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Enhancing Project Scheduling with Neutrosophic Sets: New Solution Approaches for Solving Neutrosophic CPM/PERT

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Abstract

The Critical Path Method (CPM) is an important tool in project management. However, the traditional form of CPM deals with complications associated with the ambiguity and uncertainty in estimating the duration of activities. This paper presents two new methods to solve the Neutrosophic Critical Path Method (Neu-CPM), utilizing Triangular Neutrosophic Numbers (TNNs) to define activity durations under indeterminacy. The methods are designed to conduct a forward pass and backward pass simultaneously to find the earliest and latest time for each event while at the same time to find the total float for each activity, enabling project scheduling under uncertain conditions. Neu-CPM provides a more improved approach to handling non-precise and incomplete data compared to the traditional fuzzy or intuitionistic approaches, based on its inclusion of membership degrees of truth, indeterminacy, and falsity. A numerical example is provided showing the methodology's ability to identify the project's critical path in a neutrosophic environment while studying the effect of various risk elements on the critical path. The results show that Neu-CPM provides the opportunity of more flexibility, accuracy, and reliability in project scheduling in uncertain conditions, with useful applications to practice.

Keywords: Neutrosophic set, Project scheduling, Triangular neutrosophic number, Critical path method.

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1|Introduction

The Critical Path Method or CPM is an advanced project management methodology that helps you identify the longest sequence of dependent tasks (critical path), providing you with the shortest duration possible to complete the project [1]. By declaring the duration and interdependencies of tasks, CPM allows you to optimize schedules and resources and identify completion dates on tasks that could delay the overall schedule if not performed timely [2]. CPM is commonly used in construction, engineering, software development, and manufacturing, all of which require a schedule as part of the project success [3]. An important limitation of CPM has been its assumption of fixed activity durations, which can often be unrealistic in the changing world of delays and unknowns [4]. When activity durations have an expectation of uncertainty, traditional CPM can produce schedules that cannot be relied upon. In cases where unreliable activity durations are expected, traditional CPM can be augmented with methods such as a Program Evaluation and Review Technique (PERT) and the Monte Carlo method to model duration that is probabilistic in nature [5]. However, these methods require a large amount of historical data and may not sufficiently capture subjective uncertainties, which leads to the consideration of fuzzy logic-based approaches [6].

Zadeh [6] describes fuzzy set theory further developed from classical set theory, where partial membership is allowable, and by using membership functions they are able to express vague or uncertain data as well. Unlike probabilistic approaches, fuzzy logic studies uncertainty without representing the uncertainty using true probability distributions [7]. In project scheduling, fuzzy sets can represent quantities that have uncertain duration using examples of linguistic variables such as: "short", "medium" and "long"; allowing for a less structured way of making decisions when uncertainty is present. Fuzzy Critical Path Method (FCPM) integrates fuzzy set theory and traditional CPM, which allows for representation of uncertain activity durations as fuzzy numbers [8]. This method allows for more realistic scheduling in uncertain environments; however it does have limitations. The fuzzy arithmetic makes the process more computationally intensive and it may present difficulty for practicing professionals when trying to interpret the fuzzy critical paths [9]. In addition, fuzzy CPM is also very dependent on expert judgment to establish membership functions, which is also susceptible to bias [10]. Despite these issues, FCPM remains a useful tool to apply in very uncertain project environments.

Fuzzy set theory captures uncertainty well within the framework of membership functions, whereas hesitation or indeterminacy is difficult to represent [11]. One major problem is that fuzzy sets only take into account the membership (μ) of an element. Fuzzy sets ignore the non-membership degree (ν) and the hesitation margin (π) that occurs when experts are uncertain about their assessments [11]. Without proper representations of IFS, uncertainty in settings with complexity can lead to incorrect or biased assumptions as more information tends to be missing or viewed as incomplete. With the need to address the limitations of fuzzy sets, Atanassov (1986) [11] introduced Intuitionistic Fuzzy Sets (IFS) to extend fuzzy sets by including degrees of uncertainty (i.e., membership (μ) and non-membership (ν), where $0 \leq \mu + \nu \leq 1$, with overlap reflecting the remaining value ($\pi = 1 - \mu - \nu$), in regard to unsure. Therefore, instead of a soft set, IFS can represent uncertainty, particularly in assessing the extent of incomplete or conflicting expert judgments [12]. Due to its practicality, IFS has received large attention for application in decision-making, risk assessment, and even project scheduling, where fuzzy set theory does not measure ambiguity in the real world. Although IFS is a step forward from conventional fuzzy sets using membership degree (μ), non-membership degree (ν), and hesitation degree (π), IFS still have limitations when it comes to representing indeterminate, inconsistent, or incomplete information [13]. IFS assumes that $\mu + \nu \leq 1$, but could lead to the decision-makers facing problems that they would categorize with $\mu + \nu > 1$ when they have competing evidence or indeterminate (neutral) information [14]. Neutrosophic Sets (NS) were defined and developed by Smarandache (1999) [13], and expanded upon the developing work of IFS, where they are presented in the constructs of truth-membership (T), indeterminacy-membership (I), and falsity-membership (F), where $0 \leq T + I + F \leq 3$. This offers greater flexibility with uncertainty, particularly with incomplete knowledge, contradictions, neutrality issues [15]. Neutrosophic theory has applications in many fields including decision-making, medical diagnosis, engineering, artificial intelligence and machine learning. It can also be used in cybersecurity, data analysis, sensor data fusion, risk assessment, pattern matching and information retrieval [16-22]. Its usefulness comes from being able to handle indeterminate, unknown and inconsistent information, which makes it helpful in improving analytical efforts, optimizing systems, and improving the accuracy of decision-making in complex and uncertain situations.

