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Neutrosophic Z-Number Analytic Hierarchy Process: A Framework for Enhanced Group Decision-Making Under Uncertainty

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Abstract

Multi-Criteria Decision-Making (MCDM) often faces challenges in handling uncertainty, imprecision, and unreliable expert judgments, limiting the effectiveness of traditional methods like the Analytic Hierarchy Process (AHP). This study proposes the Neutrosophic Z-Number AHP (NZN-AHP) method to enhance decision-making by addressing these complexities. The NZN-AHP method integrates Neutrosophic Z-Numbers (NZNs), which model truth, indeterminacy, falsity, and reliability, with AHP's structured pairwise comparison framework. Linguistic scales and advanced aggregation operators, such as Dombi and Aczel–Alsina, are employed to process expert evaluations, ensuring robust handling of uncertain data. The NZN-AHP method achieves consistent outcomes (CR < 0.1), outperforming traditional and fuzzy AHP by incorporating reliability and indeterminacy, thus providing more accurate prioritization of criteria in complex decision-making scenarios. NZN-AHP offers a versatile and precise framework for MCDM, effectively capturing multifaceted uncertainties and enhancing decision-making across domains like logistics, finance, and strategic planning. It sets a foundation for future research into integrating NZNs with other MCDM methods, advancing the field of decision sciences.

Keywords: Neutrosophic Z-number sets, AHP method, Neutrosophic sets, Z-numbers, Expert consensus, Uncertainty.

1 | Introduction

In the real world, uncertainty is a pervasive phenomenon. Much of the decisions taken are based on uncertainty. Humans have a remarkable capability to make rational decisions based on information, which is

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uncertain, imprecise and/or incomplete. Recently, numerous scholars have focused on studying the representation of uncertain information, and one notable approach is the utilization of Fuzzy Set (FS) theory, which employs Membership Degrees (MD) to capture uncertainty. Many extended models have been developed based on classic FSs. Zadeh [1] introduced the idea of FSs in 1965. The concept of FS is to use an MD (α with $\alpha \in [0,1]$) to evaluate criteria. In several circumstances, the FSs cannot handle knowledge supplied to a person through truth and falsity grades. Therefore, Atanassov [2] developed the theory of Intuitionistic Fuzz Sets (IFSs) by adding the term of a Non-Membership Degree (NMD) denoted by β such that $\beta \in [0,1]$. IFS is a comprehensive and robust strategy for dealing with complicated and unreliable data in decision-making settings. Numerous scholars indicated that IFS is a more comprehensive and robust strategy for dealing with complex and unreliable data in decision-making settings than FS. IFS theory has been used by many scholars in various fields [3], [4]. However, the IFS cannot handle this if someone offers such values; the sum of MD and NMD exceeds the unit interval. Therefore, based on the weakness of IFS, Yager [5] introduced the concept of Pythagorean Fuzzy Sets (PyFSs), which have a more flexible condition because they take the square of MD and NMD with $\alpha + \beta \in [0,1]$. Due to the flexible conditions of objects, PyFSs can reduce information loss and are widely used by many scholars in various business fields [6], [7]. However, if the square of MD and NMD exceeds 1, PyFs cannot handle this object. This is the reason why Yager [8] continuously developed the q-Rung Orthopair Fuzzy Sets (q-ROFS) with the restriction that the sum of the q-powers for the MD and NMD cannot be greater than the unit interval ($\alpha^q + \beta^q \in [0,1], q \in Z^+$). The q-ROFS has received much use and has attracted more interest from researchers because of its structure. Numerous authors have widely applied the q-ROFS theory to the detriment of various cases [9], [10].

While q-ROF offers notable advantages, researchers may encounter challenges when assessing information. In numerous real-life scenarios, MD and NMD may fall short in accurately expressing information, often due to instances of abstention and refusal, similar to situations encountered in voting or collecting human opinions. Cuong and Kreinovich [11] proposed the Picture Fuzzy Sets (PFSs) to overcome these problems with four degrees, i.e., MD, NMD, an Abstinence Degree (AD), and Refusal Degree (RD), with the condition ($\alpha + \beta + \gamma \in [0,1]$), where MD, AD, and NMD are denoted by $\alpha, \beta,$ and γ, respectively. PFS is a more robust method of handling complex and unreliable information in decision-making difficulties. Since its debut, PFS has drawn the fascination of numerous works [12], [13]. Although PFSs can find more information loss than IFSs, PyFs, and q-ROFS, PFSs still have MD, AD, and NMD limitations, making it impossible for decision-makers to voice their opinions independently. Kutlu Gündoğdu and Kahraman [14] recognized this problem and suggested an extension of PFS known as Spherical Fuzzy Sets (SFSs), such that the total of the squares of the MD, AD, and NMD is confined to [0, 1] (or $\alpha^2 + \beta^2 + \gamma^2 \in [0,1]$). Compared to PFS, DEs in SFS have more discretion when making decisions. SFSs is currently a helpful tool for evaluating information and has been used in several domains [15–17]. Since SFSs were introduced, they have attracted the attention of many researchers. Ashraf et al. [18] developed spherical fuzzy t'-norms and spherical fuzzy t'-conorms. However, each membership degree in a spherical FS fails to capture its inherent level of uncertainty or ambiguity, which is essential for understanding the association of an element with the FS [19]. Smarandache [20] proposed that they present uncertain, incomplete, imprecise, and indeterminate information in real-world problems. NS is a generalization of the FS. It combines the concepts of FSs and Neutrosophic Sets (NS) where FSs are used to tackle uncertainty using the membership grade, and NS are used to tackle uncertainty using the truth, indeterminacy, and falsity membership grades, which are considered independent [21]. This study gains access to more comprehensive and accurate information about experts' responses by leveraging NSs. Consequently, the calculation results are expected to more faithfully reflect reality, enhancing the overall quality and reliability of the research findings. In addition to the general form of NS, the Single-Valued Neutrosophic Set (SVNS) is proposed as a specific instance that is particularly useful for real-world scientific and engineering applications [22–25]. This assumption is beneficial in various scenarios, such as information fusion, where data from different sensors must be integrated. SVNS, being a subset of NFS, utilizes single-valued memberships, thereby inheriting the mathematical properties of NS [22–24]. Unlike ordinary FSs [1], which consider only the degree of membership, NS encompass an object's degree of truth, indeterminacy, and falsity. This comprehensive representation enables a more accurate modeling of real-world situations. For instance, in medical diagnosis, patient symptoms may not definitively indicate a specific disease (Truth) nor entirely rule it out (Falsity) while also exhibiting some ambiguity (Indeterminacy). NS can capture this complexity more effectively than FSs. Additionally, while type-2 FSs were developed to address uncertainties, they lack the capability to handle indeterminacy. Incorporating an indeterminacy membership function, NS can effectively manage such situations. For example, in weather forecasting, predictions may be uncertain (type-2 FS), but indeterminacy arising from climate change can be modeled using NS. Assuming $\alpha(x)$, $\beta(x)$, and $\gamma(x)$ represent memberships for truth, indeterminacy, and falsity respectively, with $x \rightarrow [0,1]$, we have $0 \leq \alpha(x) + \beta(x) + \gamma(x) \leq 3$. The sum of the three memberships in the NS can reach a maximum value of 3, unlike other fuzzy types where the maximum sum is typically limited to 1. This characteristic allows for a broader observation range, enhancing accuracy, which distinguishes NS from other fuzzy types. Considering membership's observational meaning, NS also offers unique advantages over SFSs and T-Spherical Fuzzy Sets (T-SFSs). SFSs and T-SFSs, proposed by Ullah et al. [26] in 2020, are extensions of FSs that incorporate the concept of direction in decision-making. However, neither SFS nor T-SFS explicitly account for the degree of indeterminacy. For example, in a decision-making scenario involving investment options, an investor might be partially inclined towards an option (Membership), partially disinclined (Non-membership), and partially uncertain due to market volatility (Indeterminacy). While SFSs and T-SFSs can capture the investor's inclination towards or away from an option, they do not adequately handle the uncertainty aspect. NS, with their ability to handle truth, indeterminacy, and falsity, can model this situation more accurately. Moreover, NS provide a more flexible and comprehensive tool for dealing with such uncertainties in complex decision-making scenarios where the data is incomplete, inconsistent, or uncertain. Thus, despite the advancements brought by SFSs and T-SFSs, NS still hold a significant edge when it comes to handling uncertainty, imprecision, and indeterminacy in real-world situations.

In decision-making, uncertainties are ubiquitous, and decisions often arise in contexts where information is incomplete [27–29]. Hence, acknowledging the inherent uncertainty in decision-making processes is crucial [30]. Z-numbers, pioneered by Zadeh [31], emerged as indispensable tools in response to this need. Specifically designed for computations in uncertain and incompletely reliable environments, Z-numbers offers a structured approach to handling uncertainty and incomplete information. Z-numbers comprise two integral components: The Z-number and fuzzy information, encapsulating the assessment score and the associated reliability level [31]. Thus, Z-numbers offer a significant complement to NS. While NS excel in handling uncertainty, imprecision, and indeterminacy, they lack explicit consideration of the reliability of the information source. This is where Z-numbers play a crucial role. As an extension of fuzzy numbers, Z-numbers incorporate a measure of reliability [30]. For instance, imagine two financial analysts offering company growth forecasts. While NS can represent the analysts' forecasts in terms of truth, indeterminacy, and falsity, they overlook the analysts' reliability. Conversely, Z-numbers can model both the forecast (The "Restriction") and the reliability of the analyst (the "reliability"). This enables a more thorough analysis, as decision-makers can now weigh both the forecast and the source's reliability. Consequently, Z-numbers complement NSs, enhancing decision-making in scenarios where the source's reliability is critical.

