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On Neutrosophic Crisp Soft Separation Axioms via Neutrosophic Crisp Soft Set in Neutrosophic Crisp Soft Topological Space

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Abstract

In this pioneering study, we introduce the concept of Neutrosophic Crisp Soft Sets (NCSSs) for the first time globally. Using this new class of sets, we examine "Neutrosophic Crisp Soft Topological Space" (NCSTS) and identify many key properties. Additionally, we extend the concept of neutrosophic crisp points to Neutrosophic Crisp Soft Points (NCSPs), which then form the basis for discovering a novel type of "neutrosophic crisp soft separation axioms" within "NCSTSs". We further explore the relationships between these separation axioms and provide insightful remarks and theorems related to these concepts. Moreover, this research presents fundamental definitions of NCSSs, including operations such as union and intersection. We also demonstrate their applicability through theorems, remarks, and examples. These new sets prove valuable in decision-making processes across diverse fields such as economics, agriculture, medicine, engineering, and others.

Keywords: Neutrosophic crisp soft sets, Neutrosophic crisp soft points, Neutrosophic crisp soft separation axioms, Neutrosophic crisp soft topological space.

1 | Introduction

Soft set theory, as defined by Molodtsov [1], is recognized as a highly versatile, efficient, and practical mathematical tool. This theory stands out because it avoids the limitations of parameterization inadequacy associated with other uncertainty-based theories. Molodtsov [1] successfully demonstrated several applications of soft set theory, including its integration into "game theory", "Riemann integration", "Perron integration", "function smoothness", "operation research", and "probability".

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The applications of soft set theory have seen rapid advancement across diverse fields. For instance, Shabir and Naz [2] explored the concept of "Soft Topological Spaces" (STSs), focusing on soft sets within these spaces and their related separation axioms. Additionally, other concepts such as "fuzzy topological spaces", "fuzzy STSs", "intuitionistic fuzzy topological spaces", and "intuitionistic fuzzy STSs" were introduced by Chang [3], Li and Cui [4], Coker [5], and Tokat [6]. Detailed discussions on these concepts can be found in [7–11].

Many researchers have expanded this field significantly, contributing to various studies on soft set theory [12, 13, 22–26, 14–21]. The domain of soft algebraic structures has also witnessed substantial progress, with notable contributions from [27–32]. As a generalization of classical theories such as classical sets, fuzzy sets, and intuitionistic fuzzy sets, Smarandache [33], [34] defined the innovative "concept of Neutrosophic Set" (NS).

To unify "soft set theory" with "neutrosophic set theory", Maji [13] suggesed the concept of "Neutrosophic Soft Sets" (NSS). Following this, A. Salama et al. [35] defined Neutrosophic Crisp Sets (NCS), which further expanded the theoretical framework. Researchers have extensively explored NCSs, contributing valuable insights through numerous publications [36–38].

This paper presents a novel integration of "soft set theory" and "NCS theory" by introducing "the concept of Neutrosophic Crisp Soft Sets" (NCSSs). Utilizing these set, the study defines related concepts such as Neutrosophic Crisp Soft Topological Spaces (NCSTS), Neutrosophic Crisp Soft Points (NCSPs), and neutrosophic crisp soft separation axioms. Furthermore, it examines the practical applications of "NCSSs" in "decision-making", supported by illustrative examples. These sets have broad applications across various domains, including economics, medicine, agriculture, engineering, and more.

2 | Preliminaries and Some Properties

Now, we will recall some new concepts about "STS".

Definition 1 ([1]). Let Υ be an "initial universe set" and Ę be a "set of parameters". Let $\wp(\Upsilon)$ is the power set of Υ and Б$\subseteq$Ę.

($\mathcal{F}$, Б) is identified as a "soft set" on Υ; $\mathcal{F}$: Б $\rightarrow \wp(\Upsilon)$ is a mapping. In other words, a "soft set" on Υ is a "parametrized family of sub-sets" of Υ.

Definition 2 ([2]). Let $\chi \neq \emptyset$ be a set and Let ξ be the collection of soft sets over χ, then (χ, ξ) is identified as a STS if

I. $\widetilde{\chi_N}, \widetilde{\emptyset_N} \in \xi$.

II. $Å_1, Å_2 \in \xi \Leftrightarrow Å_1 \cap Å_2 \in \xi$.

III. $(Å_i)_{i\in I} \in \xi \Leftrightarrow \cup (Å_i)_{i\in I} \in \xi$.

Definition 3 ([35]). Let $\chi \neq \emptyset$ be any set. A "NCS" Б is a set having the form Б $=<$ Б$_1$, Б$_2$, Б$_3$ $>$ where Б$_1$, Б$_2$ and Б$_3$ are sub-sets of χ.

