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On Neutrosophic Crisp Soft Separation Axioms via Neutrosophic Crisp Soft Set in Neutrosophic Crisp Soft Topological Space

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Abstract


In this pioneering study, we introduce the concept of Neutrosophic Crisp Soft Sets (NCSSs) for the first time globally. Using this new class of sets, we examine “Neutrosophic Crisp Soft Topological Space” (NCSTS) and identify many key properties. Additionally, we extend the concept of neutrosophic crisp points to Neutrosophic Crisp Soft Points (NCSPs), which then form the basis for discovering a novel type of “neutrosophic crisp soft separation axioms” within “NCSTS”. We further explore the relationships between these separation axioms and provide insightful remarks and theorems related to these concepts. Moreover, this research presents fundamental definitions of NCSSs, including operations such as union and intersection. We also demonstrate their applicability through theorems, remarks, and examples. These new sets prove valuable in decision-making processes across diverse fields such as economics, agriculture, medicine, engineering, and others.


Keywords: Neutrosophic crisp soft sets, Neutrosophic crisp soft points, Neutrosophic crisp soft separation axioms, Neutrosophic crisp soft topological space.

1 | Introduction

Soft set theory, as defined by Molodtsov [1], is recognized as a highly versatile, efficient, and practical mathematical tool. This theory stands out because it avoids the limitations of parameterization inadequacy associated with other uncertainty-based theories. Molodtsov [1] successfully demonstrated several applications of soft set theory, including its integration into “game theory”, “Riemann integration”, “Perron integration”, “function smoothness”, “operation research”, and “probability”.

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The applications of soft set theory have seen rapid advancement across diverse fields. For instance, Shabir and Naz [2] explored the concept of “Soft Topological Spaces” (STSs), focusing on soft sets within these spaces and their related separation axioms. Additionally, other concepts such as “fuzzy topological spaces”, “fuzzy STSs”, “intuitionistic fuzzy topological spaces”, and “intuitionistic fuzzy STSs” were introduced by Chang [3], Li and Cui [4], Coker [5], and Tokat [6]. Detailed discussions on these concepts can be found in [7–11].

Many researchers have expanded this field significantly, contributing to various studies on soft set theory [12, 13, 22–26, 14–21]. The domain of soft algebraic structures has also witnessed substantial progress, with notable contributions from [27–32]. As a generalization of classical theories such as classical sets, fuzzy sets, and intuitionistic fuzzy sets, Smarandache [33], [34] defined the innovative “concept of Neutrosophic Set” (NS).

To unify “soft set theory” with “neutrosophic set theory”, Maji [13] suggested the concept of “Neutrosophic Soft Sets” (NSS). Following this, A. Salama et al. [35] defined Neutrosophic Crisp Sets (NCS), which further expanded the theoretical framework. Researchers have extensively explored NCSs, contributing valuable insights through numerous publications [36–38].

This paper presents a novel integration of “soft set theory” and “NCS theory” by introducing “the concept of Neutrosophic Crisp Soft Sets” (NCSSs). Utilizing these set, the study defines related concepts such as Neutrosophic Crisp Soft Topological Spaces (NCSTS), Neutrosophic Crisp Soft Points (NCSPs), and neutrosophic crisp soft separation axioms. Furthermore, it examines the practical applications of “NCSSs” in “decision-making”, supported by illustrative examples. These sets have broad applications across various domains, including economics, medicine, agriculture, engineering, and more.

2 | Preliminaries and Some Properties

Now, we will recall some new concepts about “STS”.

Definition 1 ([1]). Let Y be an “initial universe set” and E be a “set of parameters”. Let $\wp(Y)$ is the power set of Y and $B \subseteq E$.

(\mathcal{F}, B) is identified as a “soft set” on Y ; $\mathcal{F}: B \rightarrow \wp(Y)$ is a mapping. In other words, a “soft set” on Y is a “parametrized family of sub-sets” of Y .

Definition 2 ([2]). Let $\chi \neq \emptyset$ be a set and Let ξ be the collection of soft sets over χ , then (χ, ξ) is identified as a STS if

- I. $\widetilde{\chi}_N, \widetilde{\emptyset}_N \in \xi$.
- II. $\check{A}_1, \check{A}_2 \in \xi \Leftrightarrow \check{A}_1 \cap \check{A}_2 \in \xi$.
- III. $(\check{A}_i)_{i \in I} \in \xi \Leftrightarrow \cup (\check{A}_i)_{i \in I} \in \xi$.

Definition 3 ([35]). Let $\chi \neq \emptyset$ be any set. A “NCS” B is a set having the form $B = \langle B_1, B_2, B_3 \rangle$ where B_1, B_2 and B_3 are sub-sets of χ .

Definition 4 ([36]). Types of \emptyset_N and χ_N in χ :

- I. \emptyset_N defined as follows (in many ways):

- $\emptyset_N = \langle \emptyset, \emptyset, \chi \rangle$ (Type 1)
- $\emptyset_N = \langle \emptyset, \chi, \emptyset \rangle$ (Type 2)
- $\emptyset_N = \langle \emptyset, \chi, \chi \rangle$ (Type 3)
- $\emptyset_N = \langle \emptyset, \emptyset, \emptyset \rangle$ (Type 4)

- II. χ_N defined as the follows:

- $\chi_N = \langle \chi, \emptyset, \emptyset \rangle$ (Type 1)
- $\chi_N = \langle \chi, \chi, \emptyset \rangle$ (Type 2)
- $\chi_N = \langle \chi, \emptyset, \chi \rangle$ (Type 3)
- $\chi_N = \langle \chi, \chi, \chi \rangle$ (Type 4)

Definition 5 ([35]). Let $\chi \neq \emptyset$ be a set and Let ξ be the collection of NCSs over χ , then (χ, ξ) called a NCTS if

- I. $\chi_N, \emptyset_N \in \xi$.
- II. $\mathring{A}_1, \mathring{A}_2 \in \xi \Leftrightarrow \mathring{A}_1 \cap \mathring{A}_2 \in \xi$.
- III. $(\mathring{A}_i)_{i \in I} \in \xi \Leftrightarrow \cup (\mathring{A}_i)_{i \in I} \in \xi$.