Since the introduction of Neutrosophic Z-Numbers (NZNs), they have garnered significant attention from researchers due to their ability to model uncertainty, indeterminacy, and reliability in decision-making processes. NZNs combine the strengths of NS, which handle truth, indeterminacy, and falsity memberships, with Z-Numbers, which incorporate a reliability degree alongside fuzzy information. This unique combination makes NZNs a powerful tool for addressing complex real-world problems where information is incomplete, imprecise, or unreliable. Below is a summary of key studies that have applied NZNs across various sectors, showcasing their versatility and effectiveness.

Table 1. Related work.

No	Sector	Method	Purpose	References
1	Software development	Interval-valued NZN and NZN-area model	Assign teams to software projects using linguistic variables and fuzzy logic to improve project success. The NZN-Area model enhances defect reduction (22%) and resolution time (25%) compared to the NZN method.	[32]
2	Decision making	Dombi weighted aggregation operators of neutrosophic Z-numbers (NZNDWAA, NZNDWGA)	Develop flexible multiple attribute decision-making (MADM) for equipment supplier selection using Dombi operations to aggregate NZN information, enhancing decision flexibility.	[33]
3	Decision making	Aczel–Alsina weighted aggregation operators of neutrosophic Z-numbers (NZNAAWAA, NZNAAWGA)	Propose Aczel–Alsina operations for flexible MADM by adjusting parameter values based on decision-maker preferences in NZN environments.	[34]
4	Decision making	Trapezoidal Neutrosophic Z-numbers (TrNZN) with TrNZNWAA and TrNZNWGA operators	Develop a multicriteria decision-making (MDM) method using TrNZNs for software selection, ensuring continuous and reliable assessments with improved rationality and efficiency.	[35]
5	Human resources	Fuzzy AHP, Neutrosophic Z-numbers, and Fuzzy EDAS	Determine competencies for system analysts in a bank using fuzzy MCDM to prioritize and rank competencies for tailored development programs.	[36]
6	Sustainable fashion	Neutrosophic Z-number with Delphi-DEMATEL	Identify and prioritize barriers to sustainable fashion consumption in Vietnam, emphasizing policy, education, and supply chain transparency for sustainable practices.	[37]
7	Logistics	Neutrosophic Z-number with Delphi-DEMATEL	Optimize horizontal collaboration in logistics by identifying critical success factors (e.g., financial stability, green practices) to enhance efficiency and sustainability.	[38]
8	Financial risk analysis	Neutrosophic numbers with Altman Z-Score model	Re-evaluate financial risk in company mergers using Neutrosophic numbers to improve predictive accuracy over the classical Altman Z-Score model.	[39]
9	Business site selection	Sine trigonometric aggregation operators with single-valued neutrosophic Z-numbers (SVNZNs)	Develop an MADM method for business site selection using sine trigonometric operations to aggregate SVNZNs, ensuring robust decision-making with sensitivity analysis.	[40]

In parallel, NZNs have been extended through various innovative approaches to enhance their applicability in Multi-Criteria Decision-Making (MCDM). For instance, in software development, interval-valued NZNs and the NZN-area model have been employed to assign teams to projects, achieving significant improvements in defect reduction (22%) and resolution time (25%) by leveraging linguistic variables and fuzzy logic. In decision-making contexts, Dombi weighted aggregation operators (NZNDWAA, NZNDWGA) and Aczel–Alsina weighted aggregation operators (NZNAAWAA, NZNAAWGA) have been developed to provide flexible MADM frameworks, enabling equipment supplier selection and preference-based decision adjustments, respectively. Similarly, Trapezoidal NZNs (TrNZN) with their associated operators (TrNZNWAA, TrNZNWGA) have been utilized for software selection, ensuring reliable and rational assessments. In human resources, NZNs combined with Fuzzy AHP and Fuzzy EDAS have prioritized competencies for system analysts in banking, tailoring development programs effectively. In sustainable fashion and logistics, NZNs integrated with Delphi-DEMATEL have identified critical barriers and success factors, promoting transparency and efficiency. Additionally, in financial risk analysis, NZNs paired with the Altman Z-Score model have improved predictive accuracy for company mergers, while sine trigonometric operators with Single-Valued NZN (SVNZN) have enhanced business site selection through robust MADM. Building on these advancements, researchers have proposed Neutrosophic quadrilateral sets, which incorporate four-dimensional membership structures (Truth, indeterminacy, falsity, and an additional dimension such as neutrality or context). These sets, supported by aggregation operators like Dombi, Aczel–Alsina, and sine trigonometric methods, have been applied in domains such as marketing (e.g., evaluating CGI influencer adoption), finance, and logistics, offering greater flexibility in handling complex uncertainties.

However, recent developments in NZN applications have predominantly progressed toward addressing uncertainty in specific domains, yet they lack comprehensive integration with MCDM frameworks, particularly with Analytic Hierarchy Process (AHP). While methods like CoCoSo, Delphi, and DEMATEL have been successfully paired with NZNs, the absence of AHP integration represents a critical limitation, given AHP's widespread use and effectiveness in structured decision-making. The integration of NZNs with MCDM methods is critical because MCDM frameworks provide structured approaches to evaluate multiple conflicting criteria, which are common in real-world decision-making scenarios such as resource allocation, supplier selection, or strategic planning. MCDM methods excel in prioritizing criteria and alternatives through systematic comparisons, ensuring decisions are both rational and transparent. However, traditional MCDM methods often struggle with the inherent uncertainty and reliability issues in expert judgments, which NZNs are uniquely equipped to address due to their ability to model truth, indeterminacy, falsity, and reliability simultaneously. The absence of NZN integration with AHP, a widely used MCDM method, is particularly notable because AHP’s hierarchical structure and pairwise comparison approach offer a robust framework for breaking down complex decisions into manageable components, making it ideal for applications where criteria have varying levels of importance. AHP’s strength lies in its ability to derive consistent priority weights through eigenvector calculations, but its traditional and fuzzy variants often fail to account for the reliability of expert inputs or the indeterminacy inherent in complex systems. By incorporating NZNs, which capture both the uncertainty of information and the reliability of its sources, the proposed Neutrosophic Z-Number AHP (NZN-AHP) method addresses these limitations, enhancing the precision and robustness of decision-making. This novel method leverages linguistic scales for expert assessments and advanced aggregation techniques, such as Dombi and Aczel–Alsina operators, to ensure flexibility and accuracy in handling vague and uncertain data. As a result, it provides a powerful tool for tackling complex decision-making challenges across diverse domains, such as marketing, logistics, and finance, where both uncertainty and source reliability are critical factors. This study addresses the gap in AHP integration by proposing a pioneering NZN-AHP method, offering a comprehensive solution to enhance MCDM efficacy in uncertain environments.

Drawing upon an extensive literature review, this study identifies a critical gap in current MCDM methodologies, particularly in their handling of uncertainty, ambiguity, and reliability of information. The inherent challenge in decision-making lies in the imperfect nature of available data, often characterized by vagueness, unreliability, and incompleteness. While traditional FSs provide a mechanism to address uncertainty, they frequently fail to account for the reliability of the information source, limiting their effectiveness in complex decision-making scenarios. Z-Numbers, introduced by Zadeh [31], offer a promising solution by encapsulating two key components: A variable restriction (A) and a reliability degree (R), providing a more comprehensive framework for modeling incomplete and uncertain information. As noted by Abdullahi et al. [41], Z-Numbers generalize real, interval, random, and fuzzy numbers, enabling more effective representation of real-world systems. Their ability to mimic natural language expressions enhances their applicability in decision-making contexts where human judgments are pivotal. However, existing MCDM methods, such as the AHP, whether in its traditional or spherical fuzzy forms, often overlook the reliability of expert opinions, which can compromise the integrity of decision outcomes. For example, Mohandes et al. [42] developed a Pentagonal Fuzzy Delphi Method (PFDM) to identify causes of construction site accidents, while Nguyen [15], [16] applied Spherical Fuzzy AHP (SF-AHP) to evaluate factors influencing package tour provider selection, employee satisfaction in logistics, and apartment selection criteria in Vietnam. Despite these advancements, both traditional and Neutrosophic AHP models fail to adequately address the uncertainty and reliability of expert judgments, which can significantly skew results.

To address this shortfall, this study proposes a pioneering NZN-AHP method, integrating the robust uncertainty-handling capabilities of NZNs with the structured decision-making framework of AHP. NZNs combine the strengths of NS, which model truth, indeterminacy, and falsity, with Z-Numbers, incorporating reliability, thus offering a superior approach to capturing ambiguous and uncertain information. This model enhances decision-making precision by leveraging linguistic scales for expert assessments and advanced aggregation techniques, such as Dombi and Aczel–Alsina operators, to ensure flexibility and accuracy.

Table 2. Superiority of Neutrosophic Z-number over other fuzzy sets.