Recent studies have illustrated that neutrosophic logic is a viable technique for dealing with uncertainties arising from project scheduling. The traditional techniques used in project scheduling, such as CPM and PERT, typically rely on deterministic or fuzzy methods that merely fail to account for ambiguity in real-world situations. for example, Mohamed et al. [23] pioneered neutrosophic CPM using triangular neutrosophic numbers (TNNs), which is now a logical framework to deal with ambiguous activity durations and exhibit the best decision-making study. Priyadharsini et al. [24] also extended neutrosophic logic by introducing the Triangular Neutrosophic PERT (TNP) allowing for uncertainties within textile project development, which exhibited a more accurate predictive measure of time and cost. Again, this study was further developed by Pratyusha and Kumar [25] who complemented decision making through the use of neutrophysics logic and time-cost tradeoff analysis to demonstrate the feasibility of an integrated platform in conditions of uncertainty. Similarly, Sinika and Ramesh [26] introduced trapezoidal neutrosophic sets for an extended analysis of critical paths further allowing the deportation of uncertainty. In another study, Secretariat [27] developed a Neutrosophic CPM (NCPM) in Python software to demonstrate an ability to conduct computations for larger projects in terms of feasibility and consistency. In addition, Romero et al. [28] developed neutrosophic statistics protocols applied to PERT enabled improvements for forecasting durations for IT projects demonstrating the effective use of neutrosophic representations for uncertainties.

These studies revealed the advantages of using neutrosophic methods as uncertainty representation, advantages to scheduling benefits, and improved decision-making. however, challenges to this streamline approach remain with regards to computational efficiencies and real-world implications. Future research should envisage new hybrid model combinations of neutrosophic logic being used with machine learning constructing hybrid protocols for project planning into a dynamic and evolving project environment.

The primary purpose of the article is explained below

(1) The Neutrosophic Critical Path Method (Neu-CPM) is designed by extending the Critical Path Method, utilising Neutrosophic Sets (NS) to account for uncertainties in the durations and costs of tasks of project management.

(2) Neu-CPM, using Triangular Neutrosophic Numbers (TNNs) adds an additional level of uncertainty as it better describes the vagueness which enhances the reality of project planning and established schedules.

(3) This study proposes two techniques for calculating the critical path based on varying levels of risk associated with a risk factor $\delta \in [0, 1]$.

(4) The possibility mean function is then employed to convert the Neutrosophic activity into the relevant crisp activity.

The subsequent sections of this article have been organized in a definitive way: Section 2 contains the definitions, results, and theorems related to the Neutrosophic Set. Section 3 discusses the development of Neutrosophic CPM by modeling the activity as a Triangular Neutrosophic Number. Section 4 discusses two ways to solve CPM problems and gives the algorithms, and Section 5 include two numerical examples to demonstrate the applied nature of the methodology. Finally, Section 6 concludes with comments and thoughts about the findings as well as possible directions for future research.

2|Preliminary Concepts

Preliminary. [[6]] The fuzzy set (FS) $\hat{F}$ is defined as

$$\hat{F} = \{\langle x; \mu_F \rangle : x \in \Omega\}, \tag{1}$$

where $\mu_F : \Omega \to [0, 1]$ is the membership grade function. [[11]] The Intuitionistic fuzzy set (IFS) $\hat{I}$ is defined as

$$\hat{I} = \{\langle x; \mu_I, \nu_I \rangle : x \in \Omega\}, \tag{2}$$

where $\mu_I, \nu_I : \Omega \to [0, 1]$ is the membership and non-membership grades and satisfied $0 \leq \mu_I + \nu_I \leq 1$. [[13]] The Neutrosophic set (NS) $\hat{N}$ is defined as

$$\hat{N} = \{\langle x; \phi_N, \varphi_N, \psi_N \rangle : x \in \Omega\}, \tag{3}$$

where $\phi_N, \varphi_N, \psi_N : \Omega \to [0,1]$ are the truth, indeterminate and falsity membership grades and satisfy $0 \leq \mu_N + \nu_N + \omega_N \leq 3$.

[[29]] Triangular neutrosophic number (TNN) is denoted by $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$, in which there are three membership grades of x are given below:

$$\mathfrak{T}(x) = \begin{cases} \dfrac{x - x^L}{x^M - x^L}\phi_x, \ x^L \leq x \leq x^M \\ \phi_x, \ x = x^M \\ \dfrac{x^U - x}{x^U - x^M}\phi_x, \ x^M \leq x \leq x^U \\ 0, \ otherwise \end{cases} \qquad \mathfrak{I}(x) = \begin{cases} \dfrac{x^M - x + \varphi_x(x - x^L)}{x^M - x^L}, \ x^L \leq x \leq x^M \\ \varphi_x, \ x = a^M \\ \dfrac{x - x^M + \varphi_x(x^U - x)}{x^U - x^M} \ x^M \leq x \leq x^U \\ 1, \ otherwise \end{cases}$$

$$\mathfrak{F}(x) = \begin{cases} \dfrac{x^M - x + \psi_x(x - x^L)}{x^M - x^L}, \ x^L \leq x \leq x^M \\ \psi_x, \ x = x^M \\ \dfrac{x - x^M + \psi_x(x^U - x)}{x^U - x^M}, \ x^M \leq x \leq x^U \\ 1, \ otherwise \end{cases}$$

where $0 \leq \mathfrak{T}(x) + \mathfrak{I}(x) + \mathfrak{F}(x) \leq 3, \ x \in \Omega$.