3 | Neutrosophic Crisp Soft Set

In this part we introducing the concept of NCSSs for the first time globally. Using this new class of sets, we present fundamental definitions of NCSSs, including operations such as union and intersection. Moreover, we examine identify several key properties. These new sets prove valuable in decision-making processes across diverse fields such as economics, agriculture, medicine, engineering, and others.

Definition 6. If U is nonempty sets, and $A \neq \emptyset$ be a subset of E (where E be a set of parameters) and

$\mathcal{F}: A \rightarrow (\wp(U), \wp(U), \wp(U))$ is a “mapping”.

Then a pair (\mathcal{F}, A) is called a NCSS over U , where $(\mathcal{F}, A) = \{ \langle p^1(e), p^2(e), p^3(e) \rangle : e \in A \} \subseteq \{ \langle \wp(U), \wp(U), \wp(U) \rangle \}$.

Definition 7. $(\mathcal{F}, A)^c$ is complement of a NCSS (\mathcal{F}, A) where

$\mathcal{F}^c: A \rightarrow (\wp(U), \wp(U), \wp(U))$ and $\mathcal{F}^c(e) = \{ \langle U - p^1(e), U - p^2(e), U - p^3(e) \rangle : e \in A \} \subseteq (\wp(U), \wp(U), \wp(U))$: for all $e \in A$.

Example 1. Let $A = \{e_1, e_3, e_5\} \subseteq E$.

Let (\mathcal{F}, A) be NCSSs on $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$ and

- $\mathcal{F}(e_1) = \{ \langle \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\} \rangle \}$,
- $\mathcal{F}(e_3) = \{ \langle \{\beta_4, \beta_5, \beta_6\}, \{\beta_5, \beta_6\}, \{\beta_1, \beta_2, \beta_3\} \rangle \}$,
- $\mathcal{F}(e_5) = \{ \langle \{\beta_1, \beta_3\}, \{\beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} \rangle \}$.

The complement of (\mathcal{F}, A) is denoted by $(\mathcal{F}, A)^c$

$\mathcal{F}^c: A \rightarrow (\wp(U), \wp(U), \wp(U))$ where

- $\mathcal{F}^c(e_1) = \{ \langle \{\beta_4, \beta_5, \beta_6\}, \{\beta_2, \beta_4, \beta_5, \beta_6\}, \{\beta_3, \beta_4, \beta_5, \beta_6\} \rangle \}$,
- $\mathcal{F}^c(e_3) = \{ \langle \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_4, \beta_5, \beta_6\} \rangle \}$,
- $\mathcal{F}^c(e_5) = \{ \langle \{\beta_2, \beta_4, \beta_5, \beta_6\}, \{\beta_1, \beta_4, \beta_5, \beta_6\}, \{\beta_4, \beta_5, \beta_6\} \rangle \}$.

Definition 8. For two NCSSs $(\mathcal{F}, \mathcal{D})$ and $(\mathcal{G}, \mathcal{W})$ on U , then $(\mathcal{F}, \mathcal{D})$ is a “neutrosophic crisp soft sub-set” of $(\mathcal{G}, \mathcal{W})$ $((\mathcal{F}, \mathcal{D}) \subseteq (\mathcal{G}, \mathcal{W}))$ if

- I. $\mathcal{D} \subseteq \mathcal{W}$.
- II. $f^1(\eta) \subseteq g^1(\eta), f^2(\eta) \subseteq g^2(\eta), f^3(\eta) \supseteq g^3(\eta)$ or $f^1(\eta) \subseteq g^1(\eta), f^2(\eta) \supseteq g^2(\eta), f^3(\eta) \supseteq g^3(\eta)$, for all $\eta \in \mathcal{D}$.

Example 2. Let $\mathcal{D} = \{e_1, e_3, e_5\} \subseteq E$, and $\mathcal{W} = \{e_1, e_2, e_3, e_5\} \subseteq E$.

Let (F, \mathcal{D}) and (G, \mathcal{W}) be two NCSSs over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$;

$$\begin{aligned} p(e_1) &= \{ \langle \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\} \rangle, \\ p(e_3) &= \{ \langle \{\beta_4, \beta_5, \beta_6\}, \{\beta_5, \beta_6\}, \{\beta_1, \beta_2, \beta_3\} \rangle, \\ p(e_5) &= \{ \langle \{\beta_1, \beta_3\}, \{\beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} \rangle, \\ G(e_1) &= \{ \langle \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_1, \beta_3, \beta_5\}, \{\beta_2\} \rangle, \\ G(e_3) &= \{ \langle \{\beta_4, \beta_5, \beta_6\}, \{\beta_4, \beta_5, \beta_6\}, \{\beta_1, \beta_3\} \rangle, \\ G(e_5) &= \{ \langle \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} \rangle, \\ G(e_2) &= \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} \rangle. \end{aligned}$$

It is clear that $(p, \mathcal{D}) \subseteq (G, \mathcal{W})$.

Definition 9. Let (F, A) and (G, \mathcal{W}) be two NCSS, then $(G, \mathcal{W}) = (p, A)$ if $(p, A) \subseteq (G, \mathcal{W})$ and $(G, \mathcal{W}) \subseteq (p, A)$.

Definition 10.

- I. A NCSS (F, A) is identified as a neutrosophic crisp null soft set over χ , denoted by a $\widetilde{\emptyset}_N$, if $F(e) = \emptyset_N$: for all $e \in A$.
- II. A NCSS (F, A) is identified as a neutrosophic crisp absolute s soft set over χ , denoted by a $\widetilde{\chi}_N$, if $F(e) = \chi_N$: for all $e \in A$.