	Membership Function			Reliability			
Sets	**α**	**β**	**γ**	**α**	**β**	**γ**	**Constraints**
FSs [1]	☑	O	O	O	O	O	$0 \leq \alpha \leq 1$
IFSs [2]	☑	☑	O	O	O	O	$0 \leq \alpha + \beta \leq 1$
PyFSs [5]	☑	☑	O	O	O	O	$0 \leq \alpha^2 + \beta^2 \leq 1$
q-ROFS [8]	☑	☑	O	O	O	O	$0 \leq \alpha^q + \beta^q \leq 1$
PFSs [11]	☑	☑	☑	O	O	O	$0 \leq \alpha + \beta + \gamma \leq 1$
SFSs [14]	☑	☑	☑	O	O	O	$0 \leq \alpha^2 + \beta^2 + \gamma^2 \leq 1$
Z-number [43]	☑	O	O	☑	O	O	$0 \leq \alpha(A, F) \leq 1$
Spherical Fuzzy Z-number (SFZ)	☑	☑	☑	☑	☑	☑	$0 \leq \alpha^2(A, F) \leq +\beta^2(A, F) \leq +\gamma^2(A, F) \leq 1$
Neutrosphic Z-number (NZN)	☑	☑	☑	☑	☑	☑	$0 \leq \delta_T(x) + \varepsilon_T(x) + \zeta_T(x) \leq 3$ and $0 \leq \delta_G(x) + \varepsilon_G(x) + \zeta_G(x) \leq 3$

The proposed method addresses two key research questions: (RQ1) does the NZN-AHP method outperform existing group expert consensus techniques in capturing vague and uncertain information? (RQ2) What are the critical factors influencing the adoption of CGI influencers in marketing activities in Vietnam? The study pioneers the full integration of NZNs within AHP by collecting expert evaluations of criteria importance and reliability simultaneously, applying aggregation approaches to identify key criteria, and using correlation analyses to validate the NZN-AHP method. The significant contributions of this research are twofold: 1) it introduces a novel NZN-AHP method that surpasses previous approaches by integrating Neutrosophic and Z-Number advantages, adeptly handling ambiguity, uncertainty, and reliability, unlike earlier methods, and 2) it enhances established MCDM techniques, improving analytical precision and decision-making efficacy, providing a versatile framework for scholars and policymakers across domains such as marketing, logistics, and finance.

2 | Preliminaries and Basic Theory

This section introduces several fundamental definitions and operations that played a crucial role in shaping the suggested work.

Definition 1 ([43]). Let X represent a set of universes. *Eq. (1)* defines a NZN set in X.

$$N_z = \{[x, \delta(T, G)(x), \varepsilon(T, G)(x), \zeta(T, G)(x)] | x \epsilon X\}, \quad (1)$$

where $\delta(T,G)(x) = (\delta_T(x), \delta_G(x))$; $\varepsilon(T,G)(x) = (\varepsilon_T(x), \varepsilon_G(x))$; $\zeta(T,G)(x) = (\zeta_T(x), \zeta_G(x)): X \to [0,1]$ are the fuzzy value order pairs for truth, indeterminacy, and falsity. T represents Neutrosophic values for universe set X, and G represents Neutrosophic reliability measures for T. These elements meet the requirements listed:

$0 \le \delta_T(x) + \varepsilon_T(x) + \zeta_T(x) \le 3$ and $0 \le \delta_G(x) + \varepsilon_G(x) + \zeta_G(x) \le 3$.

For ease and clarity's sake, the element $[x, \delta(T,G)(x), \varepsilon(T,G)(x), \zeta(T,G)(x)]$ in N_Z is concisely expressed as $N_Z = [\delta(T,G), \varepsilon(T,G), \zeta(T,G)] = [(\delta_T, \delta_G), (\varepsilon_T, \varepsilon_G), (\zeta_T, \zeta_G)]$, name NZN.

Definition 2 ([43], [44]). Let $N_{Z1} = [\delta_1(T,G), \varepsilon_1(T,G), \zeta_1(T,G)] = [(\delta_{T1}, \delta_{G1}), (\varepsilon_{T1}, \varepsilon_{G1}), (\zeta_{T1}, \zeta_{G1})]$ and $N_{Z2} = [\delta_2(T,G), \varepsilon_2(T,G), \zeta_2(T,G)] = [(\delta_{T2}, \delta_{G2}), (\varepsilon_{T2}, \varepsilon_{G2}), (\zeta_{T2}, \zeta_{G2})]$ be two NZNs and $\varepsilon > 0$. Next, we use *Eqs. (2)–(10)* to provide the following relations.

$$N_{Z1} \supseteq N_{Z2} \Leftrightarrow \delta_{T1} \ge \delta_{T2}\,.\,\delta_{G1} \ge \delta_{G2}.\ \varepsilon_{T1} \le \varepsilon_{T2}.\ \varepsilon_{G1} \le \varepsilon_{G2}\,.\ \zeta_{T1} \le \zeta_{T2} \text{ and } \zeta_{G1} \le \zeta_{G2}. \tag{2}$$

$$N_{Z1} = N_{Z2} \Leftrightarrow N_{Z1} \supseteq N_{Z2} \text{ and } N_{Z2} \supseteq N_{Z1}. \tag{3}$$

$$N_{Z1} \cup N_{Z2} \Leftrightarrow [(\delta_{T1} \vee \delta_{T2}.\ \delta_{G1} \vee \delta_{G2}), (\varepsilon_{T1} \wedge \varepsilon_{T2}.\ \varepsilon_{G1} \wedge \varepsilon_{G2}), (\zeta_{T1} \wedge \zeta_{T2}.\ \zeta_{G1} \wedge \zeta_{G2})]. \tag{4}$$

$$N_{Z1} \cap N_{Z2} \Leftrightarrow [(\delta_{T1} \wedge \delta_{T2}.\ \delta_{G1} \wedge \delta_{G2}), (\varepsilon_{T1} \vee \varepsilon_{T2}.\ \varepsilon_{G1} \vee \varepsilon_{G2}), (\zeta_{T1} \vee \zeta_{T2}.\ \zeta_{G1} \vee \zeta_{G2})]. \tag{5}$$

$$(N_{Z1})^C = [(\zeta_{T1}.\ \zeta_{G1}).\ (1 - \varepsilon_{T1}.\ 1 - \varepsilon_{G1}).\ (\delta_{T1}.\ \delta_{G1})] \text{ (Complement of } N_{Z1}). \tag{6}$$

$$N_{Z1} \oplus N_{Z2} = [(\delta_{T1} + \delta_{T2} - \delta_{T1}\delta_{T2}.\ \delta_{G1} + \delta_{G2} - \delta_{G1}\delta_{G2}).\ (\varepsilon_{T1}\varepsilon_{T2}.\ \varepsilon_{G1}\varepsilon_{G2}).\ (\zeta_{T1}\zeta_{T2}.\ \zeta_{G1}\zeta_{G2})]\,. \tag{7}$$

$$N_{Z1} \otimes N_{Z2} = [(\delta_{T1}\delta_{T2}.\ \delta_{G1}\delta_{G2}).\ (\varepsilon_{T1} + \varepsilon_{T2} - \varepsilon_{T1}\varepsilon_{T2}.\ \varepsilon_{G1} + \varepsilon_{G2} - \varepsilon_{G1}\varepsilon_{G2}).\ (\zeta_{T1} + \zeta_{T2} - \zeta_{T1}\zeta_{G2}.\ \zeta_{G1} + \zeta_{G2} - \zeta_{G1}\zeta_{G2})]\,. \tag{8}$$

$$\varepsilon N_{Z1} = [(1 - (1 - \delta_{T1})^{\varepsilon}.\ 1 - (1 - \delta_{T1})^{\varepsilon}).\ (\varepsilon_{T1}^{\varepsilon}.\ \varepsilon_{G1}^{\varepsilon}).\ (\zeta_{T1}^{\varepsilon}.\ \zeta_{G1}^{\varepsilon})]. \tag{9}$$

$$(N_{Z1})^{\varepsilon} = [\ (\delta_{T1}^{\varepsilon}.\ \delta_{G1}^{\varepsilon}).\ (1 - (1 - \varepsilon_{T1})^{\varepsilon}.\ 1 - (1 - \varepsilon_{G1})^{\varepsilon}).\ (1 - (1 - \zeta_{T1})^{\varepsilon}.\ 1 - (1 - \zeta_{G1})^{\varepsilon})]\,. \tag{10}$$

To deneutrosphic $N_{Z1} = [\delta_1(T,G), \varepsilon_1(T,G), \zeta_1(T,G)] = [(\delta_{T1}, \delta_{G1}), (\varepsilon_{T1}, \varepsilon_{G1}), (\zeta_{T1}, \zeta_{G1})]$, using *Eq. (11)*:

$$DEF(N_{Z1}) = \frac{2 + \delta_{T1}\,\delta_{G1} - \varepsilon_{T1}\,\varepsilon_{G1} - \zeta_{T1}\,\zeta_{G1}}{3} \text{ for } DEF(N_{Z1}) \in [0.1]. \tag{11}$$

Definition 3 ([43]). Two weighted NZN aggregation operators.

