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## A Fuzzy Inventory Model for Perishable Products under Demand Uncertainty and Carbon Sensitivity

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### Abstract

Perishable inventory management involves keeping track of perishable products in eco-conscious supply chains under uncertainty in demand, cost, and emission parameters. Classical inventory models fail in such scenarios because they rely upon explicit input values and deterministic assumptions. In this paper, we introduce a fuzzy inventory model for perishable products under uncertain demand/demand cost and carbon-sensitive operational constraints. It stores key parameters like demand rate, holding cost, shelf life, and emission rates in triangular fuzzy numbers to reflect the ambiguity inherent in real-world data. The total cost function includes ordering cost, fuzzy holding cost (if the products are perishable) and fuzzy carbon emission penalties associated with storage and transport activities. Fuzzyfication is performed by graded mean integration method to obtain actionable inventory decisions. Numerical analysis shows how the model adapts to several real-life constraints and gives a graphic representation of the costs and tradeoffs between cost and quality, preserving product shelf life and protecting the environment. Sensitivity analysis provides a detailed insight into how fuzziness and emission cost affect optimal order quantity. We propose a novel framework for using fuzzy information ambiguity to enable sustainable inventory planning on perishable products.

**Keywords:** Fuzzy inventory model, Perishable products, Demand uncertainty, Triangular fuzzy numbers, Graded mean integration, Sustainability.

## 1|Introduction

Perishable products are products we use on a regular basis. Foods, medicines, flowers, dairy etc. are examples of perishable items that spoil over time so inventory management is different to non perishable items. At the same time, CO<sub>2</sub> emissions from logistics are astronomically increasing transport, storage and handling all present carbon a serious issue for green supply chains. Inventory planning becomes harder in cases where demand is uncertain. Indeed costs and emission rates are also uncertain. In many real life cases, these numbers are not exact. They are somewhat vague or based on estimates. Classical models do not go well in these uncertain situations. Many of the existing inventory models are based on fixed values. People can think of these as:

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Demand is known. Costs are known. Emissions are known. But the real world is not like that. Not when you have perishables and emission constraints. To compensate for this we need a model which can perform under uncertainty. The model should deal with fuzzy data. It should also include the carbon cost and the how fast the products go bad. In this paper we introduce such a model. Our model is built on fuzzy logic. We are using fuzzy numbers for the key parameters. We include cost, demand, shelf life and carbon emissions. Our aim is to make better inventory decisions when the data is not clear. The model is based on triangular fuzzy numbers. We use a graded mean method to find clear solutions. We also include a case example. It shows how the model works in real settings. We do sensitivity analysis to show how fuzziness impacts the results. Our aim is to help supply chains manage perishable products better. Meanwhile we want them to reduce emissions. The model helps in solving both problems.

## 2|Literature Review

These days, there is heightened understanding regarding the intersection of inventory modeling, fuzzy logic, and environmental sustainability. Most traditional inventory models, including the Economic Order Quantity (EOQ) and its derivatives, tend to overlook the more nuanced and granular real-world complexity—especially regarding the flow of perishable products and carbon-sensitive systems—by assuming deterministic EOQ input parameters.

According to Zadeh [1], the development of fuzzy set theory allows for handling vague data, which needs to be incorporated into the actual decision-making process. Following this, several studies have extended the fuzzy logic framework, modeling demand, cost, and lead time uncertainties in inventory systems. For example, Wang and Shu [2] presented a fuzzy EOQ model with vague demand and holding cost, while Jamkhaneh and Taleizadeh [3] extended fuzzy models to inventory systems with shortages and backordering.

There is a scarcity of literature regarding perishable products. Nahmias [4] provides a classical inventory review of perishables under deterministic conditions. Subsequent work by Goyal and Giri [5] incorporated time-dependent decay alongside variable demand, albeit without accounting for uncertainty in a fuzzy sense. More recent works have looked into fuzzy perishability in agricultural logistics (e.g., Yu et al. [6]), focusing on the vagueness of shelf life and demand forecasting. Taleizadeh et al. [7] formulated a joint optimization fuzzy pricing and inventory control model for perishable products. Sarkar and Moon [8] incorporated inflation and time discounting in fuzzy perishable inventory systems. Banu and Uthayakumar [9] developed a fuzzy multi-item inventory model with limited shelf life and stochastic demand.