Definition 11. Union of two NCSS (F, A) and (G, \mathcal{W}) $((p, A) \widetilde{\cup} (G, \mathcal{W}))$ on the common universe χ is the NCSS (H, C) , where $C = A \cup \mathcal{W}$, and

$$H: A \cup \mathcal{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $\eta \in C$

$$\begin{aligned} H(\eta) &= \{ \langle f^1(\eta), f^2(\eta), f^3(\eta) \rangle : \eta \in A \\ &= \{ \langle g^1(\eta), g^2(\eta), g^3(\eta) \rangle : \eta \in B \\ &= \{ \langle f^1(\eta) \cup g^1(\eta), f^2(\eta) \cup g^2(\eta), f^3(\eta) \cap g^3(\eta) \rangle : \eta \in A \cap \mathcal{W} \text{ or} \\ &= \{ \langle f^1(\eta) \cup g^1(\eta), f^2(\eta) \cap g^2(\eta), f^3(\eta) \cap g^3(\eta) \rangle : \eta \in A \cap \mathcal{W} \end{aligned}$$

Example 3. In *Example 2*, $(p, A) \widetilde{\cup} (G, \mathcal{W}) = (H, C)$, where $C = A \cup \mathcal{W}$, and

$$H: A \cup \mathcal{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $e \in C$,

$$\begin{aligned} H(e_1) &= \{ \langle \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_1, \beta_3, \beta_5\}, \{\beta_2\} \rangle, \\ H(e_3) &= \{ \langle \{\beta_4, \beta_5, \beta_6\}, \{\beta_4, \beta_5, \beta_6\}, \{\beta_1, \beta_3\} \rangle, \\ H(e_5) &= \{ \langle \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} \rangle, \\ H(e_2) &= \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} \rangle. \end{aligned}$$

Definition 12. The intersection of two “soft sets” of (F, A) and (G, \mathcal{W}) on χ is the “soft sets” (H, C) ; $C = A \cap \mathcal{W}$, and

$$H: A \cap \mathcal{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $\eta \in C$

$$\begin{aligned} H(\eta) &= \{ \langle f^1(\eta) \cap g^1(\eta), f^2(\eta) \cap g^2(\eta), f^3(\eta) \cup g^3(\eta) \rangle : \eta \in A \cap B \text{ or} \\ &= \{ \langle f^1(\eta) \cap g^1(\eta), f^2(\eta) \cup g^2(\eta), f^3(\eta) \cup g^3(\eta) \rangle : \eta \in A \cap B. \end{aligned}$$

Example 4. In *Example 2*, $(p, A) \tilde{\cap} (G, \mathfrak{B}) = (H, C)$; $C = A \cap B$;

$$H: A \cap B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $e \in C$

$$H(e_1) = \{ \langle \{b_1, b_2, b_3\}, \{b_1, b_3\}, \{b_1, b_2\} \rangle,$$

$$H(e_3) = \{ \langle \{b_4, b_5, b_6\}, \{b_5, b_6\}, \{b_1, b_2, b_3\} \rangle,$$

$$H(e_5) = \{ \langle \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\} \rangle.$$

Definition 13. Let (F, A) and (G, B) are two NCSS then “ (p, A) and (G, B) ” denoted by $(p, A) \wedge (G, B)$ is defined by $(p, A) \wedge (G, B) = (H, A \times B)$; $H(\ddot{u}, \varsigma) = p(\ddot{u}) \cap G(\varsigma)$; for all $(\ddot{u}, \varsigma) \in A \times B$, which mean:

$$\begin{aligned} H(\ddot{u}, \varsigma) &= p(\ddot{u}) \cap G(\varsigma) \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \} \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \}. \end{aligned}$$

Definition 14. Let (F, A) and (G, B) are two NCSS then “ (p, A) or (G, B) ” $((p, A) \vee (G, \mathfrak{B}))$; $(p, A) \vee (G, \mathfrak{B}) = (Q, A \times B)$; $Q(\ddot{u}, \varsigma) = p(\ddot{u}) \cup G(\varsigma)$; for all $(\ddot{u}, \varsigma) \in A \times B$, which mean:

$$\begin{aligned} Q(\ddot{u}, \varsigma) &= p(\ddot{u}) \cup G(\varsigma) \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \} \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \}. \end{aligned}$$

Proposition 1.

- I. $((p, A) \wedge (G, \mathfrak{B}))^c = (p, A) \vee (G, \mathfrak{B})$.
- II. $((p, A) \vee (G, \mathfrak{B}))^c = (p, A) \wedge (G, \mathfrak{B})$.

Proof:

- I. $(p, A) \wedge (G, \mathfrak{B}) = (H, A \times \mathfrak{B})$

$$\begin{aligned} &\text{where } H(\ddot{u}, \varsigma) = p(\ddot{u}) \cap G(\varsigma) \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \} \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \}. \\ &\text{therefore } ((p, A) \wedge (G, \mathfrak{B}))^c = \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \}^c \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) \rangle \}^c \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \} \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \} \\ &= (p, A) \vee (G, \mathfrak{B}). \text{ Therefore} \\ &((p, A) \wedge (G, \mathfrak{B}))^c = (p, A) \vee (G, \mathfrak{B}). \end{aligned}$$

- II. $(p, A) \vee (G, \mathfrak{B}) = (H, A \times \mathfrak{B})$

$$\begin{aligned} &\text{where } H(\ddot{u}, \varsigma) = p(\ddot{u}) \cup G(\varsigma) \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \} \text{ or} \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \}. \\ &\text{therefore } ((p, A) \vee (G, \mathfrak{B}))^c = \\ &= \{ \langle f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) \rangle \}^c \text{ or} \end{aligned}$$

$$\begin{aligned}
 &= \{ \langle f^1(\ddot{u}) \cup g^1(\zeta), f^2(\ddot{u}) \cap g^2(\zeta), f^3(\ddot{u}) \cap g^3(\zeta) \rangle \}^c . \\
 &= \{ \langle f^1(\ddot{u}) \cap g^1(\zeta), f^2(\ddot{u}) \cap g^2(\zeta), f^3(\ddot{u}) \cup g^3(\zeta) \rangle \} \text{ or} \\
 &= \{ \langle f^1(\ddot{u}) \cap g^1(\zeta), f^2(\ddot{u}) \cup g^2(\zeta), f^3(\ddot{u}) \cup g^3(\zeta) \rangle \} . \\
 &= (p, A) \wedge (G, \mathfrak{B}) . \text{ Therefore} \\
 &((p, A) \vee (G, \mathfrak{B}))^c = (p, A) \wedge (G, \mathfrak{B}) .
 \end{aligned}$$

3.1 | Application of Neutrosophic Crisp Soft Sets in Decisions-Making

The following examples highlight the practical role of “NCSSs” in decision-making processes. A notable application is in the field of “home marketing”, where these sets assist in evaluating and selecting optimal marketing strategies based on uncertain or multi-parameter data.

