathfrak{R}}(b_j))}}$$
$$= \frac{a(a-1)}{2} \cdot \frac{1}{\sqrt{v_3 v_3 x_3 x_3}}$$
$$= \frac{a(a-1)}{2 v_3 x_3}$$

Therefore, $\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{P}}(G), \mathfrak{RI}_{\mathfrak{Q}}(G), \mathfrak{RI}_{\mathfrak{R}}(G)) = \frac{a(a-1)}{2}\big(\frac{1}{v_1x_1}, \frac{1}{v_2x_2}, \frac{1}{v_3x_3}\big)$.

4 | Application of Randic index in construction

A government agency intends to construct a waste processing plant in an area that includes several medium-sized cities. The ideal location should have public and administrative backing, be easily accessible to neighboring cities for efficient waste transportation, have existing infrastructure (partial facilities, land, staff) and minimize environmental and logistical concerns.

The example demonstrates how to utilize a neutrosophic graph and the Randic index to select the best city for building a waste management plant. The selection process takes into account a variety of parameters, including public support, infrastructure availability, road connection and environmental concerns, which are represented as nodes (cities) and edges (roads) with neutrosophic weights signifying a truth, indeterminacy and falsity-membership function. Cities such as C1, C2, C3, C4 and C5 are examined based on their population numbers and imprecise a truth, indeterminacy and falsity-membership ratings. The Randic index, a graph-theoretic metric, is used to determine node centrality and significance.

Table 1. Weight of vertices in G.

G	City 1	City 2	City 3	City 4	City 5
$(\mathfrak{P}_S, \mathfrak{Q}_S, \mathfrak{R}_S)$	(0.1,0.5,0.6)	(0.4,0.8,0.5)	(0.3,0.6,0.7)	(0.2,0.7,0.8)	(0.2,0.5,0.4)

Table 2. Weight of edges in G.

G	C1-C2	C1-C4	C2-C3	C2-C4	C3-C4	C3-C5	C4-C5
$(\mathfrak{P}_S, \mathfrak{Q}_S, \mathfrak{R}_S)$	(0.1,0.8,0.7)	(0.1,0.8,0.8)	(0.3,0.8,0.8)	(0.2,0.9,0.9)	(0.1,0.9,0.8)	(0.2,0.7,0.8)	(0.1,0.7,0.9)

Table 3. The population of cities.

G	City 1	City 2	City 3	City 4	City 5
Population	72567	92566	132890	159463	69548

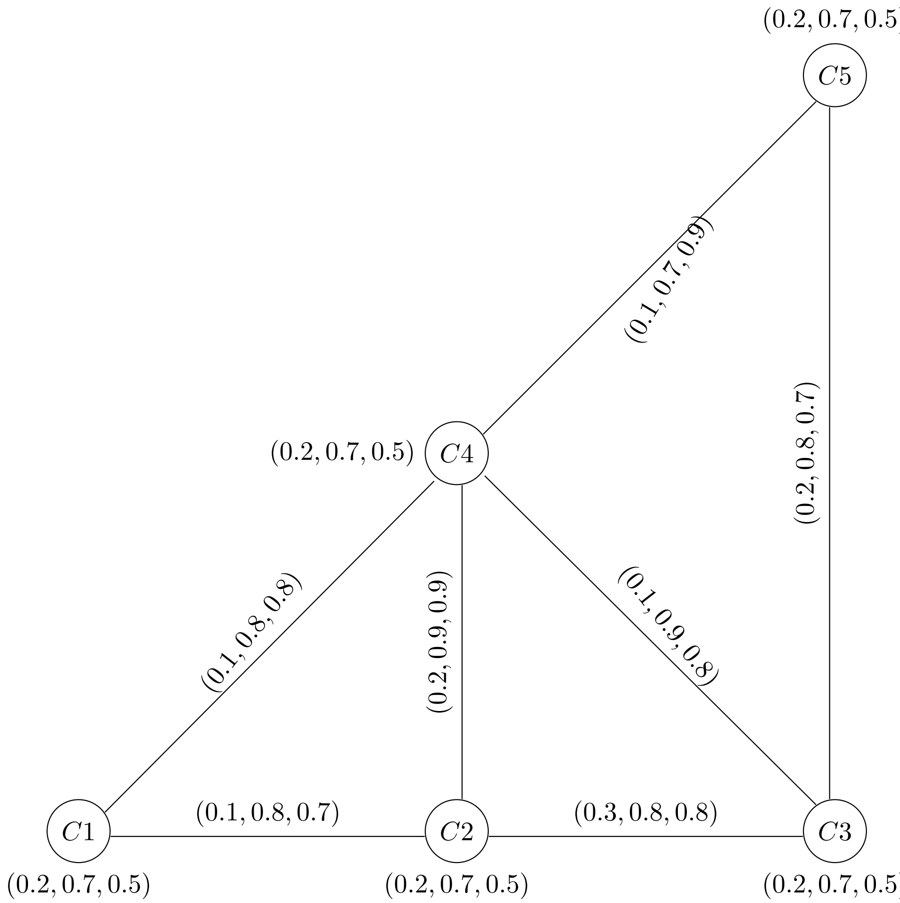

Fig. 5. Neutrosophic graph G.