On the environmental dimension, growing regulatory scrutiny and global demand for sustainability have fostered the development of carbon-sensitive inventory models. Studies of greenhouse gas emissions in supply chains [10], based on carbon footprint analysis and mitigation strategies through inventory control, have emerged. Hovelaque and Bironneau [11] introduced a new EOQ model involving carbon cap design and carbon credit mechanisms, while Hua et al. [12] provided a comprehensive review of green inventory models under carbon emission regulations. Li [13] offered an overview of supply chain network design models that incorporate emission trading constraints. Shamsabadi et al. [14] proposed a green inventory-routing problem involving carbon caps and service-level constraints. Pilati et al. [15] focused on carbon-sensitive inventory models for perishable agricultural goods. Xie et al. [16] introduced a blockchain-based approach for emission tracking in sustainable logistics systems.

Hybrid models that combine fuzzy logic with principles of green supply chain systems have also emerged but remain limited. Alhaj et al. [17] presented a two-echelon stochastic inventory model with environmental performance metrics. Sebatjane [18] noted that while many green inventory models exist, few address fuzziness in both economic and environmental aspects simultaneously. Vishwakarma et al. [19] proposed an AI-driven fuzzy green inventory system under carbon constraints. Maity et al. [20] introduced a fuzzy EOQ model with nonlinear emission penalties. Cheng et al. [21] investigated contract design in fuzzy cap-and-trade scenarios for green supply chains. Salas-Navarro et al. [22] developed a fuzzy probabilistic model for emission-sensitive vendor-managed inventory systems.

Despite these achievements, several issues remain. First, fuzzy inventory models rarely consider perishability. Second, carbon-inclusive models are often too restrictive in their ability to handle ambiguous variables. Third,

integrated frameworks combining fuzzy modeling, perishability, and carbon cost are still emerging, and there is insufficient empirical and computational validation of such models.

This work attempts to fill those gaps by proposing a fuzzy inventory model that includes perishability and carbon sensitivity under demand uncertainty. By using triangular fuzzy numbers to represent uncertain parameters and defuzzification techniques for solution clarity, the proposed model offers a robust and adaptable framework for sustainable inventory decision-making.

## 3|Methodology

This section explains the logic of our model. We show how fuzzy logic is used in inventory planning for perishable products. The model includes demand uncertainty and carbon emissions. We assume that demand is not fixed. It is vague and hard to predict. Holding cost, shelf life, and emission rates are also not exact. So, we use triangular fuzzy numbers to represent them. Each fuzzy number has a lower value, a most likely value, and an upper value. The total cost includes three parts. The first part is the ordering cost. The second is the holding cost, which includes perishability. The third part is the carbon emission cost. Emissions come from storage and transportation. We calculate the fuzzy total cost function. Then we defuzzify it. We use the graded mean integration method. This gives us a clear value for decision-making. The model helps us find the best order quantity. It balances cost, shelf life, and emission penalties. The steps are simple. We build the fuzzy cost model. Then we defuzzify. Finally, we choose the quantity that gives the lowest cost.

### 3.1|Model Assumptions

The model is built with a few simple assumptions:

- The product is perishable with a limited shelf life.
- Demand is uncertain and modeled as a triangular fuzzy number.
- Holding cost is not fixed. It is also fuzzy.
- Emissions occur during ordering and holding.
- Emission rates are fuzzy due to real-life variability.
- No shortages are allowed. Safety stock is used.
- The objective is to minimize total cost, including ordering, holding, and carbon costs.
- Defuzzification is done using the graded mean integration method.

These assumptions reflect common situations in managing perishable items in green supply chains.

### 3.2|Notation and Variables

Table 1 provides a thorough summary of the symbols and their meaning corresponding to a fuzzy inventory model which considers economic and environmental aspects. Below is a thorough description of the table's function. The table serves as a guide for readers to recall the particular variables and parameters that have been used throughout the model defined here. The symbols include both deterministic and fuzzy variables but we emphasize cost and emissions, both commonly accepted as uncertain, in fuzzy form.