Example 5. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be set consists of several houses, let the set $\ddot{E} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ give specifications to this houses (\ddot{E} is the set of parameters);

e_1 means the “expensive”.

e_2 means “beautiful”.

e_3 means “wooden”.

e_4 means “cheap”.

e_5 means in “the green surroundings”.

e_6 means “modern”.

e_7 means “in good repair”.

The “soft set” (F, \ddot{E}) describes the represents the appeal of various houses that Mr. X is considering for purchase. Where:

$$\begin{aligned}
 H: A &\rightarrow (\wp(U), \wp(U), \wp(U)) \\
 H(e_1) &= \{ \langle \emptyset, \emptyset, \emptyset \rangle \}, \\
 H(e_2) &= H(e_4) = \{ \langle U, \emptyset, \emptyset \rangle \}, \\
 H(e_3) &= \{ \langle \{u_1, u_2, u_6\}, \emptyset, \emptyset \rangle \}, \\
 H(e_5) &= \{ \langle \{u_1, u_2, u_3, u_4, u_6\}, \emptyset, \emptyset \rangle \}, \\
 H(e_6) &= \{ \langle \{u_1, u_2, u_6\}, \emptyset, \emptyset \rangle \}, \\
 H(e_7) &= \{ \langle \{u_1, u_3, u_6\}, \emptyset, \emptyset \rangle \}.
 \end{aligned}$$

Let $A = \{e_2, e_3, e_4, e_5, e_7\}$ be the set of descending home by Mr.X.

We put $u_{ij} = 1$ when $u_i \in H(e_j)$. Otherwise, $u_{ij} = 0$.

In the following diagram of the NCSS (the first part of this set), which contributes to the introduction of data to the computer, which allows the development of an algorithm to solve the issue and its programming.

$\begin{matrix} A \\ U \end{matrix}$	e_2	e_3	e_4	e_5	e_7	v_i^1
u_1	1	1	1	1	1	5
u_2	1	1	1	1	0	4
u_3	1	1	0	1	1	4
u_4	1	0	1	1	0	3
u_5	1	0	1	0	0	2
u_6	1	1	1	1	1	5

For every house u_i , we give $v_i^1 = \sum_j u_{ij}$, then $v_k^1 = \text{mak}(v_i^1)$ indicates the optimal option (choice) for the issue raised. In the previous example, Mr. X can choose u_1 or u_6 . in this example, there is no need for diagram of the second and third part of this set, because it is empty.

Remark 1. The previous “*Example 5*” can be regarded as representing a soft set in the classical sense because, in this case, it aligns with the classical definition; $\beta^2(u_j) = \beta^3(u_j) = \emptyset$; for all $j = 1, 2, 3, 4, 5, 6, 7$.

However, “*Example 5*” also highlights that a “NCSS” serves as a generalization of the classical soft set. In other words, the classical soft set can be viewed as a specific case of the NCSS. In “*Example 5*”, the focus was narrowed to a single aspect of interest for a merchant seeking to purchase a house. This explains why only one parameter was considered. where $\beta^2(u_j) = \beta^3(u_j) = \emptyset$; for all $j = 1, 2, 3, 4, 5, 6, 7$.

However, we could expand the analysis to incorporate all aspects relevant to the merchant’s decision-making process, such as the property’s description, location within the city, area, price, and other factors. From these, we could prioritize the three most significant aspects, as will be elaborated in the next example.

Example 6. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the set consists of several houses, and let the set $\tilde{E} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ give specifications to this houses (\tilde{E} is the set of parameters);

u_1 means the “expensive”.

u_2 means “beautiful”.

u_3 means “wooden”.

u_4 means “cheap”.

u_5 means in “the green surroundings”.

u_6 means “modern”.

u_7 means “in good repair”.

(F, \tilde{E}) describes the represents the appeal of various houses, which Mr. X (say) is going to buy. Where:

$$H: A \rightarrow (\emptyset(U), \emptyset(U), \emptyset(U));$$

$$H(u_1) = \{ \langle U, \{u_1, u_2, u_6\}, \emptyset \rangle \}$$

$$H(u_2) = H(u_4) = \{ \langle \{u_2, u_4, u_6\}, \{u_2, u_6\}, \{u_1, u_6\} \rangle \},$$

$$H(u_3) = \{ \langle \{u_3, u_4, u_6\}, \{u_1, u_2\}, \{u_1, u_4\} \rangle \},$$

$$H(u_5) = \{ \langle \{u_3, u_4, u_5\}, \{u_1, u_2, u_6\}, \{u_1, u_2, u_6\} \rangle \}$$

$$H(u_6) = \{ \langle \{u_1, u_2, u_3, u_4, u_5\}, \emptyset, \{u_4\} \rangle \}$$

$$H(u_7) = \{ \langle \{u_2, u_3, u_4\}, \{u_1, u_6\}, \{u_5, u_6\} \rangle \}$$

Let $A = \{u_2, u_3, u_4, u_5, u_7\}$ be the set of descending home by Mr.X.