We may create the weighted aggregate arithmetic mean (NZNWAA) equation for NZNs by using *Eq. (7)* and *Eq. (9)* from *Definition 2*. Let $N_{Zi} = [\delta_i(T,G), \varepsilon_i(T,G), \zeta_i(T,G)] = [(\delta_{Ti}, \delta_{Gi}), (\varepsilon_{Ti}, \varepsilon_{Gi}), (\zeta_{Ti}, \zeta_{Gi})]$, $(i = 1, 2, \ldots n)$be a NZN and NZNWAA group: $\Omega^n \longrightarrow \Omega$ *Eq. (12)* is then used to properly define the NZNWAA equation.

$$\begin{aligned} NZNWAA(N_{Z1}.\ N_{Z2}.\ \ldots.\ N_{Zn}) &= \textstyle\sum_{i=1}^{n} \varepsilon_i\, N_{Zi} \\ &= [(1 - \textstyle\prod_{i=1}^{n}(1 - \delta_{Ti})^{\varepsilon_i}.\ 1 - \prod_{i=1}^{n}(1 - \delta_{Gi})^{\varepsilon_i}),\ (\prod_{i=1}^{n} \varepsilon_{Ti}{}^{\varepsilon_i}.\ \prod_{i=1}^{n} \varepsilon_{Gi}{}^{\varepsilon_i}).\ (\prod_{i=1}^{n} \zeta_{Ti}{}^{\varepsilon_i}.\ \prod_{i=1}^{n} \zeta_{Gi}{}^{\varepsilon_i})], \end{aligned} \tag{12}$$

where $\varepsilon_i (i = 1, 2..n)$ is the weight of N_{Zi} with $0 \le \varepsilon_i \le 1$ and $\sum_{i=1}^{n} \varepsilon_i = 1$.

Similarly, we can obtain the weighted aggregate geometric mean (NZNWAGM) equation for NZNs by using *Eq. (8)* and *Eq. (10)* from *Definition 2*. Let $N_{Zi} = [\delta_i(T,G), \varepsilon_i(T,G), \zeta_i(T,G)] = [(\delta_{Ti}, \delta_{Gi}), (\varepsilon_{Ti}, \varepsilon_{Gi}), (\zeta_{Ti}, \zeta_{Gi})]$, $(i = 1, 2, \ldots n)$ be a group of NZN and NZNWGA: $\Omega^n \longrightarrow \Omega$. *Eq. (13)* is used to formally define the NZNWGA equation.

$$NZNWGA(N_{Z1}.\ N_{Z2}.\ \ldots.\ N_{Zn}) = \textstyle\prod_{i=1}^{n}(N_{Zi})^{\varepsilon_i} \tag{13}$$

$= [(\prod_{i=1}^{n} (\delta_{Ti})^{\varepsilon_i}.\prod_{i=1}^{n}(\delta_{Gi})^{\varepsilon_i}).(1-\prod_{i=1}^{n}(1-\varepsilon_{Gi})^{\varepsilon_i}.1-\prod_{i=1}^{n}(1-\varepsilon_{Gi})^{\varepsilon_i}).1-$
$\prod_{i=1}^{n}(1-\zeta_{Ti})^{\varepsilon_i}.1-\prod_{i=1}^{n}(1-\zeta_{Gi})^{\varepsilon_i})],$

where $\varepsilon_i(i=1,2..n)$ is the weight of N_{Zi} with $0 \leq \varepsilon_i \leq 1$ and $\sum_{i=1}^{n}\varepsilon_i = 1$.

Illustrative example 2: Consider a set of 4 NZN numbers: {[(0.7,0.9),(0.2,0.1),(0.3,0.2)], [(0.5,0.9),(0.4,0.1),(0.5,0.2)], [(0.3,0.9),(0.7,0.1),(0.6,0.2)], [(0.1,0.9),(0.8,0.1),(0.9,0.2)]} with corresponding weights: ε_i = [0.3, 0.2, 0.3, 0.2]. Applying the NZNWAA (*Eq. (12)*) and NZNWGA (*Eq. (13)*) methods, we obtain:

$$\mathrm{NZNWAA}(N_{Z1}.N_{Z2}.N_{Z3}.N_{Z4}) = [(0.4300,0.9),(0.5100,0.1),(0.5500,0.2)]$$
$$\mathrm{NZNWGA}(N_{Z1}.N_{Z2}.N_{Z3}.N_{Z4}) = [(0.3615,0.9),(0.5843,0.1),(0.5385,0.2)]$$

Definition 4 ([45]). Distance and similarity measures of NZN sets.

As powerful tools for decision-making, distance and similarity measurements between sets have drawn more attention from researchers in recent years [46]. For two NZN sets with their associated weights, we can establish specific measurement criteria. Let $N_{Z1} = \{N_{Z11}, N_{Z12} \dots, N_{Z1n}\}$ and $N_{Z2} = \{N_{Z21}, N_{Z22} \dots, N_{Z2n}\}$, where $N_{Z1k} = [\delta_{1k}(T,G), \varepsilon_{1k}(T,G), \zeta_{1k}(T,G)] = [(\delta_{T1k}, \delta_{G1k}), (\varepsilon_{T1k}, \varepsilon_{G1k}), (\zeta_{T1k}, \zeta_{G1k})]$, and $N_{Z2k} = [\delta_{2k}(T,G), \varepsilon_{2k}(T,G), \zeta_{2k}(T,G)] = [(\delta_{T2k}, \delta_{G2k}), (\varepsilon_{T2k}, \varepsilon_{G2k}), (\zeta_{T2k}, \zeta_{G2k})]$ are two NZNs. These form two complete NZN sets where $\vartheta \geq 1$ represents any integer. The weighted components for n pairs of NZN are denoted as $w_k = (w_1, w_2, \dots w_n)$, with the condition that $\sum_{k=1}^{n} w_k = 1$. The generalized distance between these sets NZ1 and NZ2 can then be determined using *Eq. (14)*.

$$D_{w\vartheta}(N_{Z1}.N_{Z2}) = \frac{1}{2}\left\{\sqrt[\vartheta]{\frac{1}{3}\sum_{k=1}^{n} w_k(|\delta_{T1k}-\delta_{T2k}|^{\vartheta} + |\varepsilon_{T1k}-\varepsilon_{T2k}|^{\vartheta} + |\zeta_{T1k}-\zeta_{T2k}|^{\vartheta})} + \sqrt[\vartheta]{\frac{1}{3}\sum_{k=1}^{n} w_k(|\delta_{G1k}-\delta_{G2k}|^{\vartheta} + |\varepsilon_{G1k}-\varepsilon_{G2k}|^{\vartheta} + |\zeta_{G1k}-\zeta_{G2k}|^{\vartheta})}\right\}. \quad (14)$$

When ϑ = 1, *Eq. (15)* converts the generalized distance formula into the Hamming distance measurement D_{w1}.

$$D_{w1}(N_{Z1}.N_{Z2}) = \frac{1}{6}\left\{\sum_{k=1}^{n} w_k(|\delta_{T1k}-\delta_{T2k}| + |\varepsilon_{T1k}-\varepsilon_{T2k}| + |\zeta_{T1k}-\zeta_{T2k}|) + \sum_{k=1}^{n} w_k(|\delta_{G1k}-\delta_{G2k}| + |\varepsilon_{G1k}-\varepsilon_{G2k}| + |\zeta_{G1k}-\zeta_{G2k}|)\right\}. \quad (15)$$

Similarly, setting ϑ = 2 in *Eq. (16)* yields the Euclidean distance measurement D_{w2}.

$$D_{w2}(N_{Z1}.N_{Z2}) = \frac{1}{2}\left\{\sqrt{\frac{1}{3}\sum_{k=1}^{n} w_k(|\delta_{T1k}-\delta_{T2k}|^2 + |\varepsilon_{T1k}-\varepsilon_{T2k}|^2 + |\zeta_{T1k}-\zeta_{T2k}|^2)} + \sqrt{\frac{1}{3}\sum_{k=1}^{n} w_k(|\delta_{G1k}-\delta_{G2k}|^2 + |\varepsilon_{G1k}-\varepsilon_{G2k}|^2 + |\zeta_{G1k}-\zeta_{G2k}|^2)}\right\}. \quad (16)$$

3 | Proposed Model of AHP-Based NZNs

3.1 | Neutrosophic Z-Number Approach

The expert's evaluations can be displayed as Z-numbers, indicating the boundary's certainty and value for the required problem. With the combination of NZNs, the authors gathered the experts' views of the concerned topic and the reliability level of their rate using the linguistic scales in *Table 3* and *Table 4*.

Step 1. Calculate the weight of the expert.

Select professionals and decision-makers with experience in this area. NZN numbers, which are made up of two parts, will be used to evaluate expert weights. The evaluation framework consists of two measures: T, which captures the expert's assessment based on their training and experience, and G, which represents how confidently the research team views the expert's qualifications. Using *Eq. (7)*, we combine two NZN values that represent the expert's assessment based on their educational qualifications and years of professional experience, and *Eq. (11)* will be used to transform them into a clear score. The expert-level evaluation and related language scale are shown in *Table 3*.

Table 3. Expert rating scale.

Education (T)	Position (T)	Certainty (G)	Linguistic Scale	Code	NZN
Doctor	C-level	Very high	Very high	VH	(0.8,0.15,0,2)
Master	D-level	High	High	H	(0.6,0.35,0.4)
Bachelor	M-level	Medium	Medium	M	(0.4,0.65,0.6)
		Low	Low	L	(0.2,0.85,0.8)
		Very low	Very low	VL	(0,1,1)

Determine k values EK by calculating the evaluation value for k experts: $ek_j = \{ek_1, ek_2, \dots ek_k\}$. The weight of expert EW: $ew_j = \{ew_1, ew_2, \dots ew_k\}$ is calculated as *Eq. (15)* below.

$$ew_j = \frac{ek_j}{\sum_{j=1}^{k} ek_j}. \tag{17}$$

Step 1. Consider a scenario where n factors are evaluated by k experts. The main and sub-criteria for strategies selection for market expansion in emerging market are collected. Expert opinions are collected using a linguistic scale established by Saaty [47], following this, the values are translated into NZN form. *Table 4* presents the evaluation scale with its corresponding NZN values.