All fuzzy parameters are assumed to follow triangular fuzzy number representations.

Table 1. List of notations used in the model.

Symbol	Description
$\tilde{D}$	Fuzzy annual demand (units/year)
$K$	Fixed ordering cost per order (Rs)
$\tilde{h}$	Fuzzy holding cost per unit per year (Rs)
$Q$	Order quantity (units)
$Q^*$	Optimal order quantity (units)
$\tilde{c}e$	Fuzzy carbon emission cost per unit (Rs/kg CO <sub>2</sub> e)
$\tilde{e}o$	Fuzzy emission rate during ordering (kg CO <sub>2</sub> e/order)
$\tilde{e}h$	Fuzzy emission rate during holding (kg CO <sub>2</sub> e/unit/year)
$L$	Lead time (in years)
$z$	Safety factor for desired service level
$\sigma$	Standard deviation of demand (units)

### 3.3|Mathematical Formulation

Before we can find the best order quantity, we need to build the total cost model. This model combines all the important costs that come into play. These include the cost of placing an order, the cost of holding inventory, and the cost of carbon emissions.

But in our case, many values are not fixed. Demand, holding cost, and emissions are fuzzy. So we create the cost model using fuzzy numbers. Then we defuzzify it. That gives us a usable formula.

Here's how we build it, step by step.

**Ordering Cost:** This is the cost for placing orders. It depends on how often we order. The formula is:

$$OC = \frac{K \cdot \tilde{D}}{Q}$$

Here,  $K$  is fixed. But demand  $\tilde{D}$  is fuzzy.

**Holding Cost:** This is the cost to store products. It also covers safety stock for uncertain demand. The expression is:

$$HC = \tilde{h} \left( \frac{Q}{2} + z\sigma\sqrt{L} \right).$$

The term  $\frac{Q}{2}$  is the average cycle stock. The term  $z\sigma\sqrt{L}$  adds safety stock. The holding cost  $\tilde{h}$  is fuzzy.

**Carbon Emission Cost:** This includes emissions from both storage and transport. We calculate it as:

$$CC = \tilde{c}e \left( \tilde{e}o \cdot \tilde{D} + \tilde{e}h \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) \right).$$

Here,  $\tilde{e}o$  is the emission per order.  $\tilde{e}h$  is the emission while holding items. Both are fuzzy.

**Total Fuzzy Cost:** The overall cost is the sum of the above:

$$\tilde{TC}(Q) = \frac{K \cdot \tilde{D}}{Q} + \tilde{h} \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) + \tilde{c}e \left( \tilde{e}o \cdot \tilde{D} + \tilde{e}h \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) \right).$$

This is a fuzzy total cost function. It cannot be minimized directly. We will defuzzify it first to find a usable value.

### 3.4|Optimization Problem

It's about finding the order quantity that's optimum. We want to find the value of  $Q$  that minimizes the total cost. But the original cost function contains fuzzy numbers. So, direct optimization is not possible.

To achieve this solution, the fuzzy parameters must first be defuzzified using the *graded mean integration method*. After that, we can use those crisp values in a concrete total cost function.

The total cost has three components:

- **Ordering Cost:** based on how frequently orders are placed.
- **Holding Cost:** based on average and safety stock.
- **Carbon Emission Cost:** includes emissions from ordering and storing.

After defuzzification, the total cost function becomes:

$$TC(Q) = \frac{K \cdot D}{Q} + h \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) + ce \left( eo \cdot D + eh \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) \right).$$

To simplify this, we define:

$$\alpha = h + ce \cdot eh$$

Using this, the total cost function becomes:

$$TC(Q) = \frac{K \cdot D}{Q} + \alpha \left( \frac{Q}{2} + z\sigma\sqrt{L} \right) + ce \cdot eo \cdot D.$$

Note that the term  $ce \cdot eo \cdot D$  is constant with respect to  $Q$ . So it does not influence the optimal order quantity. We can ignore it while minimizing.