$\mathbb{D}(C1) = (0.2.1.6, 1.5)$,$\mathbb{D}(C2) = (0.6, 2.5, 2.4)$,$\mathbb{D}(C3) = (0.6, 2.4, 2.4)$,$\mathbb{D}(C4) = (0.5, 3.3, 3.4)$ and $\mathbb{D}(C5) = (0.3, 1.4, 1.7)$.

$$\sum_{i\neq j, C_iC_j\in E} \frac{1}{\sqrt{(\mathfrak{P}_S(C_i)\mathfrak{P}_S(C_j)\mathbb{D}_{\mathfrak{P}}(C_i)\mathbb{D}_{\mathfrak{P}}(C_j))}} = 14.434 + 22.361 + 4.811 + 6.455 + 7.454 + 9.623 + 12.910$$

$$= 78.048$$

$$\sum_{i\neq j, C_iC_j\in E} \frac{1}{\sqrt{(\mathfrak{Q}_S(C_i)\mathfrak{Q}_S(C_j)\mathbb{D}_{\mathfrak{Q}}(C_i)\mathbb{D}_{\mathfrak{Q}}(C_j))}} = 0.747 + 0.736 + 0.589 + 0.465 + 0.548 + 0.996 + 0.786$$

$$= 4.867$$

$$\sum_{i\neq j, C_iC_j\in E} \frac{1}{\sqrt{(\mathfrak{R}_S(C_i)\mathfrak{R}_S(C_j)\mathbb{D}_{\mathfrak{R}}(C_i)\mathbb{D}_{\mathfrak{R}}(C_j))}} = 0.962 + 0.639 + 0.704 + 0.554 + 0.468 + 0.936 + 0.735$$

$$= 4.998$$

So,

$$\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{P}}(G), \mathfrak{RI}_{\mathfrak{Q}}(G), \mathfrak{RI}_{\mathfrak{R}}(G)) = (78.048, 4.867, 4.998)$$

Then,

$$\begin{aligned}
&\mathfrak{RI}_{\mathfrak{P}}(G - C1) = 78.048 - 12.434 - 22.361 = 41.250,\\
&\mathfrak{RI}_{\mathfrak{P}}(G - C2) = 78.048 - 14.434 - 4.811 - 6.455 = 52.348,\\
&\mathfrak{RI}_{\mathfrak{P}}(G - C3) = 78.048 - 4.811 - 7.454 - 9.623 = 56.160,\\
&\mathfrak{RI}_{\mathfrak{P}}(G - C4) = 78.048 - 22.361 - 6.455 - 7.454 - 12.910 = 28.868\\
&\mathfrak{RI}_{\mathfrak{P}}(G - C5) = 78.048 - 9.623 - 12.910 = 55.515.
\end{aligned}$$

$$\begin{aligned}
&\mathfrak{RI}_{\mathfrak{Q}}(G - C1) = 4.867 - 0.747 - 0.736 = 3.384,\\
&\mathfrak{RI}_{\mathfrak{Q}}(G - C2) = 4.867 - 0.747 - 0.589 - 0.465 = 3.066,\\
&\mathfrak{RI}_{\mathfrak{Q}}(G - C3) = 4.867 - 0.589 - 0.548 - 0.996 = 2.944,\\
&\mathfrak{RI}_{\mathfrak{Q}}(G - C4) = 4.867 - 0.736 - 0.465 - 0.548 - 0.786 = 2.602,\\
&\mathfrak{RI}_{\mathfrak{Q}}(G - C5) = 4.867 - 0.996 - 0.786 = 3.327.
\end{aligned}$$

$$\begin{aligned}
&\mathfrak{RI}_{\mathfrak{R}}(G - C1) = 4.998 - 0.962 - 0.639 = 3.397,\\
&\mathfrak{RI}_{\mathfrak{R}}(G - C2) = 4.998 - 0.962 - 0.704 - 0.554 = 2.778,\\
&\mathfrak{RI}_{\mathfrak{R}}(G - C3) = 4.998 - 0.704 - 0.468 - 0.936 = 2.890,\\
&\mathfrak{RI}_{\mathfrak{R}}(G - C4) = 4.998 - 0.639 - 0.554 - 0.468 - 0.735 = 2.602,\\
&\mathfrak{RI}_{\mathfrak{R}}(G - C5) = 4.998 - 0.936 - 0.735 = 3.327.
\end{aligned}$$

The computation above shows that avoid nodes C1, C2, C3 and C5, this isn't appropriate for constructing a waste processing plant. The most important and significant factor in preserving a high randix index is city C4, then the largest population and most transportation routes are found in city C4. the best location for the waste management facility is city C4.

5 | Conclusion

Neutrosophic graphs are more precise, flexible, and compatible than fuzzy graphs. Neutrosophic graphs are commonly used in social networks to identify the most effective individuals within a group or organisation. The Connectivity Index has numerous uses in psychology, medical science, social groupings, and computer networks. This work presents randic indexes for neutrosophic graphs and subgraphs, along with their properties. a truth, indeterminacy and falsity-membership function of the randic index of vague graphs are investigated using some isomorphic properties. Additionally, the randic index of directed neutrosophic graphs is introduced. In the future, we plan to explore Eccentricity index and wiener index for neutrosophic incidence graphs.

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Data Availability

This manuscript has no associated data.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings. Funders played no role in the study's design, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results.

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