We put $u_{ij} = 1$ or 2 or 3 if u_i belong to one or two or three subset of $\{\beta^1(u_j), \beta^2(u_j), \beta^3(u_j)\}$. Otherwise, $u_{ij} = 0$.

In the following diagram of the NCSS (the first, second, third part of this set), which contributes to the introduction of data to the computer, which allows the development of an algorithm to solve the issue and its programming.

A \ U	u ₂	u ₃	u ₄	u ₅	u ₇	v _i
u ₁	1	2	1	2	1	7
u ₂	2	1	2	2	1	8
u ₃	0	1	0	1	1	3
u ₄	1	2	1	1	1	7
u ₅	0	0	0	1	1	2
u ₆	3	1	3	2	2	11

For every house u_i , we give $v_i = \sum_j u_{ij}$, then $v_k = \text{mak}(v_i)$ indicates the optimal option (choice) for the issue raised. In the previous example, Mr. X can choose u_6 .

4 | Neutrosophic Crisp Soft Point

In this part we introduced the concept of NCSPs.

Definition 15. Let x belong to χ , A special type of NCSS, such as $(F, A = \{x\})$ is called a “NCSP” related to x type i (NCSP(i)) over χ , where $F: \{x\} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$ is a mapping, where $(F, \{x\})^i = \{ \langle f^1(e), f^2(e), f^3(e) \rangle : e \in A \} \subseteq \wp(\chi), \wp(\chi), \wp(\chi) ; f^i(x) \neq \emptyset \text{ and } f^j(e) = \emptyset, \text{ if } i \neq j; i, j = 1, 2, 3.$

Example 7. Let $A = \{u_1\} \subseteq E$, and $B = \{u_2\} \subseteq E$.

$(F, A)^1$ and $(G, B)^1$ is two “NCSPs” over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$ type 1;

$$p(u_1) = \{ \langle \{\beta_1\}, \emptyset, \emptyset \rangle \},$$

$$G(u_2) = \{ \langle \{\beta_2\}, \emptyset, \emptyset \rangle \},$$

(H, A) is “NCSPs” over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$ type 2;

$$H(u_1) = \{ \langle \emptyset, \{\beta_1\}, \emptyset \rangle \},$$

(T, A) is “NCSPs” over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$ type 3;

$$t^1(u_1) = \emptyset, \quad t^2(u_1) = \emptyset, \quad t^3(u_1) = \{\beta_1\}.$$

Definition 16. Let (F, A) be a NCSS over U and $(G, \{v\})^i$ be a NCSP type i over U . We say that $(G, \{v\})^i \in (F, A)$ whenever $v \in A$ and $g^i(v) \subseteq f^i(v)$. Note that $(G, \{v\})^i \notin (F, A)$ if $v \notin A$ or $g^i(v) \not\subseteq f^i(v); i = 1, 2, 3.$

Example 8. Let $A = \{u_1, u_3, u_4, u_5\} \subseteq E$, and $B = \{u_1, u_2, u_3, u_5\} \subseteq E$.

If (p, A) and (G, B) be two “NCSSs” over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$;

$$p(u_1) = \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_4\}, \{\beta_4, \beta_2\} \rangle \},$$

$$p(u_3) = \{ \langle \{\beta_2, \beta_3, \beta_6\}, \{\beta_3, \beta_4\}, \{\beta_1, \beta_3\} \rangle \},$$

$$p(u_5) = \{ \langle \{\beta_2, \beta_3\}, \{\beta_2, \beta_4\}, \{\beta_1, \beta_3\} \rangle \},$$

$$p(u_4) = \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_4\}, \{\beta_2, \beta_3\} \rangle \},$$

$$G(u_1) = \{ \langle \{\beta_2, \beta_3, \beta_4\}, \{\beta_2, \beta_3, \beta_5\}, \{\beta_2\} \rangle \},$$

$$G(u_3) = \{ \langle \{\beta_4, \beta_6\}, \{\beta_5, \beta_6\}, \{\beta_3\} \rangle \},$$

$$G(u_5) = \{ \langle \{\beta_2, \beta_3\}, \{\beta_2, \beta_3\}, \{\beta_2, \beta_3\} \rangle \},$$

$$G(u_2) = \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} \rangle \}.$$

Let $A_1 = \{U_1\} \subseteq E$, and $A_2 = \{U_2\} \subseteq E$.

$(p_1, A_1)^1$ and $(p_2, A_2)^1$ is two NCSPs over the same universe U type 1;

$$p(U_1) = \{ \langle \{B_1\}, \emptyset, \emptyset \rangle \},$$

$$G(U_2) = \{ \langle \{B_2\}, \emptyset, \emptyset \rangle \},$$

$$(p_1, A_1)^1 \in (p, A), (p_1, A_1)^1 \notin (G, \mathfrak{B}), (p_2, A_2)^1 \notin (p, A), (p_2, A_2)^1 \notin (G, \mathfrak{B})$$

$(p_3, A_1)^2$ is NCSP over the same universe U type 2;

$$p_3(U_1) = \{ \langle \emptyset, \{B_1\}, \emptyset \rangle \},$$

$$(p_3, A_1)^2 \in (F, A), (p_3, A_1)^2 \notin (G, B)$$

$(T, A_1)^3$ is NCSP over the same universe U type 3;

$$t^1(U_1) = \emptyset, \quad t^2(U_1) = \emptyset, \quad t^3(U_1) = \{B_1\}.$$

$$(T, A_1)^3 \notin (p, A), (T, A_1)^3 \notin (G, \mathfrak{B})$$

5 | Separation Axioms in a “Neutrosophic Crisp Soft Topological Space”

Here, we will define the concept “neutrosophic crisp STS” and studied some concepts connected to this new space, and finally, we studied separation axioms in this space.

Definition 17. A Neutrosophic Crisp Soft Topology (NCST) on a set $\chi \neq \emptyset$ is a family \mathfrak{T} of “neutrosophic crisp soft subsets” in χ satisfying the following conditions:

- I. $\widetilde{\emptyset}_N, \widetilde{\chi}_N \in \mathfrak{T}$
- II. If $\mathfrak{A}, \mathfrak{B}$ belong to \mathfrak{T} then $A \cap B \in \mathfrak{T}$.
- III. The union of any number of sets in \mathfrak{T} belongs to \mathfrak{T} .