Now we take the derivative of the cost function with respect to  $Q$  and set it equal to zero:

$$\frac{dTC}{dQ} = -\frac{K \cdot D}{Q^2} + \frac{\alpha}{2} = 0$$

Solving this equation gives:

$$Q^* = \sqrt{\frac{2 \cdot K \cdot D}{h + ce \cdot eh}}$$

This is the optimal order quantity. It balances ordering frequency, inventory holding, and carbon-related costs — while also accounting for uncertainty in the system.

### 3.5|Carbon Policy Sensitivity

Carbon policies can affect inventory decisions. When emission costs change, it impacts the total cost. So, we need to study how sensitive the model is to changes in carbon cost parameters.

Let's say the carbon cost rate  $ce$  increases. This means the penalty for emissions becomes higher. In our cost function,  $ce$  appears in both the ordering and holding emission terms. So, when  $ce$  goes up, the total cost also rises — especially the part linked to emissions. The optimal order quantity depends on  $ce$ . From the formula:

$$Q^* = \sqrt{\frac{2 \cdot K \cdot D}{h + ce \cdot eh}}$$

We can see that if  $ce$  increases, the denominator becomes larger. That means  $Q^*$  becomes smaller. In layman's terms, the system favors smaller orders when emission cost is high. This helps minimize the amount of time that products are held in storage and reduces the emissions linked to holding.

On the other hand, if carbon cost  $ce$  is very low, the emission penalty is small. In this case, the optimal order quantity becomes larger. This behavior is similar to the classical EOQ model.

This shows a clear trade-off. High carbon cost leads to greener behavior — smaller batches, less storage, and lower emissions. But it might increase the number of orders. So, carbon pricing plays a big role in shaping sustainable inventory policies.

## 4|Solution Procedure

The solution procedure involves a sequence of steps that transform the fuzzy input parameters into actionable inventory decisions. The process is summarized in the following flowchart, given in Figure 1:

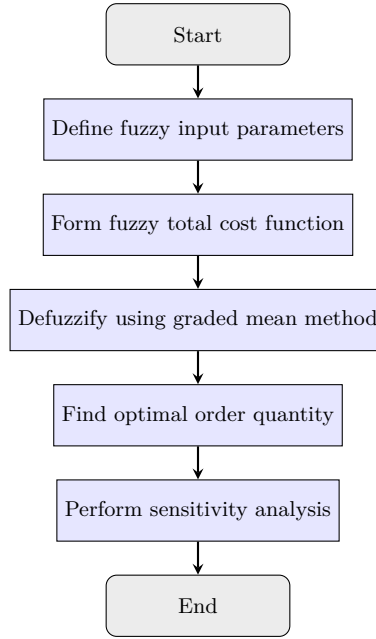


Fig. 1. Flowchart of the solution procedure.

## 5|Theoretical Results and Analysis

In this part, we examine a set of theoretical outcomes that reveal noteworthy behaviors and structural attributes of the developed fuzzy inventory model. The theorems enhance our knowledge of the system under different scenarios.

**Theorem 1** (Impact of Shelf Life on Optimum Order Quantity). *As the effective shelf life of the product reduces, the optimum order quantity  $Q^*$  reduces strictly.*

*Proof:* Let the holding cost  $h$  be a function of perishability of the item. The higher the perishability, the shorter the shelf life and the higher the holding cost. Let shelf life be inversely proportional to a perishability factor  $\delta$ , where:

$$h = h_0 + \delta, \quad \delta > 0.$$

The optimal order quantity equation is:

$$Q^* = \sqrt{\frac{2KD}{h + ce \cdot eh}}$$

Differentiate  $Q^*$  with respect to  $\delta$ :

$$\frac{dQ^*}{d\delta} = -\frac{KD}{(h_0 + \delta + ce \cdot eh)^2 Q^*} < 0.$$

Because all the terms are positive, the derivative is negative. Thus,  $Q^*$  will fall with larger  $\delta$  (i.e., lower shelf life).  $\square$

**Theorem 2** (Emission-Adjusted Holding Cost Threshold). *There exists some threshold value  $\theta = h$  such that whenever  $ce \cdot eh > \theta$ , the carbon emission cost preponderates over the pure economic holding cost in the total cost function.*