[[29]] Suppose $\widehat{X}_1 = \langle x_1^L, x_1^M, x_1^U; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle$ and $\widehat{X}_2 = \langle x_2^L, x_2^M, x_2^U; \phi_{x_2}, \varphi_{x_2}, \psi_{x_2}\rangle$ are two TNNs. The arithmetic operations are given as

(1) $\widehat{X}_1 \oplus \widehat{X}_2 = \langle x_1^L + x_2^L, x_1^M + x_2^M, x_1^U + x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.

(2) $\widehat{X}_1 - \widehat{X}_2 = \langle x_1^L - x_2^U, x_1^M - x_2^M, x_1^U - x_2^L; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.

(3) $\widehat{X}_1 \otimes \widehat{X}_2 = \langle x_1^L x_2^L, x_1^M x_2^M, x_1^U x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.

(4) $\lambda\widehat{X}_1 = \begin{cases} \langle \lambda x_1^L, \lambda x_1^M, \lambda x_1^U; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, \ \lambda > 0 \\ \langle \lambda x_1^U, \lambda x_1^M, \lambda x_1^L; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, \ \lambda < 0 \end{cases}$
where $a \wedge b = \min(a,b)$ and $a \vee b = \max(a,b)$.

[[30]] The α, β & $\gamma-$cut for a TNN $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$, is defined as

$$\widehat{X}^{(\alpha,\beta,\gamma)} = \{x : \phi_x \geq \alpha, \varphi_x \leq \beta, \psi_x \leq \gamma\}, \tag{4}$$

where $0 \leq \alpha \leq \phi_x, \varphi_x \leq \beta \leq 1$ and $\psi_x \leq \gamma \leq 1$.

[[31]] The possibility means over the risk parameter $\delta \in [0,1]$ of truth, indeterminate, and falsity membership grades of $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$ are redefined as:

$$\widetilde{\Im}(\widehat{X}) = \delta \ \mu(\widehat{X}_\alpha) + (1-\delta)\Big(\frac{\mu(\widehat{X}_\beta) + \mu(\widehat{X}_\gamma)}{2}\Big). \tag{5}$$

That implies

$$\widetilde{\Im}(\widehat{X}) = \delta\Big(\frac{x^L + 4x^M + x^U}{6}\Big)\phi_x^2 + \frac{1-\delta}{2}\Bigg(\frac{2[x^L + x^M + x^U] - [x^L - 2x^M + x^U]\varphi_x - [x^L + 4x^M + x^U]\varphi_x^2}{6} + \frac{2[x^L + x^M + x^U] - [x^L - 2x^M + x^U]\psi_x - [x^L + 4x^M + x^U]\psi_x^2}{6}\Bigg). \tag{6}$$

[[31]] For two TNNs $\widehat{X_1}$ and $\widehat{X_2}$, we say that

(1) $\widehat{X_1} \preccurlyeq \widehat{X_2}$ if and only if $\widetilde{\Im}(\widehat{X_1}) \leq \widetilde{\Im}(\widehat{X_2})$,

(2) $\widehat{X_1} \prec \widehat{X_2}$ if and only if $\widetilde{\Im}(\widehat{X_1}) < \widetilde{\Im}(\widehat{X_2})$,

(3) $\widehat{X_1} \approx \widehat{X_2}$ if and only if $\widetilde{\Im}(\widehat{X_1}) = \widetilde{\Im}(\widehat{X_2})$,

where $\widetilde{\Im}(.)$ is the possibility mean function for TNN.

3|Development of Neutrosophic CPM

Consider network $N = \langle E_{ij} \rangle$, being a project model, is given. E is asset of events(nodes) and $A \subset E \times E$ is a set of activities. The set $E = \{1, 2, \ldots, n\}$ is labeled in such a way that the following condition holds: $(i, j) \in A$ and $i < j$. The activity times in the network are determined by T_{ij}. The following are the essential notations in the Neu-CPM which generalizes the classical approach to project scheduling by adding uncertainty through Neutrosophic sets.

T_{ie} =Earliest occurrence time of predecessor event i,
T_{il} = Latest occurrence time of predecessor event i,
T_{je}=Earliest occurrence time of successor event j,
T_{jl} = Latest occurrence time of successor event j
ET = Earliest start time of an activity ij,
LT =Earliest finish time of an activity ij,
T_{ijl} Start=Latest start time of an T_{il} activity ij,
T_{ijl} Finish t = Latest finish time of an activity ij,
T_{ij} = Duration time of activity ij,

Earliest and Latest occurrence time of an event
T_{je} = maximum $(T_{je} + T_{ij})$, calculate all T_{je} for jth event, select maximum value.
T_{il} = minimum $(T_{jl} - T_{ij})$, calculate all T_{il} for ith event, select minimum value.
$ET = T_{ie}$
$LT= T_{ie} + T_{ij}$,
$ET = T_{jl}$,
$ET = T_{jl} - Tij$,
TS_{ij} = The neutrosophic fuzzy total float of the activity i-j.

An abstract diagram is given in Figure 1, which shows that the Neutrosophic CPM/PERT for better understand. The activities are considered as triangular Neutrosophic number.