Then (χ, \mathfrak{T}) is said to be a “NCSTS”.

Definition 18. If (χ, \mathfrak{T}) is a (NCSTS) in χ , The elements in \mathfrak{T} are said to be “Neutrosophic Crisp Soft Open Sets” (NCSOS), a “NCSS” F is closed NCSCS if F^c (its complement) is an NCSOS.

Example 9. Let $A = \{U_1, U_3\} \subseteq E$, and $B = \{U_1, U_2\} \subseteq E$.

Let (p, A) and (G, \mathfrak{B}) be two NCSS over $\chi = \{B_1, B_2, B_3, B_4, B_5, B_6\}$;

$$p(U_1) = \{ \langle \{B_1, B_3\}, \{B_1, B_4\}, \{B_4, B_2\} \rangle \},$$

$$p(U_3) = \{ \langle \{B_2, B_3, B_6\}, \{B_3, B_4\}, \{B_1, B_3\} \rangle \},$$

$$G(U_1) = \{ \langle \{B_2, B_3, B_4\}, \{B_2, B_3, B_5\}, \{B_2\} \rangle \},$$

$$G(U_2) = \{ \langle \{B_1, B_3\}, \{B_1, B_2\}, \{B_2, B_3\} \rangle \}.$$

$$(p, A) \cup (G, \mathfrak{B}) = (H, C),$$

where $C = AU\mathfrak{B}$, and

$$H: A \cup \mathfrak{B} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $U \in C$,

$$H(\mathcal{U}_1) = p(\mathcal{U}_1) \cup G(\mathcal{U}_1) = \{ \langle \{ \beta_1, \beta_2, \beta_3, \beta_4 \}, \{ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \}, \{ \beta_2 \} \rangle,$$

$$H(\mathcal{U}_3) = p(\mathcal{U}_3) = \{ \langle \{ \beta_2, \beta_3, \beta_6 \}, \{ \beta_3, \beta_4 \}, \{ \beta_1, \beta_3 \} \rangle,$$

$$H(\mathcal{U}_2) = G(\mathcal{U}_2) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_2 \}, \{ \beta_2, \beta_3 \} \rangle.$$

$$(p, A) \cap (G, \mathfrak{B}) = (H, C)$$

where $C = A \cap \mathfrak{B}$, and

$$K: A \cap \mathfrak{B} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $\mathcal{U} \in C$,

$$K(\mathcal{U}_1) = \{ \langle \{ \beta_3 \}, \emptyset, \{ \beta_2, \beta_4 \} \rangle,$$

$$\mathfrak{T} = \{ \emptyset_N, \chi_N, (A, p), (\mathfrak{B}, G), (A \cap \mathfrak{B}, K), (A \cup \mathfrak{B}, H) \}$$

Then (χ, \mathfrak{T}) is a NCSTS.

Example 10. Let $A = \{ \mathcal{U}_1, \mathcal{U}_3, \mathcal{U}_4, \mathcal{U}_5 \} \subseteq E$, and $\mathfrak{B} = \{ \mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_5 \} \subseteq E$.

Let (F, A) and (G, \mathfrak{B}) be two NCSS over $\chi = \{ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6 \}$ such that

$$p(\mathcal{U}_1) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_4 \}, \{ \beta_4, \beta_2 \} \rangle,$$

$$p(\mathcal{U}_3) = \{ \langle \{ \beta_2, \beta_3, \beta_6 \}, \{ \beta_3, \beta_4 \}, \{ \beta_1, \beta_3 \} \rangle,$$

$$p(\mathcal{U}_5) = \{ \langle \{ \beta_2, \beta_3 \}, \{ \beta_2, \beta_4 \}, \{ \beta_1, \beta_3 \} \rangle,$$

$$p(\mathcal{U}_4) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_4 \}, \{ \beta_2, \beta_3 \} \rangle,$$

$$G(\mathcal{U}_1) = \{ \langle \{ \beta_2, \beta_3, \beta_4 \}, \{ \beta_2, \beta_3, \beta_5 \}, \{ \beta_2 \} \rangle,$$

$$G(\mathcal{U}_3) = \{ \langle \{ \beta_4, \beta_6 \}, \{ \beta_5, \beta_6 \}, \{ \beta_3 \} \rangle,$$

$$G(\mathcal{U}_5) = \{ \langle \{ \beta_2, \beta_3 \}, \{ \beta_2, \beta_3 \}, \{ \beta_2, \beta_3 \} \rangle,$$

$$G(\mathcal{U}_2) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_2 \}, \{ \beta_2, \beta_3 \} \rangle.$$

$$(p, A) \cup (G, \mathfrak{B}) = (H, C)$$

where $C = A \cup \mathfrak{B}$, and

$$H: A \cup \mathfrak{B} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $\mathcal{U} \in C$,

$$H(\mathcal{U}_1) = \{ \langle \{ \beta_1, \beta_2, \beta_3, \beta_4 \}, \{ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \}, \{ \beta_2 \} \rangle,$$

$$H(\mathcal{U}_3) = \{ \langle \{ \beta_6 \}, \emptyset, \{ \beta_5, \beta_6 \} \rangle,$$

$$H(\mathcal{U}_4) = p(\mathcal{U}_4) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_4 \}, \{ \beta_2, \beta_3 \} \rangle,$$

$$p(\mathcal{U}_5) = \{ \langle \{ \beta_2, \beta_3 \}, \{ \beta_2, \beta_3, \beta_4 \}, \{ \beta_3 \} \rangle,$$

$$(p, A) \cap (G, \mathfrak{B}) = (H, C)$$

where $C = A \cap \mathfrak{B}$, and

$$K: A \cap \mathfrak{B} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $\mathcal{U} \in C$,

$$K(\mathcal{U}_1) = \{ \langle \{ \beta_3 \}, \emptyset, \{ \beta_2, \beta_4 \} \rangle,$$

$$K(\mathcal{U}_3) = \{ \langle \{ \beta_6 \}, \emptyset, \{ \beta_1, \beta_3 \} \rangle,$$

$$K(\mathcal{U}_5) = \{ \langle \{ \beta_2, \beta_3 \}, \{ \beta_2 \}, \{ \beta_3 \} \rangle.$$

$$\mathfrak{X} = \{ \emptyset_N, \chi_N, (A, p), (\mathfrak{B}, G), (A \cap \mathfrak{B}, K), (A \cup \mathfrak{B}, H) \}$$

Then (χ, \mathfrak{X}) is a “NCSTS”.