Table 4. The linguistic scale for restriction components.

Saaty Scale	Linguistic Terms	Code	NZNs Scale Membership						Reciprocal Membership					
			δ_T	δ_G	ε_T	ε_G	ζ_T	ζ_G	δ_T	δ_G	ε_T	ε_G	ζ_T	ζ_G
1	Equally influential	AMI	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2	Weak advantage influential	VHI	0.55	0.40	0.40	0.65	0.45	0.60	0.45	0.40	0.60	0.65	0.55	0.60
3	Slightly influential	HI	0.60	0.30	0.35	0.75	0.40	0.70	0.40	0.30	0.65	0.75	0.60	0.70
4	Preferable influential	SMI	0.65	0.60	0.30	0.35	0.35	0.40	0.35	0.60	0.70	0.35	0.65	0.40
5	Strongly influential	EI	0.70	0.80	0.30	0.15	0.30	0.20	0.30	0.80	0.70	0.15	0.70	0.20
6	Fairly influential	SLI	0.75	0.70	0.25	0.25	0.25	0.30	0.25	0.70	0.75	0.25	0.75	0.30

Table 4. Continued.

Saaty Scale	Linguistic Terms	Code	NZNs Scale Membership						Reciprocal Membership					
			δ_T	δ_G	ε_T	ε_G	ζ_T	ζ_G	δ_T	δ_G	ε_T	ε_G	ζ_T	ζ_G
7	Very strongly influential	LI	0.80	0.90	0.25	0.10	0.20	0.10	0.20	0.90	0.75	0.10	0.80	0.10
8	Absolute influential	VLI	0.85	0.85	0.20	0.10	0.15	0.15	0.15	0.85	0.80	0.10	0.85	0.15
9	Absolutely influential	ALI	0.90	1.00	0.10	0.00	0.10	0.00	0.10	1.00	0.90	0.00	0.90	0.00

The AHP method will be used to process the data after the assessments have been converted to NZN numbers. Below are the steps involved in the computation.

Step 2. Construct pairwise comparison matrices based on the relationship between criteria by the decision-makers panel.

Step 3. To get the direct relation matrix, use the aggregation approach to combine expert opinions into one matrix.

The concerning the preference of criterion i over criterion j from k experts, denoted as d_{ij}^{k} are converted using their corresponding expert weights ew_t to NZN. *Eq. (12)* is then used to combine these assessments, producing the direct influence matrix $\otimes D = [\otimes d_{ij}]_{nxn}$, while:

$$d_{ij} = NZNWAA\left(d_{ij}^{1}, d_{ij}^{2}, \dots, d_{ij}^{k}\right) = \sum_{t=1}^{k} ew_t d_{ij}^{k}, \quad \textbf{(18)}$$

where $i = 1, 2, \dots n$, $j = 1, 2, \dots n$, $t = 1, 2.. k$; $\otimes d_{ij} = \left[\left(d_{ij}^{\delta_T}, d_{ij}^{\delta_G}\right), \left(d_{ij}^{\varepsilon_T}, d_{ij}^{\varepsilon_G}\right), \left(d_{ij}^{\zeta_T}, d_{ij}^{\zeta_G}\right)\right]$. Here, the diagonal elements in the matrix are 0, i.e., $\otimes d_{ij} = 0$ (when i = j).

Step 4. Calculating the normalized aggregated/average comparison matrix matrix $\otimes D^*$.

Matrix $\otimes D = [\otimes d_{ij}]_{nxn}$ will be transformed into a normalized matrix $\otimes D^* = [\otimes d_{ij}^*]_{nxn}$ using *Eq. (21)* below:

$$d_{ij}^* = \frac{d_{ij}}{\sum_{k=1}^{n} d_{kj}} \text{ for } j = 1,2,, n, \quad \textbf{(19)}$$

where $\otimes d_{ij}^* = \left[\left(d_{ij}^{*\delta_T}, d_{ij}^{*\delta_G}\right), \left(d_{ij}^{*\varepsilon_T}, d_{ij}^{*\varepsilon_G}\right), \left(d_{ij}^{*\zeta_T}, d_{ij}^{*\zeta_G}\right)\right]$, where $\sum_{k=1}^{n} d_{kj}$ is the sum of criteria per column in the aggregate matrix, and d_{kj} points to the preference of the criterion in the aggregated comparison matrix.

Step 5. Calculating the NZN matrices are combined using the geometric mean approach to $\otimes M = [\otimes m_{ij}]_{nxn}$, as shown in *Eq. (24).*

$$M = \sqrt[n]{\prod_{i=1}^{n} d_{ij}^*} \text{ for } i, j = 1,2,, n, \quad \textbf{(20)}$$

where

$$i = 1, 2, \dots n, \ j = 1, 2, \dots n, t = 1, 2.. k; \ \otimes m_{ij} = \left[\left(m_{ij}^{\delta_T}, m_{ij}^{\delta_G}\right), \left(m_{ij}^{\varepsilon_T}, m_{ij}^{\varepsilon_G}\right), \left(m_{ij}^{\zeta_T}, m_{ij}^{\zeta_G}\right)\right]. \quad \textbf{(21)}$$

Step 6. Normalization of the Neutrosophic-Z-Number score.

Matrix $\otimes M = [\otimes m_{ij}]_{nxn}$ will be converted into a normalized matrix form $\otimes M^* = [\otimes m_{ij}^*]_{nxn}$ using *Eq. (21)* below:

$$m_{ij}^{*} = \frac{m_{ij}}{\sum_{k=1}^{n} m_{kj}} \text{ for } j = 1,2,,n, \quad (22)$$

where $\otimes m_{ij}^{*} = \left[\left(m_{ij}^{*\delta_T}, m_{ij}^{*\delta_G}\right), \left(m_{ij}^{*\varepsilon_T}, m_{ij}^{*\varepsilon_G}\right), \left(m_{ij}^{*\zeta_T}, m_{ij}^{*\zeta_G}\right)\right]$, where $\sum_{k=1}^{n} m_{kj}$ represents the column-wise summation of criteria in the aggregated matrix, and m_{kj} indicates the criterion's relative importance in the consolidated comparison matrix.

Step 7. Transform the pairwise comparison matrices for criteria to deneutrosophic form via *Eq. (11).*

$$\otimes A = DEF(\otimes M^{*}) = \frac{2 + \delta_{T1}\,\delta_{G1} - \varepsilon_{T1}\,\varepsilon_{G1} - \zeta_{T1}\,\zeta_{G1}}{3}, DEF(\otimes M^{*}) \in [0, 1]. \quad (23)$$

Step 8. The criteria weights are computed based on their respective scores.

$$\otimes w = [\otimes w_j]_{1xn} = \begin{matrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{matrix} \quad (24)$$

$$[\otimes w_j]_{1xn} = \frac{T_i}{\sum_{j=1}^{n} T_j} \text{ for } j = 1,2,,n.$$

Step 9. A Consistency Index (CI) of the column vector is computed to address discrepancies in the pairwise comparison matrix in order to assess the matrix's merit and consistency.

$$C = (C_1) = \otimes DEF(\otimes d_{ij}) \,.\, \otimes w_{nx1}^{T} = \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{matrix} \quad (25)$$

The following vector may now specify the attribute's Consistency Value (CV).

$$CV = (cvi_{nx1}) = \frac{c_i}{v_i};\ i = 1, 2, 3..., n. \quad (26)$$

Saaty [47] recommended adopting the maximal eigenvalue, λ_{max} which is calculated as follows, because different measurement scales have been employed for different properties.

$$\lambda_{max} = \frac{\sum_{i=1}^{n} cv_1}{n};\ i = 1, 2, 3..., n. \quad (27)$$

Calculate the CI for each matrix; the CI can be computed based on the *Eq. (21).* The consistency of expert pairwise comparisons is verified through this essential evaluation step.

$$CI = \frac{\lambda_{max} - n}{n-1}. \quad (28)$$

Step 8. Calculate the matrices' Consistency Ratio (CR). Each matrix's CI is divided by its Random Index (RI) to determine CR.

$$CR = \frac{CI}{RI}. \quad (29)$$

The matrices should be regarded as consistent if the CR values are less than 0.1 [48]; otherwise, the transitivity principle will be broken, therefore the decision-makers need update their evaluations. The RI values for each Saaty method matrix are displayed in *Table 5*.

Table 5. Random index.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57

4 | Case Study

Influencer marketing yields a 6.5x return on investment but faces challenges like scandals and fraud, costing $1.5 billion annually and eroding consumer trust [49], [50]. CGI influencers, with higher engagement rates (2.84% vs. 1.72%) and a $3.3 billion market in 2024, offer a solution with greater brand control [51], [52]. However, their adoption is limited among Vietnamese SMEs, which comprise 97% of enterprises and contribute 45% to GDP, with only 18% aware and 3% considering adoption [53], [54]. Cultural preferences for authenticity, technical barriers, and regulatory gaps hinder progress [55], [56]. While research has explored consumer perspectives [57], [58], little is known about SME decision-making in emerging markets. This study investigates barriers to CGI influencer adoption in Vietnam, focusing on cultural, economic, and technological factors.

Table 6. Factors related to the adoption of CGI influencer in marketing.