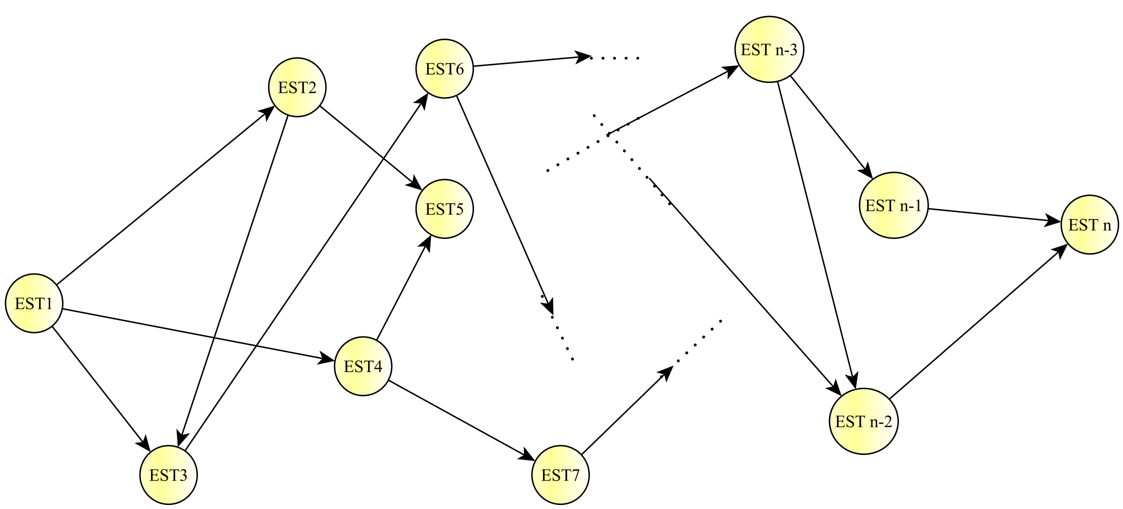

Fig. 1. Conceptual diagram for Neutrosophic CPM.

4|Solution Techniques for Neutrosophic CPM/PERT

There are two technique proposed to solve the Neutrosophic CPM/PERT. These techniques are used for finding the CPM/PERT problem under Neutrosophic environment.

4.1|Solution Technique 1

Forward Pass Calculation

(1) Choose the risk factor $\delta \in [0,1]$

(2) Calculate the possibility activity during by using possibility mean function.

(3) The computation begins from the start node and move towards the end node. For easiness, the forward pass computation starts by assuming the earliest occurrence time of zero for the initial project event.

(4) Earliest starting time of activity (i,j) is the earliest event time of the tail end event i.e. $(Es)_{ij} = E_i$

(5) Earliest finish time of activity (i,j) is the earliest starting time + the activity time i.e $(Ef)_{ij} = (Es)_{ij} + T_{ij}$ or $(Ef)_{ij} = E_i + T_{ij}$

(6) Earliest event time for event j is the maximum of the earliest finish times of all activities ending in to that event i.e $E_j = max[(Ef)_{ij}$ for all immediate predecessor of $(i,j)]$ or $E_j = max[E_i + T_{ij}]$

Backward Pass Calculation

(1) Choose the risk factor $\delta \in [0,1]$.

(2) Calculate the possibility activity duration by using possibility mean function.

(3) For ending event assume E=L Remember that all E's have been computed by forward pass computations.

(4) Latest finish time for activity (i,j) is equal to the j i.e. $(Lf)_{ij} = L_j$

(5) Latest starting time of activity (i,j) = the latest completion time of (i,j) - the activity time or $(Ls)_{ij} = (Lf)_{ij} - T_{ij} or (Ls)_{ij} = L_j - T_{ij}$

(6) Latest event time for 'i' is the minimum of the latest start time of all activities originating from that event i.e. $L_i = min[(Ls)_{ij} forallimmediatesuccessorof(i,j)] = min[(Lf)_{ij} - T_i j] = min[L_j - T_{ij}]$

4.2|Solution Technique 2

Forward Pass Calculation

(1) Construct the fuzzy project network using Neutrosophic numbers as activity durations and assign event numbers accordingly.

(2) Represent each Neutrosophic activity duration in Neutrosophic Triangular Number.

(3) Assume that the Neutrosophic earliest starting time of the initial event is zero, i.e.,

$$EST_1 = \langle 0, 0, 0; 1, 0 \rangle.$$

(4) Compute the earliest start time for each event using:

$$EST_j = \max\{EST_i \oplus T_{ij} \mid i \in NP(j), j \neq 1\}, \quad j = 2, 3, \ldots, n,$$

where $NP(j)$ represents the set of all predecessor nodes of event j.

(5) Using the ranking function equation (**??**), Select the maximum value of EST_j.

(6) Compute the Neutrosophic earliest finish time for each activity using:

$$EFT_j = EST_j \oplus T_{ij}.$$

Backward Pass Calculation Forward Pass Calculation

(1) Calculate fuzzy project network having Neutrosophic Fuzzy numbers as activity duration and numbering the events.

(2) Represent each Neutrosophic fuzzy activity duration into Triangular Neutrosphic Number.

(3) Assume that Neutrosophic fuzzy earliest starting time of initial event is zero, i.e., $ET_J = \langle 0,0,0;1,0,0\rangle$ for j=1.

(4) Compute the earliest start for each events by using

$$ET_j = max\ ET_j \bigoplus T_{ij}, forj \neq 1, j = 2,3,....n.$$

(5) By using the possibility mean function, Select the maximum value of ET_j.