Definition 19. If (χ, \mathfrak{X}) be an NCSTS. $(G, \{x\})^i$ is a NCSP(i), and $(H, B) \in \mathfrak{X}$ is a NCSOS, (H, B) is defined to be neutrosophic crisp soft open nhd of $(G, \{x\})^i$ in (χ, \mathfrak{X}) if $(G, \{x\})^i \in (H, B)$

Definition 20. Let (χ, \mathfrak{X}) be an NCSTS. $(G, \{x\})^i$ is NCSP(i), and (H, B) is a NCSS, (H, B) is defined to be neutrosophic crisp soft nhd of $(G, \{x\})^i$ in (χ, \mathfrak{X}) if there is a NCSOS (F, A) containing $(G, \{x\})^i$ such that $(F, A) \subseteq (H, B)$.

Remark 2. Every neutrosophic crisp soft open nhd of $(G, \{x\})^i$ is be neutrosophic crisp soft nhd of $(G, \{x\})^i$, but the opposite is not true (in general).

Now, we study “Separation axioms” in a NCSTS.

Definition 21. A NCSTS (χ, \mathfrak{X}) is called

- I. $N_1S\text{-}\mathfrak{X}_0\text{-space}$ if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi \exists$ a NCSOS (F, N) in χ such that $(G, \{x\})^1 \in (F, N)$, $(H, \{y\})^1 \notin (F, N)$ or $(G, \{x\})^1 \notin (F, N)$, $(H, \{y\})^1 \in (F, N)$.
- II. $N_2S\text{-}\mathfrak{X}_0\text{-space}$ if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi \exists$ a NCSOS (F, N) in χ such that $(G, \{x\})^2 \in (F, N)$, $(H, \{y\})^2 \notin (F, N)$ or $(G, \{x\})^2 \notin (F, N)$, $(H, \{y\})^2 \in (F, N)$.
- III. $N_3S\text{-}\mathfrak{X}_0\text{-space}$ if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi \exists$ a NCSOS (F, N) in χ such that $(G, \{x\})^3 \in (F, N)$, $(H, \{y\})^3 \notin (F, N)$ or $(G, \{x\})^3 \notin (F, N)$, $(H, \{y\})^3 \in (F, N)$.
- IV. $N_1S\text{-}\mathfrak{X}_1\text{-space}$ if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^1 \in (F_1, N_1)$, $(H, \{y\})^1 \notin (F_2, N_2)$ and $(G, \{x\})^1 \notin (F_1, N_1)$, $(H, \{y\})^1 \in (F_2, N_2)$.
- V. $N_2S\text{-}\mathfrak{X}_1\text{-space}$ if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^2 \in (F_1, N_1)$, $(H, \{y\})^2 \notin (F_2, N_2)$ and $(G, \{x\})^2 \notin (F_1, N_1)$, $(H, \{y\})^2 \in (F_2, N_2)$.
- VI. $N_3S\text{-}\mathfrak{X}_1\text{-space}$ if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^3 \in (F_1, N_1)$, $(H, \{y\})^3 \notin (F_2, N_2)$ and $(G, \{x\})^3 \notin (F_1, N_1)$, $(H, \{y\})^3 \in (F_2, N_2)$.
- VII. $N_1S\text{-}\mathfrak{X}_2\text{-space}$ if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^1 \in (F_1, N_1)$ and $(H, \{y\})^1 \in (F_2, N_2)$ with $(F_1, N_1) \cap (F_2, N_2) = \widetilde{\emptyset}_N$.
- VIII. $N_2S\text{-}\mathfrak{X}_2\text{-space}$ if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^2 \in (F_1, N_1)$ and $(H, \{y\})^2 \in (F_2, N_2)$ with $(F_1, N_1) \cap (F_2, N_2) = \widetilde{\emptyset}_N$.
- IX. $N_3S\text{-}\mathfrak{X}_2\text{-space}$ if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi \exists$ a two NCSOS $(F_1, N_1), (F_2, N_2)$ in χ such that $(G, \{x\})^3 \in (F_1, N_1)$ and $(H, \{y\})^3 \in (F_2, N_2)$ with $(F_1, N_1) \cap (F_2, N_2) = \widetilde{\emptyset}_N$.

Example 11. Let $N = \{ \mathcal{U}_1, \mathcal{U}_3 \} \subseteq E$, and $B = \{ \mathcal{U}_1, \mathcal{U}_2 \} \subseteq E$.