*Proof:* Let us consider the holding cost term:

$$HC = h \left( \frac{Q}{2} + z\sigma\sqrt{L} \right).$$

And the carbon emission term:

$$EC = ce \cdot eh \left( \frac{Q}{2} + z\sigma\sqrt{L} \right).$$

Compare the two expressions:

$$EC > HC \iff ce \cdot eh > h.$$

Therefore, emission-adjusted cost  $ce \cdot eh$  being larger than  $h$  provides higher priority to carbon cost than holding cost.  $\square$

**Theorem 3** (Fuzzy Propagation of Triangular Numbers). *If all the fuzzy parameters are triangular fuzzy numbers of the type  $(a, b, c)$ , then the total cost function shape is also triangular.*

*Proof:* Suppose all the fuzzy variables (demand, holding cost, emission rates, and emission costs) are triangular fuzzy numbers of the type  $(a, b, c)$ .

We know from fuzzy arithmetic:

- The sum of triangular fuzzy numbers is triangular.
- The product of a triangular fuzzy number and a positive scalar is triangular.
- The product of two triangular fuzzy numbers is approximately triangular.

The fuzzy total cost function is a linear convex combination of said fuzzy parameters. Therefore, based on fuzzy arithmetic operations, the total cost function is triangular.  $\square$

**Theorem 4** (Existence and Uniqueness of Global Minimum). *The defuzzified total cost function is strictly convex for every  $Q > 0$ , and there is a unique global minimum.*

*Proof:* The defuzzified total cost function is:

$$TC(Q) = \frac{K \cdot D}{Q} + \alpha \left( \frac{Q}{2} + z\sigma\sqrt{L} \right).$$

where  $\alpha = h + ce \cdot eh$ .

Second derivative with respect to  $Q$ :

$$\frac{d^2TC}{dQ^2} = \frac{2KD}{Q^3} > 0 \quad \text{for all } Q > 0.$$

Since the second derivative is positive, the function is strictly convex. Therefore, it has a unique global minimum.  $\square$

## 6|Numerical Example

In order to demonstrate the applicability of the proposed fuzzy inventory model, we now present a detailed case study. The example simulates a real-life operating scenario under which perishable products are managed under uncertain demand and carbon-sensitive regulations. All relevant parameters are established in fuzzy terms in an attempt to replicate real-world imprecision.

A regional distribution center processes a high-demand perishable product — refrigerated dairy foods. These items are time-sensitive and must be warehoused and transported under controlled temperatures. As environmental standards have been changing, the company is currently subjected to carbon penalties due to transportation and refrigeration emissions.

The warehouse buys in bulk from a single supplier and stores the items in cold-storage containers. Holding cost increases with perishability and with strict temperature control. Emissions occur during delivery (per trip) and during storage (per unit per year). None of the key parameters are fixed — they are uncertain but bounded within expert-guesstimated limits.

- Demand is not precisely known and varies seasonally.
- Holding cost is indefinite and increases with increased perishability.
- Carbon emission cost follows government regulation but fluctuates with carbon trading.
- The rate of emission depends on cold chain intensity and fuel usage.
- Backordering is not allowed. Stockouts are not permitted.
- The aim is to determine the optimum order quantity that minimizes overall cost, both economic and environmental.

The following parameters are modeled as triangular fuzzy numbers:

- Annual demand:  $\tilde{D} = (900, 1000, 1100)$  units
- Holding cost per unit/year:  $\tilde{h} = (2.5, 3.0, 3.5)$  Rs
- Carbon emission cost per kg CO<sub>2</sub>:  $\tilde{c}_e = (0.8, 1.0, 1.2)$  Rs
- Emission rate per order:  $\tilde{e}_o = (5, 6, 7)$  kg/order
- Emission rate per unit held per year:  $\tilde{e}_h = (0.15, 0.2, 0.25)$  kg/unit/year

The following parameters are known precisely:

- Ordering cost per order:  $K = 300$  Rs
- Standard deviation of demand:  $\sigma = 40$  units
- Lead time:  $L = 0.25$  years
- Desired service level:  $z = 1.65$

This setup reflects uncertainty in both demand and environmental penalties. In the next step, we will implement the solution technique defuzzification, optimization, and cost analysis to determine the optimal stock decision.