(6) Neutrosophic fuzzy earliest finish time for each activity is

$$ET_j = (ET_j \bigoplus T_{ij}).$$

Backward Pass Calculation

(1) Assume $LT_n = ET_n$, since Neutrosophic fuzzy latest finish of all the end activities are taken as the earliest completion of the project network, $LT_J, j = n-1, n-2, \cdots, 2, 1$ by using $LT_j = \min_J LT_J - T_{IJ}$

(2) By using the ranking function equation(3.6), choose the maximum value of LT_j

(3) Neutrosophic fuzzy latest start time for each activity is $LT_{ij} = (LT_j - T_{ij})$

(4) Calculate total float $TF_{ij} = min(LT_{ij} - (ET_{ij} \bigoplus T_{ij}))$

(5) The neutrosophic fuzzy activity is said to be a critical activity if and only if its total $TF_{ij} = 0$. Critical path is the longest path from initial event to terminal event in project network having maximum duration. All activities in a critical path are called critical activities.

The **critical path** is the longest path from the initial event to the terminal event in the project network, having the maximum duration. All activities in a critical path are called **critical activities**.

5|Numerical Examples

A relevant project case can validate and illustrate the computational process of Neutrosophic critical path analysis. The project activities are denoted by Neutrosophic number representations, with sequencing done by performing the forward and backward pass, along with Neutrosophic arithmetic operations and possibility mean functions. This is a plausible approach to uncertainty for a project schedule as it offers a more flexible and realistic specification of project constraints than using deterministic processes.

Suppose there is a project network with the set of node N={1, 2, 3, 4, 5, 6,7} the neutrosophic fuzzy activity time for each activity as shown in Table 1. The study highlights the use of Neutrosophic Critical Path Analysis (Neu-CPA) to handle uncertainty in project scheduling. Two different approaches were used to assess the project network as shown in Figure 2, in which activity durations are triangular neutrosophic numbers (TNNs). Technique 1 replaced neutrosophic durations with "crisp" values using a possibility mean function, while technique 2 preserved the neutrosophic format for computations.

Technique 1:
For $\delta = 0$,

(1) To calculate the earliest start time:
set $E_1 = 0$
$E_2 = E_1 \bigoplus t_{12} = 3.60$
$E_3 = E_1 \bigoplus t_{13} = 2.49$
$E_4 = max\{E_2 \bigoplus T_{24}, E_3 \bigoplus T_{34}\} = 6.46$

Table 1. The fuzzy activity time for each activity in the project network shown as figure.

Activity	Immediate Predecessors	Activity Time Duration
$1 \to 2$	$-$	$\langle 3, 4, 5, 0.5, 0.4, 0.2 \rangle$
$1 \to 3$	$-$	$\langle 2, 3, 4, 0.6, 0.5, 0.3 \rangle$
$2 \to 4$	A	$\langle 1, 4, 5, 0.4, 0.5, 0.6 \rangle$
$3 \to 4$	B	$\langle 4, 5, 6, 0.3, 0.4, 0.5 \rangle$
$3 \to 5$	B	$\langle 7, 8, 9, 0.7, 0.8, 0.9 \rangle$
$3 \to 6$	B	$\langle 5, 6, 7, 0.1, 0.7, 0.8 \rangle$
$4 \to 7$	C, D	$\langle 2, 4, 6, 0.8, 0.6, 0.1 \rangle$
$5 \to 7$	E, I	$\langle 3, 5, 8, 0.5, 0.6, 0.9 \rangle$
$6 \to 5$	F	$\langle 3, 6, 7, 0.4, 0.3, 0.1 \rangle$
$6 \to 7$	F	$\langle 4, 7, 9, 0.7, 0.6, 0.2 \rangle$

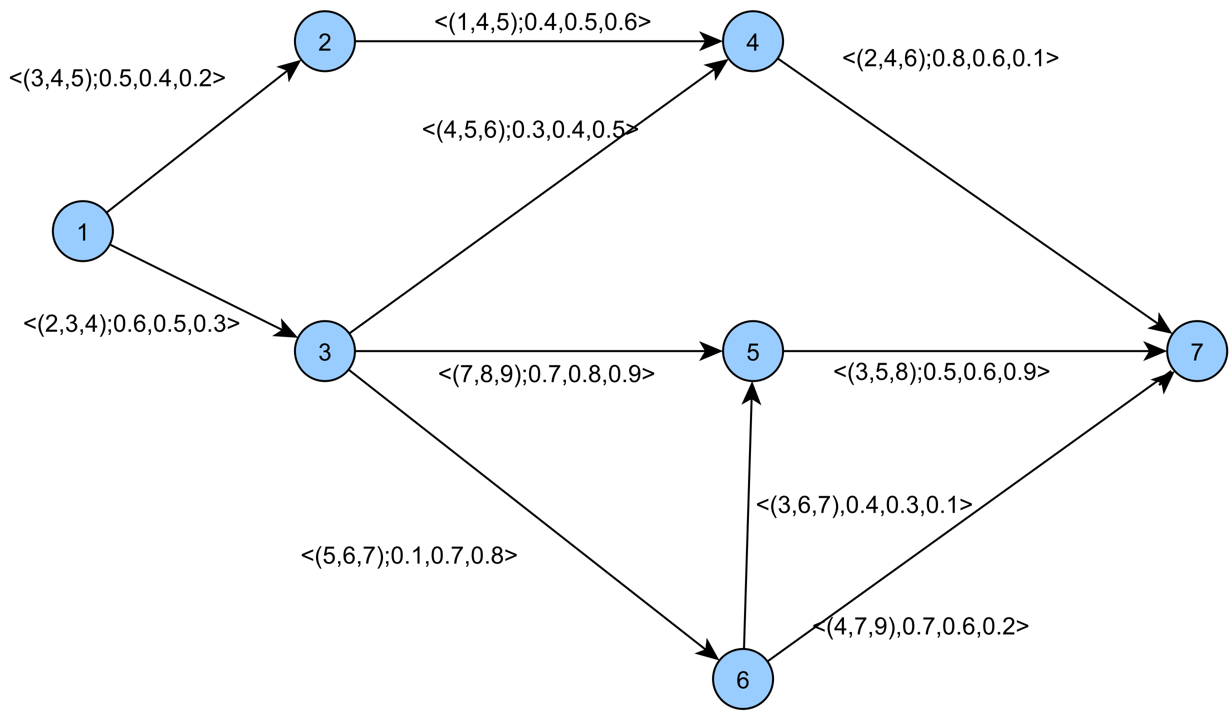

Fig. 2. Critical path analysis.