Let (F, N) and (G, B) be two NCSS over the same universe $\chi = \{ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6 \}$ such that

$$p(\mathcal{U}_1) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_4 \}, \{ \beta_4, \beta_2 \} \rangle,$$

$$p(\mathcal{U}_3) = \{ \langle \{ \beta_2, \beta_3, \beta_6 \}, \{ \beta_3, \beta_4 \}, \{ \beta_1, \beta_3 \} \rangle,$$

$$G(\mathcal{U}_1) = \{ \langle \{ \beta_2, \beta_3, \beta_4 \}, \{ \beta_2, \beta_3, \beta_5 \}, \{ \beta_2 \} \rangle,$$

$$G(\mathcal{U}_2) = \{ \langle \{ \beta_1, \beta_3 \}, \{ \beta_1, \beta_2 \}, \{ \beta_2, \beta_3 \} \rangle.$$

$$(p, N) \cup (G, B) = (H, C)$$

where $C = D \cup B$, and

$$H: N \cup B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $u \in C$,

$$H(u_1) = p(u_1) \cup G(u_1) = \{ \langle \{b_1, b_2, b_3, b_4\}, \{b_1, b_2, b_3, b_4, b_5\}, \{b_2\} \rangle,$$

$$H(u_3) = p(u_3) = \{ \langle \{b_2, b_3, b_6\}, \{b_3, b_4\}, \{b_1, b_3\} \rangle,$$

$$H(u_2) = G(u_2) = \{ \langle \{b_1, b_3\}, \{b_1, b_2\}, \{b_2, b_3\} \rangle.$$

$$(p, N) \cap (G, B) = (H, C)$$

where $C = D \cap B$, and

$$K: D \cap B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$$

and for all $u \in C$,

$$K(u_1) = \{ \langle \{b_3\}, \emptyset, \{b_2, b_4\} \rangle,$$

$$\mathfrak{X} = \{ \emptyset_N, \chi_N, (N, p), (\mathfrak{B}, G), (N \cap \mathfrak{B}, K), (N \cup \mathfrak{B}, H) \}$$

Then (χ, \mathfrak{X}) is a NCSTS.

Then (χ, \mathfrak{X}) is not $N_1S\text{-}\mathfrak{X}_0$ -space, because for $(K, \{u_1\})^1 \neq (H, \{u_3\})^1 \in \chi$

Where

$$K(u_1) = \{ \langle \{b_5\}, \emptyset, \emptyset \rangle, H(u_3) = \{ \langle \{b_5\}, \emptyset, \emptyset \rangle,$$

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, (χ, \mathfrak{X}) is not $N_1S\text{-}\mathfrak{X}_i$ -space, $i=1, 2$.

Then (χ, \mathfrak{X}) is not $N_2S\text{-}\mathfrak{X}_0$ -space, because for $(K, \{u_1\})^2 \neq (H, \{u_3\})^2 \in \chi$

Where

$$K(u_1) = \{ \langle \emptyset, \{b_6\}, \emptyset \rangle, H(u_3) = \{ \langle \emptyset, \{b_6\}, \emptyset \rangle,$$

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, (χ, \mathfrak{X}) is not $N_2S\text{-}\mathfrak{X}_i$ -space, $i=1, 2$.

Then (χ, \mathfrak{X}) is not $N_3S\text{-}\mathfrak{X}_0$ -space, because for $(K, \{u_1\})^3 \neq (H, \{u_3\})^3 \in \chi$

Where

$$K(u_1) = \{ \langle \emptyset, \emptyset, \{b_5\} \rangle, H(u_3) = \{ \langle \emptyset, \emptyset, \{b_5\} \rangle,$$

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, (χ, \mathfrak{X}) is not $N_3S\text{-}\mathfrak{X}_i$ -space, $i=1, 2$.

Example 12. Let $A = \{u_1, u_2\} \subseteq E = \{u_1, u_2\}$.

Let (F, N) be a NCSS on universe $\chi = \{b_1, b_2, b_3\}$ such that

$$p(u_1) = \{ \langle \{b_1, b_2\}, \{b_2, b_3\}, \{b_3\} \rangle,$$

$$p(\mathcal{U}_2) = \{ \langle \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_1, \beta_2\} \rangle,$$

$$J = \{ \emptyset_N, \chi_N, (N, p) \}$$

Then (χ, \mathfrak{S}) is a NCSTS

- I. Then (χ, \mathfrak{S}) is N_1 - \mathfrak{S}_o -space, but (χ, \mathfrak{S}) is not N_1 - \mathfrak{S}_i -space, $i=1, 2$.
- II. Then (χ, \mathfrak{S}) is N_2 - \mathfrak{S}_o -space, but (χ, \mathfrak{S}) is not N_2 - \mathfrak{S}_i -space, $i=1, 2$.
- III. Then (χ, \mathfrak{S}) is N_3 - \mathfrak{S}_o -space, but (χ, \mathfrak{S}) is not N_3 - \mathfrak{S}_i -space, $i=1, 2$.

Remark 3. For a NCSTS (χ, \mathfrak{S})

- I. Every N_i S- \mathfrak{S}_1 -space is N_i S- \mathfrak{S}_o -space, $i=1, 2, 3$.
- II. Every N_i S- \mathfrak{S}_2 -space is N_i S- \mathfrak{S}_1 -space, $i=1, 2, 3$.

Proof: the proof follows directly from definitions.

The converse of Remark 3 does not hold, as demonstrated in the Example 13:

Example 13. In Example 12.

- I. Then (χ, \mathfrak{S}) is N_1 S- \mathfrak{S}_o -space, (χ, \mathfrak{S}) is not N_1 S- \mathfrak{S}_i -space, $i=1, 2$.
- II. Also, (χ, \mathfrak{S}) is N_2 S- \mathfrak{S}_o -space, (χ, \mathfrak{S}) is not N_2 S- \mathfrak{S}_i -space, $i=1, 2$.
- III. Also, (χ, \mathfrak{S}) is N_3 S- \mathfrak{S}_o -space, (χ, \mathfrak{S}) is not N_3 S- \mathfrak{S}_i -space, $i=1, 2$.

6 | Conclusion

This study has successfully established a ground-breaking connection between “NCSs” and “soft sets”, introducing innovative mathematical tools and concepts that enhance our understanding of these theories. Through the definition of “NCSTSs”, “NCSSs”, “NCSPs”, and “neutrosophic crisp soft separation axioms”, we have developed a cohesive framework that bridges these areas of neutrosophic crisp soft theory. This framework paves the way for novel interdisciplinary research opportunities across various scientific domains.

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Consent for publication has been obtained from the author.

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This article does not include experiments involving humans or animals.

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The data are available from the corresponding author upon reasonable request.

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