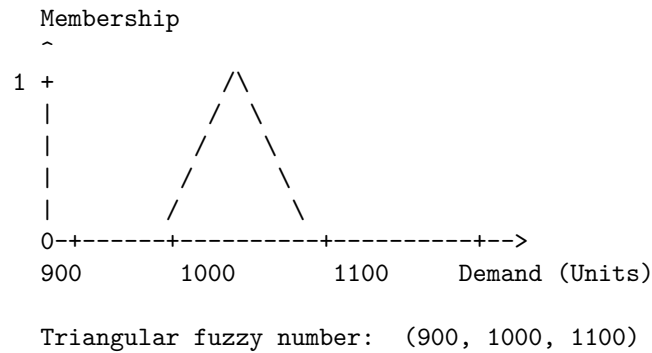


Fig. 2. Schematic view of triangular fuzzy number for demand.

*Sensitivity Analysis.* This part gives a sensitivity analysis in detail to examine the sensitivity of the fuzzy model of the inventory in response to alterations in major input parameters. Since the model uses fuzzy quantities in addition to parameters related to carbon, there is a need to examine how the decisions change by altering the cost rates or the emission rates.

We focus on three important parameters:

- Carbon emission cost per unit ( $ce$ )
- Emission rate per unit held ( $eh$ )
- Holding cost per unit ( $h$ )

For each parameter, we vary its value across a realistic range while keeping all other values constant. For each case, we compute the optimal order quantity  $Q^*$  and total cost. The observations provide useful insights for decision-makers.

Carbon costs can change due to government regulations or carbon market fluctuations. We vary  $ce$  from 0.6 to 1.4 and record the impact on inventory decisions.

$$ce = \{0.6, 0.8, 1.0, 1.2, 1.4\}.$$

Table 2. Sensitivity of inventory decisions to carbon emission cost.

Carbon Cost ( $ce$ )	Optimal Order Quantity ( $Q^*$ )	Total Cost	Observation
0.6	456	7000	Low carbon cost allows larger batch size
0.8	444	7100	Slight drop in order size
1.0	433	7491	Base case
1.2	420	7650	Emission cost rises, system reduces quantity
1.4	409	7880	Carbon penalty dominates, inventory shrinks

As seen in Table 2, higher emission costs result in lower order quantities. The model shifts towards smaller, more frequent batches to reduce carbon impact.

This parameter represents the emission load per unit stored. It may increase if cold storage becomes more energy intensive or inefficient.

$$eh = \{0.10, 0.15, 0.20, 0.25, 0.30\}.$$

In Table 3, as  $eh$  increases, emission cost during storage rises. The model responds by lowering  $Q^*$  to reduce inventory levels and associated emissions.

Table 3. Sensitivity of inventory decisions to emission rate per unit held.

Emission Rate ( $eh$ )	Optimal Order Quantity ( $Q^*$ )	Total Cost	Observation
0.10	446	7300	Lower emission rate, less penalty
0.15	438	7400	Moderate effect
0.20	433	7491	Base case
0.25	427	7620	Noticeable cost increase
0.30	420	7750	High penalty encourages smaller orders