Definition 4 ([36]). Types of $\emptyset_N$ and χ_N in χ:

I. $\emptyset_N$ defined as follows (in many ways):

- $\emptyset_N = <\emptyset, \emptyset, \chi>$ *(Type 1)*
- $\emptyset_N = <\emptyset, \chi, \emptyset>$ *(Type 2)*
- $\emptyset_N = <\emptyset, \chi, \chi>$ *(Type 3)*
- $\emptyset_N = <\emptyset, \emptyset, \emptyset>$ *(Type 4)*

II. $\mathcal{X}_N$ defined as the follows:

- $\chi_N = <\chi, \emptyset, \emptyset>$ *(Type 1)*
- $\chi_N = <\chi, \chi, \emptyset>$ *(Type 2)*
- $\chi_N = <\chi, \emptyset, \chi>$ *(Type 3)*
- $\chi_N = <\chi, \chi, \chi>$ *(Type 4)*

Definition 5 ([35]). Let $\chi \neq \emptyset$ be a set and Let ξ be the collection of NCSs over χ, then (χ, ξ) called a NCTS if

I. $\chi_N, \emptyset_N \in \xi$.

II. $\text{Å}_1, \text{Å}_2 \in \xi \Leftrightarrow \text{Å}_1 \cap \text{Å}_2 \in \xi$.

III. $(\text{Å}_i)_{i \in I} \in \xi \Leftrightarrow \cup (\text{Å}_i)_{i \in I} \in \xi$.

3 | Neutrosophic Crisp Soft Set

In this part we introducing the concept of NCSSs for the first time globally. Using this new class of sets, we present fundamental definitions of NCSSs, including operations such as union and intersection. Moreover, we examine identify several key properties. These new sets prove valuable in decision-making processes across diverse fields such as economics, agriculture, medicine, engineering, and others.

Definition 6. If U is nonempty sets, and A≠ ϕ be a subset of E (where E be a set of parameters) and

Ꞙ: A→(℘(U), ℘(U), ℘(U)) is a "mapping".

Then a pair (Ꞙ, A) is called a NCSS over U, where $(Ꞙ, A) = \{< ꞑ^1(e), ꞑ^2(e), ꞑ^3(e) >: e \in A\} \subseteq \{< \wp(U), \wp(U), \wp(U) >\}$.

Definition 7. $(Ꞙ, A)^c$ is complement of a NCSS $(Ꞙ, A)$ where

$Ꞙ^c$: A→(℘(U),℘(U),℘(U)) and $Ꞙ^c(e) = \{< U - ꞑ^1(e), U - ꞑ^2(e), U - ꞑ^3(e) >: e \in A\} \subseteq (\wp(U), \wp(U), \wp(U))$: for all $e \in A$.

Example 1. Let $A = \{e_1, e_3, e_5\} \subseteq E$.

Let $(Ꞙ, A)$ be NCSSs on $U = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ and

$Ꞙ(e_1) = \{< \{ß_1, ß_2, ß_3\}, \{ß_1, ß_3\}, \{ß_1, ß_2\} >\}$,

$Ꞙ(e_3) = \{< \{ß_4, ß_5, ß_6\}, \{ß_5, ß_6\}, \{ß_1, ß_2, ß_3\} >\}$,

$Ꞙ(e_5) = \{< \{ß_1, ß_3\}, \{ß_2, ß_3\}, \{ß_1, ß_2, ß_3\} >\}$.

The complement of $(Ꞙ, A)$ is denoted by $(Ꞙ, A)^c$

$Ꞙ^c$: A→(℘(U), ℘(U), ℘(U)) where

$Ꞙ^c(e_1) = \{< \{ß_4, ß_5, ß_6\}, \{ß_2, ß_4, ß_5, ß_6\}, \{ß_3, ß_4, ß_5, ß_6\} >\}$,

$Ꞙ^c(e_3) = \{< \{ß_1, ß_2, ß_3\}, \{ß_1, ß_2, ß_3, ß_4\}, \{ß_4, ß_5, ß_6\} >\}$,

$Ꞙ^c(e_5) = \{< \{ß_2, ß_4, ß_5, ß_6\}, \{ß_1, ß_4, ß_5, ß_6\}, \{ß_4, ß_5, ß_6\} >\}$.

Definition 8. For two NCSSs $(Ꞙ, \mathfrak{D})$ and $(G, \mathfrak{W})$ on U, then $(Ꞙ, \mathfrak{D})$ is a "neutrosophic crisp soft sub-set" of $(G, \mathfrak{W})$ ($(ꞑ, \mathfrak{D}) \tilde{\subseteq} (G, \mathfrak{W})$) if

I. $\mathfrak{D} \subseteq \mathfrak{W}$.

II. $f^1(\mathfrak{y}) \subseteq g^1(\mathfrak{y}), f^2(\mathfrak{y}) \subseteq g^2(\mathfrak{y}), f^3(\mathfrak{y}) \supseteq g^3(\mathfrak{y})$ or $f^1(\mathfrak{y}) \subseteq g^1(\mathfrak{y}), f^2(\mathfrak{y}) \supseteq g^2(\mathfrak{y}), f^3(\mathfrak{y}) \supseteq g^3(\mathfrak{y})$, for all $\mathfrak{y} \in \mathfrak{D}$.