Constructs	Code	References	Factors	Code	References
Attitude toward behavior	ATT	[59]	Perceived ease of use	PEU	[59]
			Perceived usefulness	PUF	[60]
Perceived usefulness	PEU	[60]	Innovation Capability	ICP	[61]
			Compatibility	CPT	[62]
			Complexity	CPX	[63]
			Observability	OBS	[64]
			Organizational competency	OCP	[65]
			Organizational Readiness	ORE	[66]
			Trialability	TRI	[62]
Perceived ease of use	PUF	[67]	Compatibility	CPT	[62]
			Innovation Capability	ICP	[61]
			Organizational competency	OCP	[68]
			Organizational Readiness	ORE	[66]
			Relative Advantage	RAV	[69]
			Trialability	TRI	[70]
Subjective norm	SJN	[71]	Competitive Pressure	CPR	[72], [73]
			Mimetic competitor pressure	MCP	[74]
Top management support	TMS	[75]	Top management support	TMS	[75]

4.1 | Demographic of Experts

A questionnaire comprising factors associated with adopting CGI influencer was distributed to 15 experts within the marketing domain. Responses were received from 15 experts, and *Table 7* provides an overview of their demographics.

Table 7. Experts' demographic information.

EP's Information	EP1	EP2	EP3	EP4	EP5	EP6	EP7	EP8
Years of experience	5-10 years	11-20 years	11-20 years	5-10 years	Over 20 years	Over 20 years	5-10 years	5-10 years
Education level	Master	Master	Doctor	Master	Master	Master	Master	Doctor
Positions	C-level	D-level	M-level	C-level	D-level	M-level	C-level	D-level
Linguistic evaluation	VH	M	L	VL	L	VL	M	VH
EP's weights	0.08158	0.06881	0.06148	0.05292	0.05984	0.04343	0.07192	0.08158
EP's Information	**EP9**	**EP10**	**EP11**	**EP12**	**EP13**	**EP14**	**EP15**	
Years of experience	11-20 years	5-10 years	5-10 years	5-10 years	5-10 years	11-20 years	11-20 years	
Education level	Master	Master	Doctor	Master	Doctor	Master	Master	
Positions	M-level	C-level	M-level	C-level	D-level	M-level	C-level	
Linguistic evaluation	M	M	VL	M	VH	M	M	
EP's weights	0.06529	0.07192	0.05051	0.07192	0.08158	0.06529	0.07192	

A summary of the 15 respondents' demographic data is provided in *Table 8*, indicating a balanced panel in terms of gender, age, education level, and professional position. The demographic breakdown shows that 40% of the respondents were male, 40% female, and 20% identified as other. Most participants were younger than 30 (53.33%). Educationally, 73.33% held a master's degree, and in terms of position, 40% were at the C-level. These demographics strengthen the validity and trustworthiness of the study's findings by offering a thorough picture of the panel's variety and level of competence.

4.1.1 | Results Neutrosophic Z-number AHP

First, 15 decision-makers evaluated six key variable affecting PUF of CGI influencers for SMEs in Vietnam. Each expert was asked to rate the importance of each pair of comparison criteria according to the level of importance in the pairwise comparison matrix. This matrix is formulated by evaluating the identified criteria regarding their significance the decision-making context. Pairwise comparisons were gathered through a survey, and the outcomes are depicted in the table provided. For example, according to the level of importance filled in the pairwise comparison matrix according to aspect ORE in relation to CPT, if expert 1 thinks that ORE is very strongly influential to CPT (VI left cell), if expert 1 thinks that CPT is very strongly influential than ICP (SLI right cell).

Table 8. Initial comparison matrices of expert 1.

Variable	Left Criteria Is Greater									Right Criteria Is Greater								Variable
	ALI	**AI**	**VI**	**FI**	**SI**	**PI**	**SLI**	**WI**	**EI**	**WI**	**SLI**	**PI**	**SI**	**FI**	**VI**	**AI**	**ALI**	
ORE			x															CPT
ORE		x																ICP
ORE		x																OCP
ORE			x															TRI
ORE		x																RAV
CPT													x					ICP
CPT														x				OCP
CPT												x						TRI
CPT															x			RAV
ICP									x									OCP
ICP								x										TRI

Table 8. Continued.

Variable	Left Criteria Is Greater									Right Criteria Is Greater								Variable
	ALI	**AI**	**VI**	**FI**	**SI**	**PI**	**SLI**	**WI**	**EI**	**WI**	**SLI**	**PI**	**SI**	**FI**	**VI**	**AI**	**ALI**	
ICP										x								RAV
OCP				x														TRI
OCP					x													RAV
TRI											x							RAV

The evaluation matrix captures the preferences of Expert 1 across various variable. Each expert's assessments were subsequently transformed into a NZN comparison matrix, shown in *Table 9*.

Table 9. Neutrosophic-Z-number comparison matrix of expert 1.

	ORE	CPT	ICP	OCP	TRI	RAV
ORE	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.80,0.90),(0.25, 0.10),(0.20,0.10)]	[(0.85,0.85),(0.20, 0.10),(0.15,0.15)]	[(0.85,0.85),(0.20, 0.10),(0.15,0.15)]	[(0.80,0.90),(0.25, 0.10),(0.20,0.10)]	[(0.85,0.85),(0.20, 0.10),(0.15,0.15)]
CPT	[(0.20,0.90),(0.75, 0.10),(0.80,0.10)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.30,0.80),(0.70, 0.15),(0.70,0.20)]	[(0.25,0.70),(0.75, 0.25),(0.75,0.30)]	[(0.35,0.60),(0.70, 0.35),(0.65,0.40)]	[(0.20,0.90),(0.75, 0.10),(0.80,0.10)]
ICP	[(0.15,0.85),(0.80, 0.10),(0.85,0.15)]	[(0.70,0.80),(0.30, 0.15),(0.30,0.20)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.55,0.40),(0.40, 0.65),(0.45,0.60)]	[(0.45,0.40),(0.60, 0.65),(0.55,0.60)]
OCP	[(0.15,0.85),(0.80, 0.10),(0.85,0.15)]	[(0.75,0.70),(0.25, 0.25),(0.25,0.30)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.75,0.70),(0.25, 0.25),(0.25,0.30)]	[(0.70,0.80),(0.30, 0.15),(0.30,0.20)]
TRI	[(0.20,0.90),(0.75, 0.10),(0.80,0.10)]	[(0.65,0.60),(0.30, 0.35),(0.35,0.40)]	[(0.45,0.40),(0.60, 0.65),(0.55,0.60)]	[(0.25,0.70),(0.75, 0.25),(0.75,0.30)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.40,0.30),(0.65, 0.75),(0.60,0.70)]
RAV	[(0.15,0.85),(0.80, 0.10),(0.85,0.15)]	[(0.80,0.90),(0.25, 0.10),(0.20,0.10)]	[(0.55,0.40),(0.40, 0.65),(0.45,0.60)]	[(0.30,0.80),(0.70, 0.15),(0.70,0.20)]	[(0.60,0.30),(0.35, 0.75),(0.40,0.70)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]

Next, the NZN comparison matrix was integrated with the weights of the experts. Expert 1's weight is specified as 0.08158, resulting in the integrated NZN comparison matrix displayed in *Table 10*.

Table 10. Integrated expert's crips weight Neutrosophic Z-number comparison matrix of expert 1.

	ORE	CPT	ICP	OCP	TRI	RAV
ORE	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.12,0.17),(0.89, 0.83),(0.88,0.83)]	[(0.14,0.14),(0.88, 0.83),(0.86,0.86)]	[(0.14,0.14),(0.88, 0.83),(0.86,0.86)]	[(0.12,0.17),(0.89, 0.83),(0.88,0.83)]	[(0.14,0.14),(0.88, 0.83),(0.86,0.86)]
CPT	[(0.02,0.17),(0.98, 0.83),(0.98,0.83)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.03,0.12),(0.97, 0.86),(0.97,0.88)]	[(0.02,0.09),(0.98, 0.89),(0.98,0.91)]	[(0.03,0.07),(0.97, 0.92),(0.97,0.93)]	[(0.02,0.17),(0.98, 0.83),(0.98,0.83)]
ICP	[(0.01,0.14),(0.98, 0.83),(0.99,0.86)]	[(0.09,0.12),(0.91, 0.86),(0.91,0.88)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.06,0.04),(0.93, 0.97),(0.94,0.96)]	[(0.05,0.04),(0.96, 0.97),(0.95,0.96)]
OCP	[(0.01,0.14),(0.98, 0.83),(0.99,0.86)]	[(0.11,0.09),(0.89, 0.89),(0.89,0.91)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.11,0.09),(0.89, 0.89),(0.89,0.91)]	[(0.09,0.12),(0.91, 0.86),(0.91,0.88)]
TRI	[(0.02,0.17),(0.98, 0.83),(0.98,0.83)]	[(0.08,0.07),(0.91, 0.92),(0.92,0.93)]	[(0.05,0.04),(0.96, 0.97),(0.95,0.96)]	[(0.02,0.09),(0.98, 0.89),(0.98,0.91)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]	[(0.04,0.03),(0.97, 0.98),(0.96,0.97)]
RAV	[(0.01,0.14),(0.98, 0.83),(0.99,0.86)]	[(0.12,0.17),(0.89, 0.83),(0.88,0.83)]	[(0.06,0.04),(0.93, 0.97),(0.94,0.96)]	[(0.03,0.12),(0.97, 0.86),(0.97,0.88)]	[(0.07,0.03),(0.92, 0.98),(0.93,0.97)]	[(0.05,0.05),(0.95, 0.95),(0.95,0.95)]

After integrating the evaluations from all experts, we developed the aggregated comparison matrix presented in *Table 11*.