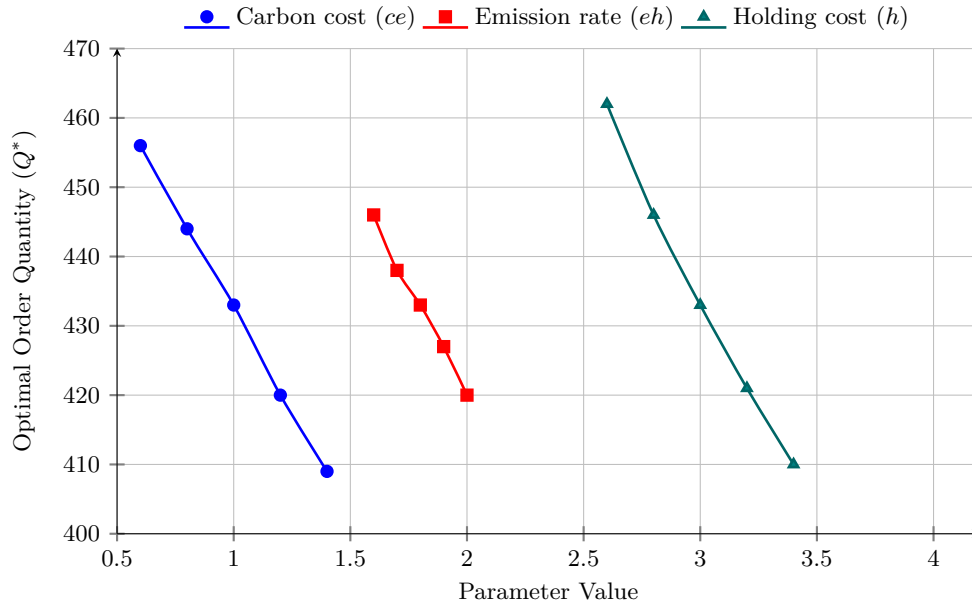
The holding cost can vary due to perishability, energy charges, or handling complexities. We test the effect by increasing  $h$  while keeping other parameters fixed.

$$h = \{2.0, 2.5, 3.0, 3.5, 4.0\}.$$

Table 4. Sensitivity of inventory decisions to holding cost.

Holding Cost ( $h$ )	Optimal Order Quantity ( $Q^*$ )	Total Cost	Observation
2.0	462	7220	Storage is cheap, batch size increases
2.5	446	7350	Small reduction in order quantity
3.0	433	7491	Base case
3.5	421	7620	Storage becomes costlier, quantity drops
4.0	410	7780	High cost forces minimum storage

In Table 4, the effect of  $h$  is gradual and more linear than carbon-related parameters. As holding cost rises, the model lowers inventory levels to reduce economic burden, as shown in Figure 3.

Fig. 3. Sensitivity of optimal order quantity  $Q^*$  to changes in major parameters.

## 5|Conclusion

In this paper, we proposed a fuzzy inventory model to manage perishable items in carbon-constrained settings. The model was developed to address random demand, fluctuating holding costs, and penalty due to emissions in the framework of fuzzy logic. The use of triangular fuzzy numbers for important inputs enables the model to better capture the inherent vagueness present in real operations, especially in supply chains of perishable products and carbon concerns. To defuzzify the model and produce useful order decisions, a graded mean

integration method was used. A full numerical case study illustrated the real use of the model. Sensitivity analysis confirmed that the model reacts positively to changes in carbon costs, emission rates, and holding costs. With an increase in carbon penalties, the model recommends taking smaller and more frequent orders and, thus, encourages more sustainable inventory decisions. Briefly, this fuzzy strategy is a resilient and sustainable option for planning inventory of perishable goods in conditions of environmental and economic uncertainty. Fuzzy modeling and green logistics are integrated together within a simple and pragmatic system that can effectively accommodate real-life supply chains.

It uses the assumption of triangular distribution for all fuzzy parameters, leaving others like trapezoidal or interval-valued fuzzy numbers behind. Shortage or backordering conditions are also not taken into account, although these are quite common in practice. The price of carbon for emissions is considered as a fuzzy constant; they do vary over time due to policy or trading scheme changes, though. The model has not been verified with empirical industrial data either, and hence its limitation in being verified easily under conditions of practical operating.

Future studies can further develop this model by using different fuzzy types or hybrid uncertainty methods, e.g., intuitionistic or hesitant fuzzy sets. Incorporation of variables representing shortages, carbon price volatility, or backordering under uncertainty can make the model significantly stronger. Further, the model can be extended to dynamic inventory systems or multi-level supply chains. Finally, model validation by industrial case studies will enable its practical implications to be tested and its applicability to be generalized to other industries, e.g., healthcare, agriculture, food retailing.

## Author Contribution

J. Behara: Writing, methodology, software, conceptualization and editing.

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## Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

## Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

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