$E_5 = max\{E_3 \bigoplus T_{35}, E_6 \bigoplus T_{65}\} = 10.21$
$E_6 = E_3 \bigoplus t_{36} = 5.1$
$E_7 = max\{E_4 \bigoplus T_{47}, E_5 \bigoplus T_{57}, E_6 \bigoplus T_{67}\} = 12.39$

(2) To calculate the latest Finish time:
$L_7 = 12.39$
$L_5 = L_7 - t_{57} = 10.21$
$L_6 = min\{L_7 - t_{67}, L_5 - t_{65}\} = 5.1$
$L_4 = L_7 - t_{47} = 9.13$
$L_3 = min\{L_4 - t_{34}, L_5 - t_{35}, L_6 - t_{36}\} = 2.49$
$L_2 = L_4 - t_{24} = 6.74$
$L_1 = min\{L_2 - t_{12}, L_3 - t_{13}\} = 0$

Similarly, for other value of the risk factor $\delta \in [0,1]$, The corresponding crisp value is calculated in Table 2. For the value of $\delta = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ the critical path remain same i.e., $1 \to 3 \to 6 \to 5 \to 7$ but the value of $\delta = 0.7, 0.8, 0.9, 1$ the critical path changed i.e., $1 \to 3 \to 5 \to 7$. So the changed critical path shown in the below Figure 3 and 4. For $\delta = 0$, we generated the earliest start (ES) and latest finish (LF) times, revealing the

Table 2. The crisp value of the Nutrosophic activity time duration.

Activity	δ=0	δ=0.1	δ =0.2	δ=0.3	δ=0.4	δ=0.5	δ=0.6	δ=0.7	δ=0.8	δ=0.9	δ=1
t_{12}	3.6	3.34	3.08	2.82	2.56	2.3	2.04	1.78	1.52	1.26	1
t_{13}	2.49	2.349	2.208	2.067	1.926	1.785	1.644	1.503	1.362	1.221	1.08
t_{24}	2.3983	2.2172	2.036	1.8548	1.6737	1.4925	1.3113	1.1302	0.949	0.7678	0.5867
t_{34}	3.975	3.6225	3.27	2.9175	2.565	2.2125	1.86	1.5075	1.155	0.8025	0.45
t_{35}	2.2	2.372	2.544	2.716	2.888	3.06	3.232	3.404	3.576	3.748	3.92
t_{36}	2.61	2.355	2.1	1.845	1.59	1.335	1.08	0.825	0.57	0.315	0.06
t_{47}	3.26	3.19	3.12	3.05	2.98	2.91	2.84	2.77	2.7	2.63	2.56
t_{57}	2.1858	2.0964	2.007	1.9176	1.8282	1.7388	1.6493	1.559	1.4705	1.3811	1.2917
t_{65}	5.1167	4.6957	4.2747	3.8537	3.4327	3.0117	2.5907	2.1697	1.7487	1.3277	0.9067
t_{67}	5.3667	5.1648	4.963	4.7612	4.5593	4.3575	4.1557	3.9538	3.752	3.5502	3.3483

critical path as $1 \to 3 \to 6 \to 5 \to 7$ with an overall project duration of 12.39 units. The critical path did not change as we increased the risk factor δ from 0 to 0.6, but the duration of the project decreased (Table 2). For $\delta \geq 0.7$, however, the critical path changed to $1 \to 3 \to 5 \to 7$ (Figures 3 and 4) demonstrating how increasing uncertainty changes the activities that act as bottlenecks of performance on the project.

Technique 2

(1) To calculate the triangular neutrosophic earliest start activity time

Set as $E_1 = \langle 0, 0, 0, 1, 0, 0 \rangle$

$E_2 = E_1 \bigoplus t_{12} = \langle 3, 4, 5, 0.5, 0.4, 0.2 \rangle$

$E_3 = E_1 \bigoplus t_{13} = \langle 2, 3, 4, 0.6, 0.5, 0.3$

$E_4 = max\{E_2 \bigoplus T_{24}, E_3 \bigoplus T_{34}\} = \langle 6, 8, 10, 0.3, 0.5, 0.5 \rangle$

$E_5 = max\{E_3 \bigoplus T_{35}, E_6 \bigoplus T_{65}\} = \langle 10, 15, 18, 0.1, 0.7, 0.8 \rangle$

$E_6 = E_3 \bigoplus t_{36} = \langle 7, 9, 11, 0.1, 0.7, 0.8 \rangle$

$E_7 = max\{E_4 \bigoplus T_{47}, E_5 \bigoplus T_{57}, E_6 \bigoplus T_{67}\} = \langle 8, 12, 16, 0.3, 0.6, 0.5 \rangle$

(2) To calculate the latest Finish time:

Set as $L_7 = E_7 = \langle 8, 12, 16, 0.3, 0.6, 0.5 \rangle$

$L_5 = L_7 - t_{57} = \langle 5, 7, 8, 0.3, 0.6, 0.9 \rangle$

$L_6 = min\{L_7 - t_{67}, L_5 - t_{65}\} = \langle 4, 5, 7, 0.3, 0.6, 0.5 \rangle$

$L_4 = L_7 - t_{47} = \langle 6, 8, 10, 0.3, 0.6, 0.5 \rangle$

$L_3 = min\{L_4 - t_{34}, L_5 - t_{35}, L_6 - t_{36}\} = \langle -2, -1, -1, 0.3, 0.8, 0.9 \rangle$

$L_2 = L_4 - t_{24} = \langle 5, 4, 5, 0.3, 0.6, 0.6 \rangle$

$L_1 = min\{L_2 - t_{12}, L_3 - t_{13}\} = \langle 2, 0, 0, 0.3, 0.6, 0.6 \rangle$

For different value of the risk factor $\delta \in [0,1]$ are calculated in Table 3.

This method kept the neutrosophic structure in calculating ES and LF in TNN form. The results (Table 3) showed that project duration varied significantly with δ, with the mean value of the possibilities decreasing with increasing uncertainty. Technique 1 informed not only better outcomes but also represented uncertainty in a more nuanced manner by preserving its truth, indeterminacy, and falsity membership degrees.

Table 4 and Figure 5 compare both techniques, and show how Technique 1 results are to have a relatively longer project durations than Technique 2 due to the crisp conversion. However, Technique 2 allows a more realistic and flexible evolution assessment under uncertainty (i.e., risk sensitivity). The change of critical paths at $\delta = 0.7$ provides further insight to project scheduling related to the importance of risk sensitivity.

This study confirms NCPA's utility for managing uncertainty in projects. If we could look at both Technique 1 and Technique 2 at the same time, we could be using Technique 1 for the simplicity of the computational

Table 3. Possibility mean value of the ET and LT.

	Activity	δ=0	δ=0.1	δ=0.2	δ=0.3	δ=0.4	δ=0.5	δ=0.6	δ=0.7	δ=0.8	δ=0.9	δ=1
E4	M1	5.1783	4.7832	4.388	3.9928	3.5977	3.2025	2.807347	2.412184	2.017021	1.621858	1.226695
	M2	6	5.472	4.944	4.416	3.888	3.36	2.832	2.304	1.776	1.248	0.72
E5	M1	3.025	3.1185	3.212	3.3055	3.399	3.4925	3.586	3.6795	3.773	3.8665	3.96
	M2	6.2967	5.6817	5.0667	4.4517	3.8367	3.2217	2.6067	1.9917	1.3767	0.7617	0.1467
E7	M1	8.34	7.614	6.888	6.162	5.436	4.71	3.984	3.258	2.532	1.806	1.08
	M2	6.8458	6.1771	5.5083	4.8396	4.1708	3.5021	2.83334	2.164594	1.495849	0.827103	0.158357
	M3	6.9083	6.2373	5.5663	4.8953	4.2243	3.5533	2.8823	2.2113	1.5403	0.8693	0.1983
L6	M1	3.6658	3.3457	3.0257	2.7056	2.3855	2.0654	1.745347	1.42527	1.105192	0.785115	0.465038
	M2	0.5258	0.4838	0.4417	0.3996	0.3575	0.3154	0.273333	0.231248	0.189162	0.147076	0.10499
L3	M1	2.085	1.9035	1.722	1.5405	1.359	1.1775	0.996	0.8145	0.633	0.4515	0.27
	M2	-0.3458	-0.3217	-0.2977	-0.2736	-0.2495	-0.2495	-0.2013	-0.1773	-0.1532	-0.1291	-0.105
	M3	-0.2583	-0.2333	-0.02083	-0.1833	-0.1583	-0.1583	-0.1083	-0.0833	-0.583	-0.0333	-0.0083
L1	M1	0.3467	0.315	0.2833	0.2517	0.22	0.22	0.1567	0.125	0.933	0.0617	0.03
	M2	-1.1708	-1.0912	-1.0117	-0.9321	-0.8525	-0.8525	-0.6933	-0.6138	-0.5342	-0.4546	-0.375

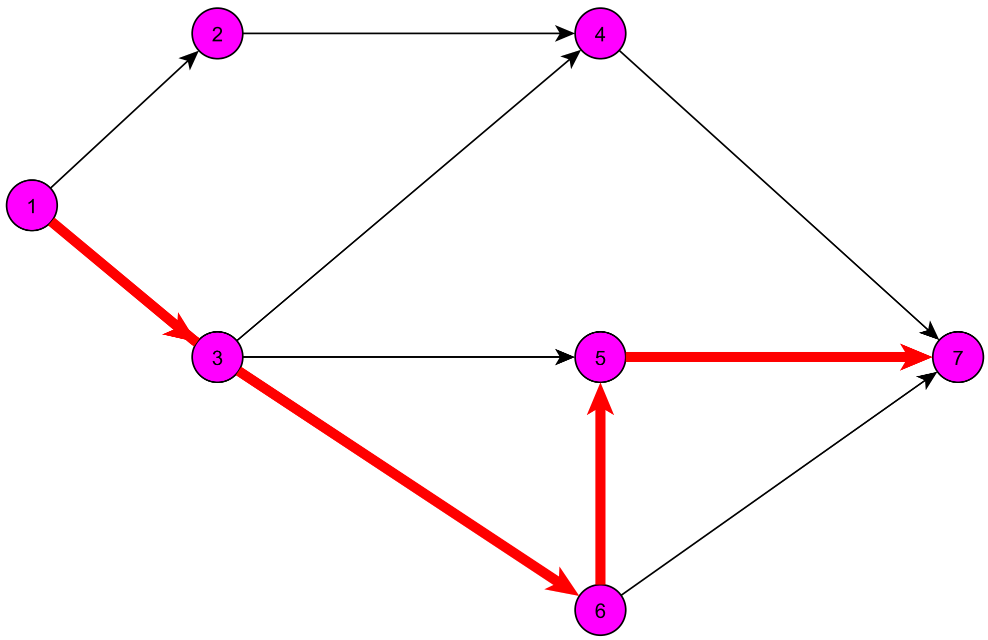

Fig. 3. Critical path when $\delta \in [0, 0.6]$.