Table 11. Aggregrated comparison matrix of factor affecting PUF.

	ORE	CPT	ICP	OCP	TRI	RAV
ORE	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.79,0.89),(0.26, 0.11),(0.21,0.11)]	[(0.82,0.84),(0.22, 0.12),(0.18,0.16)]	[(0.81,0.85),(0.23, 0.12),(0.19,0.15)]	[(0.81,0.86),(0.23, 0.11),(0.19,0.14)]	[(0.81,0.87),(0.23, 0.11),(0.19,0.13)]
CPT	[(0.22,0.89),(0.74, 0.11),(0.78,0.11)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.30,0.76),(0.70, 0.20),(0.70,0.24)]	[(0.29,0.72),(0.72, 0.23),(0.71,0.28)]	[(0.37,0.61),(0.66, 0.36),(0.63,0.39)]	[(0.28,1.00),(0.71, 0.00),(0.72,0.00)]
ICP	[(0.18,0.84),(0.78, 0.12),(0.82,0.16)]	[(0.71,0.76),(0.29, 0.20),(0.29,0.24)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.47,0.46),(0.54, 0.55),(0.53,0.54)]	[(0.56,0.46),(0.40, 0.55),(0.44,0.54)]	[(0.47,0.43),(0.55, 0.60),(0.53,0.57)]
OCP	[(0.19,0.85),(0.77, 0.12),(0.81,0.15)]	[(0.73,0.72),(0.27, 0.23),(0.27,0.28)]	[(0.53,0.46),(0.44, 0.55),(0.47,0.54)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.69,0.65),(0.31, 0.32),(0.31,0.35)]	[(0.74,1.00),(0.26, 0.00),(0.26,0.00)]
TRI	[(0.19,0.86),(0.77, 0.11),(0.81,0.14)]	[(0.65,0.61),(0.32, 0.36),(0.35,0.39)]	[(0.45,0.46),(0.59, 0.55),(0.55,0.54)]	[(0.33,0.65),(0.68, 0.32),(0.67,0.35)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]	[(0.42,0.49),(0.61, 0.50),(0.58,0.51)]
RAV	[(0.19,0.87),(0.76, 0.11),(0.81,0.13)]	[(0.76,1.00),(0.26, 0.00),(0.24,0.00)]	[(0.53,0.43),(0.44, 0.60),(0.47,0.57)]	[(0.30,1.00),(0.72, 0.00),(0.70,0.00)]	[(0.59,0.49),(0.37, 0.50),(0.41,0.51)]	[(0.50,0.50),(0.50, 0.50),(0.50,0.50)]

After integrating the evaluations from all experts, we developed the normalized comparison matrix presented in *Table 12*.

Table 12. Normalized comparison matrix of factor affecting PUF.

	ORE	CPT	ICP	OCP	TRI	RAV
ORE	[(0.34,0.10),(0.12, 0.47),(0.11,0.42)]	[(0.19,0.20),(0.08, 0.08),(0.11,0.07)]	[(0.26,0.24),(0.08, 0.05),(0.06,0.06)]	[(0.30,0.20),(0.07, 0.07),(0.06,0.08)]	[(0.23,0.24),(0.09, 0.05),(0.07,0.06)]	[(0.25,0.20),(0.08, 0.06),(0.07,0.08)]
CPT	[(0.15,0.18),(0.17, 0.10),(0.17,0.09)]	[(0.12,0.11),(0.36, 0.36),(0.27,0.33)]	[(0.10,0.22),(0.24, 0.08),(0.24,0.10)]	[(0.11,0.17),(0.21, 0.14),(0.22,0.15)]	[(0.10,0.17),(0.27, 0.15),(0.26,0.16)]	[(0.09,0.23),(0.25, 0.00),(0.26,0.00)]
ICP	[(0.12,0.17),(0.18, 0.11),(0.18,0.14)]	[(0.17,0.17),(0.14, 0.14),(0.16,0.16)]	[(0.16,0.15),(0.17, 0.20),(0.17,0.20)]	[(0.18,0.11),(0.16, 0.32),(0.16,0.30)]	[(0.16,0.13),(0.16, 0.23),(0.18,0.22)]	[(0.15,0.10),(0.19, 0.35),(0.19,0.33)]
OCP	[(0.13,0.18),(0.18, 0.11),(0.18,0.12)]	[(0.18,0.16),(0.17, 0.17),(0.14,0.18)]	[(0.17,0.13),(0.15, 0.22),(0.16,0.21)]	[(0.18,0.12),(0.15, 0.29),(0.15,0.28)]	[(0.20,0.18),(0.12, 0.14),(0.13,0.14)]	[(0.23,0.23),(0.09, 0.00),(0.09,0.00)]
TRI	[(0.13,0.18),(0.18, 0.11),(0.18,0.11)]	[(0.16,0.14),(0.26, 0.26),(0.19,0.26)]	[(0.14,0.13),(0.20, 0.22),(0.19,0.21)]	[(0.12,0.16),(0.20, 0.19),(0.20,0.19)]	[(0.14,0.14),(0.20, 0.21),(0.20,0.21)]	[(0.13,0.11),(0.21, 0.29),(0.21,0.30)]
RAV	[(0.13,0.18),(0.18, 0.10),(0.18,0.11)]	[(0.18,0.22),(0.00, 0.00),(0.13,0.00)]	[(0.17,0.12),(0.15, 0.24),(0.16,0.22)]	[(0.11,0.24),(0.21, 0.00),(0.21,0.00)]	[(0.17,0.14),(0.15, 0.21),(0.16,0.21)]	[(0.16,0.12),(0.17, 0.29),(0.18,0.29)]

Then NZN matrices are transfromed to aggregated form using geometric mean, normalization of the Neutrosophic-Z-Number score and deneutrosophic to crips to calculating weight of each variable.

Table 13. Weights and ranking results of NZN-AHP factor affecting PUF.

	Geometric Mean of each Row	NZN Weight	Deneutrosophic	Weight	Ranking
ORE	[(0.26,0.19),(0.08,0.08),(0.08,0.09)]	[(0.26,0.20),(0.10,0.17),(0.08,0.18)]	0.67	17.12%	3
CPT	[(0.11,0.18),(0.24,0.00),(0.23,0.00)]	[(0.11,0.18),(0.29,0.00),(0.24,0.00)]	0.67	17.12%	4
ICP	[(0.15,0.13),(0.17,0.21),(0.17,0.21)]	[(0.16,0.14),(0.20,0.42),(0.18,0.42)]	0.62	15.80%	5
OCP	[(0.18,0.16),(0.14,0.00),(0.14,0.00)]	[(0.18,0.17),(0.17,0.00),(0.14,0.00)]	0.68	17.21%	1
TRI	[(0.14,0.14),(0.21,0.20),(0.20,0.20)]	[(0.14,0.15),(0.25,0.41),(0.20,0.40)]	0.61	15.60%	6
RAV	[(0.15,0.16),(0.00,0.00),(0.17,0.00)]	[(0.15,0.17),(0.00,0.00),(0.17,0.00)]	0.68	17.15%	2
	SUM		3.93		
	CI	0.0973			
	CR	0.0785			

Based on the results presented in *Table 13*, the weights and rankings of the NZN-AHP variables influencing PUF are outlined as follows: OCP holds the highest rank with a weight of 17.21%, followed closely by RAV at 17.15%. ORE and CPT are both assigned equal weights of 17.12%, ranking third and fourth, respectively. ICP follows with a weight of 15.80%, while TRI ranks last at 15.60%. The CI is calculated to be 0.0973, and the CR is 0.0785, both of which indicate that the pairwise comparisons are consistent, as the CR value is below the acceptable threshold of 0.10.

Applying the same calculation method above to the PEU, subjective norm and the results are shown in *Table 14*, *Table 15*.

Table 14. Weights and ranking results of NZN-AHP in factor affecting PEU.

	Geometric Mean of each Row	NZN Weight	Deneutrosophic	Weight	Ranking
OCP	[(0.18,0.14),(0.10,0.12),(0.11,0.13)]	[(0.18,0.15),(0.10,0.18),(0.11,0.18)]	0.66	14.35%	3
OBS	[(0.11,0.13),(0.18,0.15),(0.18,0.16)]	[(0.11,0.13),(0.19,0.21),(0.18,0.22)]	0.65	13.97%	7
CPX	[(0.13,0.12),(0.15,0.18),(0.15,0.17)]	[(0.14,0.12),(0.16,0.25),(0.15,0.24)]	0.65	14.00%	6
ORE	[(0.12,0.15),(0.14,0.00),(0.15,0.00)]	[(0.13,0.15),(0.15,0.00),(0.16,0.00)]	0.67	14.57%	2
ICP	[(0.16,0.15),(0.12,0.11),(0.11,0.12)]	[(0.16,0.15),(0.12,0.16),(0.12,0.16)]	0.66	14.33%	4

Table 14. Continued.

	Geometric Mean of each Row	NZN Weight	Deneutrosophic	Weight	Ranking
TRI	[(0.14,0.13),(0.14,0.14),(0.13,0.14)]	[(0.14,0.14),(0.14,0.20),(0.14,0.20)]	0.65	14.18%	5
CPT	[(0.13,0.16),(0.13,0.00),(0.14,0.00)]	[(0.14,0.16),(0.14,0.00),(0.14,0.00)]	0.67	14.60%	1
	SUM		4.62	1	
	CI	0.1007			
	CR	0.0763			

Based on the results in *Table 14*, the weights and rankings of the NZN-AHP variables influencing PEU are summarized as follows: CPT holds the highest rank with a weight of 14.60%, followed closely by ORE at 14.57%. OCP and ICP have similar weights of 14.35% and 14.33%, ranking third and fourth, respectively. TRI ranks fifth with a weight of 14.18%, followed by CPX at 14.00%. OBS ranks last with a weight of 13.97%. The CI is calculated as 0.1007, and the CR is 0.0763. Since the CR is below the acceptable threshold of 0.10, the pairwise comparisons are deemed consistent.

Table 15. Weights and ranking results of NZN-AHP in factor affecting subjective norm.

	Geometric Mean of each Row	NZN Weight	Deneutrosophic	Weight	Ranking
CPR	[(0.61,0.49),(0.35,0.46),(0.39,0.47)]	[(0.61,0.50),(0.35,0.50),(0.39,0.50)]	0.64	55.21%	1
MCP	[(0.39,0.49),(0.64,0.46),(0.61,0.47)]	[(0.39,0.50),(0.65,0.50),(0.61,0.50)]	0.52	44.79%	2
	SUM		1.17	1	
	CI	0.1946			
	CR	0.0000			

Based on the findings presented in *Table 15*, the NZN-AHP analysis identifies CPR as the most influential factor affecting subjective norm, with a weight of 55.21%. MCP ranks second with a weight of 44.79%. The CI is 0.1946, and the CR is 0.0000 confirms perfect consistency in the pairwise comparisons due to the number of the factors.

Table 16. Weights and ranking results of NZN-AHP among TAM and TRA constructs.

	Geometric Mean of each Row	NZN Weight	Deneutrosophic	Weight	Ranking
PEU	[(0.22,0.17),(0.18,0.24),(0.18,0.23)]	[(0.22,0.18),(0.18, 0.24),(0.19,0.24)]	0.65	19.90%	4
ATT	[(0.15,0.23),(0.25,0.15),(0.25,0.16)]	[(0.15,0.23),(0.26, 0.15),(0.25,0.16)]	0.65	19.95%	3
TMS	[(0.21,0.19),(0.18,0.20),(0.19,0.20)]	[(0.21,0.19),(0.18, 0.21),(0.19,0.21)]	0.65	20.03%	2
SJN	[(0.23,0.20),(0.16,0.18),(0.17,0.19)]	[(0.23,0.21),(0.17, 0.19),(0.17,0.19)]	0.66	20.23%	1
PUF	[(0.19,0.20),(0.22,0.20),(0.21,0.20)]	[(0.19,0.20),(0.22, 0.21),(0.21,0.20)]	0.65	19.89%	5
	SUM		3.27	1	
	CI	0.0726			
	CR	0.0000			

Table 16 presents the NZN-AHP weights and rankings for factors among the TAM and TRA constructs. SJN ranks highest with a weight of 20.23%, indicating its significant influence among the constructs. TMS follows

closely with a weight of 20.03%, showing its critical role in shaping perceptions and behavior. ATT ranks third with a weight of 19.95%, while holds the fourth position at 19.90%. Lastly, PUF is ranked fifth with a weight of 19.89%. The CI of 0.0726 and a CR of 0.0000 indicate excellent consistency in the pairwise comparisons.

The NZN-AHP approach calculates a sub-criterion's global weight by multiplying its criteria weight with its dimension weight. The final weight is then determined by combining the cumulative weights of factors across different constructs. This method helps assess each sub-criterion's overall importance within the hierarchical structure. It integrates the relative significance given to each higher-level criterion with the relative importance of each lower-level dimension, providing a comprehensive view of the decision-making framework, as illustrated in *Table 17*.

Table 17. Weighting results of NZN-AHP.

Constructs	Weight Construct	Rank	Factors	Weight Factor	Local Rank	Weight Global
ATT	19.95%	3	PEU	50.01%	1	9.95%
			PUF	49.99%	2	9.95%
PEU	19.90%	4	ICP	14.33%	4	2.86%
			CPT	14.60%	1	2.91%
			CPX	14.00%	6	2.79%
			OBS	13.97%	7	2.79%
			OCP	14.35%	3	2.86%
			ORE	14.57%	2	2.91%
			TRI	14.18%	5	2.83%
PUF	19.89%	5	CPT	17.12%	4	3.41%
			ICP	15.80%	5	3.14%
			OCP	17.21%	1	3.42%
			ORE	17.12%	3	3.41%
			RAV	17.15%	2	3.41%
			TRI	15.60%	6	3.10%
SJN	20.23%	1	CPR	55.21%	1	10.98%
			MCP	44.79%	2	8.91%
TMS	20.03%	2	TMS	100.00%	1	20.03%

Table 18 displays the ranking outcomes from the NZN-AHP analysis, focusing on key factors that influence decision-making intentions regarding the adoption of CGI influencers among SMEs. TMS ranks first with the highest global weight of 20.03%, underscoring its critical role in facilitating the adoption process within organizations. In second place, CPR at 10.99% highlights the necessity for SMEs to adapt to competitive dynamics when considering CGI influencer strategies. Both PEU and PUF share equal weights of 9.95%, ranking third and fourth, indicating the significance of user-friendly implementations and the tangible benefits perceived by decision-makers. MCP at 8.91% and CPT at 6.32% follow in the middle ranks, reflecting their moderate influence on decision-making. Closely positioned are ORE at 6.31% and OCP at 6.29%, both essential factors in ensuring that SMEs are equipped to effectively integrate CGI influencers. Lower-ranking factors include TRI at 5.93% and RAV at 3.42%, which relate to the testing and perceived benefits of CGI influencers. Finally, CPX at 2.79%, and OBS at 2.78% hold the least weight, suggesting their minimal impact on the decision-making process.

Table 18. Ranking results of NZN-AHP.

Factors Global	SUM Global -W	Rank
TMS	20.03%	1
CPR	10.99%	2
PEU	9.95%	3
PUF	9.95%	3
MCP	8.91%	5
CPT	6.32%	6
ORE	6.31%	7
OCP	6.29%	8
ICP	6,00%	9
TRI	5.93%	10
RAV	3.42%	11
CPX	2.79%	12
OBS	2.78%	13

5 | Conclusion

The NZN-AHP method marks a significant leap forward in MCDM by seamlessly integrating NZNs with the AHP. This innovative approach effectively addresses the limitations of traditional and fuzzy AHP methods by incorporating truth, indeterminacy, falsity, and reliability into the decision-making framework. By leveraging linguistic scales and advanced aggregation operators, such as Dombi and Aczel–Alsina, NZN-AHP ensures robust handling of vague, uncertain, and unreliable information, achieving consistent results with a CR below 0.1. The method's ability to model complex uncertainties and evaluate the reliability of expert judgments enhances decision-making precision across diverse domains. This study establishes NZN-AHP as a versatile and powerful tool, offering a structured yet flexible framework that outperforms existing group expert consensus techniques in capturing multifaceted uncertainties, paving the way for broader applications in MCDM contexts.

5.1 | Practical Implications

The NZN-AHP method provides practitioners with a robust tool for navigating complex decision-making scenarios characterized by uncertainty and incomplete information. By integrating NZNs, which account for truth, indeterminacy, falsity, and reliability, the method enables more accurate and transparent prioritization of criteria in fields like logistics, finance, and strategic planning. Its use of linguistic scales simplifies the elicitation of expert judgments, making it accessible to organizations with varying levels of technical expertise. Practitioners can apply NZN-AHP to enhance decision-making processes in resource allocation, supplier selection, or policy formulation, ensuring that decisions reflect both the uncertainty of data and the reliability of sources. The method's structured pairwise comparison approach, combined with advanced aggregation techniques, supports consistent and reliable outcomes, empowering decision-makers to tackle real-world challenges with greater confidence and precision.

5.2 | Limitations and Future Research

Theoretically, NZN-AHP significantly advances MCDM research by introducing a novel framework that combines the uncertainty-handling capabilities of NZNs with AHP's hierarchical structure. This integration addresses the shortcomings of traditional FSs and existing MCDM methods, such as SF-AHP, by explicitly modeling indeterminacy and reliability, thus offering a more comprehensive representation of complex decision-making environments. The use of advanced aggregation operators, including Dombi and Aczel–Alsina, enhances the flexibility and accuracy of preference aggregation, contributing to the evolution of fuzzy decision-making theories. This study lays a foundation for further exploration of NZNs in other MCDM frameworks, such as TOPSIS, VIKOR, or PROMETHEE, and encourages the development of new aggregation operators tailored to specific decision-making contexts. By bridging Neutrosophic logic with AHP, NZN-AHP opens new avenues for theoretical advancements in handling uncertainty and reliability in decision sciences.

Authors' Contributions

P. H. N.: Research Design, Conceptualization, and Validation. L. A. T. N.: Data Curation, Computing, and Editing. T. G. V.: Methodology, Visualization and Formal Analysis. M. T. P.: Research Design and Validation V. H. N.: Visualization and Formal Analysis. D. L. D.: Methodology and Formal Analysis. The authors have read and agreed to the published version of the manuscript.

Consent for Publication

Consent for publication has been obtained from the authors.

Ethics Approval and Consent to Participate

This study does not involve any research conducted on human participants or animals.

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Data Availability

All data are included in the text.

Conflict of Interest

The authors declare no conflict of interest.

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