aspects, while taking advantage of Technique 2 to represent more of the uncertainty. The authors reinforce the need for risk-adjusted scheduling, since critical paths depend on the level of uncertainty.

6|Conclusion and Future Directions

The study presents Neutrosophic Critical Path Analysis (Neu-CPA) as a successful and effective way to manage uncertainty in project scheduling. The results clearly point out that traditional deterministic ways of completing project schedule research may not capture the complexities of real life. The results in this study clearly show that the critical path in fact changes at higher risk levels ($\delta \geq 0.7$) and in this context, it provides a better understanding of the need for flexible scheduling under a state of uncertainty. Adopting Technique 1 had the advantage of crisp estimates, while Technique 2 synthesizes the neutrosophic structure to consider indeterminacy within the network under analysis. Despite providing a rich contribution to project scheduling under uncertainty, the study should be mindful of its limitations: (1) reliance on triangular neutrosophic numbers already defined

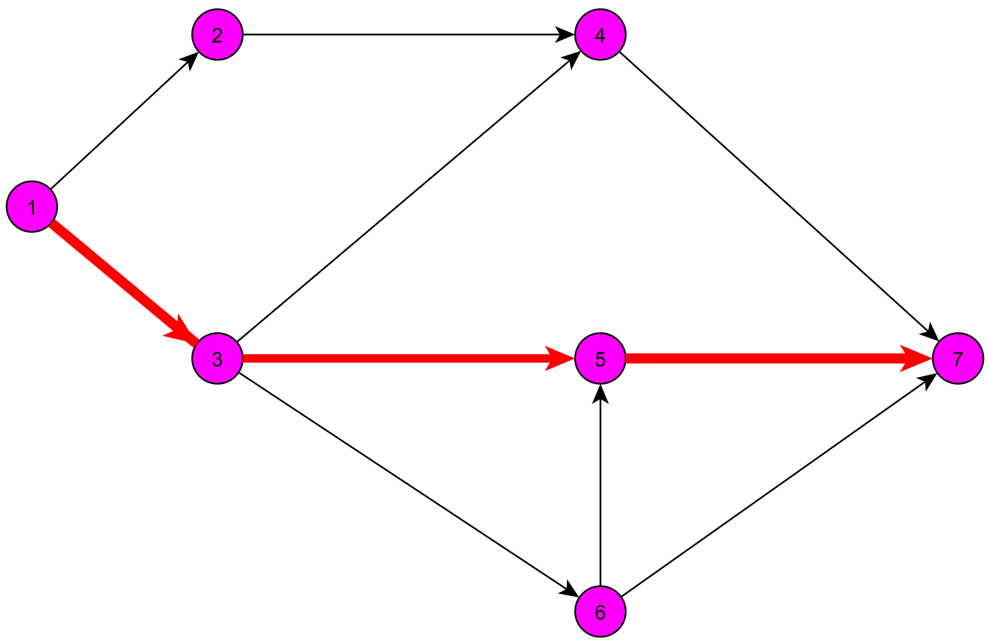

Fig. 4. Critical path when $\delta \in [0.7, 1]$.

Table 4. Critical path analysis using technique 1 and technique 2.

Risk Factor (δ)	Technique 1	Technique 2
0	12.4025	8.58583333333334
0.1	11.4961	7.74708333333334
0.2	10.5897	6.90833333333334
0.3	9.6833	6.06958333333333
0.4	8.7769	5.23083333333334
0.5	7.8705	4.39208333333334
0.6	6.964	3.55333333333333
0.7	6.466	4.17041666666667
0.8	6.4085	4.1275
0.9	6.3501	4.08458333333333
1	6.2917	4.04166666666667

Fig. 5. Comparision of project completion duration in technique 1 and technique 2 with different risk factor.

in an earlier section may not reflect all real state characteristics, (2) as network sizes grow, it can be as so cited here as complex , and (3) the provided risk factor (δ) would operate linearly when possibly real-world uncertainty may truly be nonlinear. Areas for future research should look to develop hybrid models that integrate neutrosophic logic with machine learning to report on a dynamic risk assessment level, possibly assess trapezoidal neutrosophic sets for varying degrees of uncertainty, and find optimization algorithms for reducing processing load. Ultimately, it could be useful to integrate data that projects have produced through an Internet of Things enabled project management system, for example, to develop a scheduling process that incorporates adaptivity into the scheduling process. Together, these steps can make NCPA a scalable approach to new types of project scheduling and uncertainties when working with dynamic and large-scale projects. Within this articulated, future research seeks to fully explore the gap between theoretical disciplinary frameworks and practice.

Author Contribution

P Dash: methodology, software, and editing. K. K. Mohanta: conceptualization, writing and editing. All authors have read and agreed to the published version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

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