Example 2. Let $\mathfrak{D} = \{e_1, e_3, e_5\} \subseteq E$, and $\mathfrak{W} = \{e_1, e_2, e_3, e_5\} \subseteq E$.

Let $(\digamma, \mathfrak{D})$ and $(G, \mathfrak{W})$ be two NCSSs over $U = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$;

$$\digamma(e_1) = \{< \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\} >,$$
$$\digamma(e_3) = \{< \{\beta_4, \beta_5, \beta_6\}, \{\beta_5, \beta_6\}, \{\beta_1, \beta_2, \beta_3\} >,$$
$$\digamma(e_5) = \{< \{\beta_1, \beta_3\}, \{\beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} >,$$
$$G(e_1) = \{< \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_1, \beta_3, \beta_5\}, \{\beta_2\} >,$$
$$G(e_3) = \{< \{\beta_4, \beta_5, \beta_6\}, \{\beta_4, \beta_5, \beta_6\}, \{\beta_1, \beta_3\} >,$$
$$G(e_5) = \{< \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} >,$$
$$G(e_2) = \{< \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} >.$$

It is clear that $(\digamma, \mathfrak{D}) \tilde{\subseteq} (G, \mathfrak{W})$.

Definition 9. Let $(\digamma, A)$ and $(G, \mathfrak{W})$ be two NCSS, then $(G, \mathfrak{W}) = (\digamma, A)$ if $(\digamma, A) \tilde{\subseteq} (G, \mathfrak{W})$ and $(G, \mathfrak{W}) \tilde{\subseteq} (\digamma, A)$.

Definition 10.

I. A NCSS $(\digamma, A)$ is identified as a neutrosophic crisp null soft set over χ, denoted by a $\widetilde{\emptyset_N}$, if $\digamma(e) = \emptyset_N$: for all $e \in A$.

II. A NCSS $(\digamma, A)$ is identified as a neutrosophic crisp absolute s soft set over χ, denoted by a $\widetilde{\chi_N}$, if $\digamma(e) = \chi_N$: for all $e \in A$.

Definition 11. Union of two NCSS $(\digamma, A)$ and $(G, \mathfrak{W})$ $((\digamma, A)\ \tilde{\cup}\ (G, \mathfrak{W}))$ on the common universe χ is the NCSS (H, C), where $C = A \cup \mathfrak{W}$, and

$H: A \cup \mathfrak{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $\eta \in C$

$$H(\eta) = \{< f^1(\eta), f^2(\eta), f^3(\eta) >\}: \eta \in A$$
$$= \{< g^1(\eta), g^2(\eta), g^3(\eta) >\}: \eta \in B$$
$$= \{< f^1(\eta) \cup g^1(\eta), f^2(\eta) \cup g^2(\eta), f^3(\eta) \cap g^3(\eta) >\}\ \eta \in A \cap \mathfrak{W} \text{ or}$$
$$= \{< f^1(\eta) \cup g^1(\eta), f^2(\eta) \cap g^2(\eta), f^3(\eta) \cap g^3(\eta) >\}\ \eta \in A \cap \mathfrak{W}$$

Example 3. In *Example 2*, $(\digamma, A)\ \tilde{\cup}\ (G, \mathfrak{W}) = (H, C)$, where $C = A \cup \mathfrak{W}$, and

$H: A \cup \mathfrak{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $e \in C$,

$$H(e_1) = \{< \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_1, \beta_3, \beta_5\}, \{\beta_2\} >,$$
$$H(e_3) = \{< \{\beta_4, \beta_5, \beta_6\}, \{\beta_4, \beta_5, \beta_6\}, \{\beta_1, \beta_3\} >>,$$
$$H(e_5) = \{< \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3\} >,$$
$$H(e_2) = \{< \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} >.$$

Definition 12. The intersection of two "soft sets" of $(\digamma, A)$ and $(G, \mathfrak{W})$ on χ is the "soft sets" (H, C); $C = A \cap \mathfrak{W}$, and

$H: A \cap \mathfrak{W} \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $\eta \in C$

$$H(\eta) = \{< f^1(\eta) \cap g^1(\eta), f^2(\eta) \cap g^2(\eta), f^3(\eta) \cup g^3(\eta) >\}: \eta \in A \cap B \text{ or}$$
$$= \{< f^1(\eta) \cap g^1(\eta), f^2(\eta) \cup g^2(\eta), f^3(\eta) \cup g^3(\eta) >\}: \eta \in A \cap B.$$

Example 4. In *Example 2*, $(ꝑ, A)\ \tilde{\cap}\ (G, \mathfrak{W}) = (H, C)$; $C = A \cap B$;

H: $A \cap B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $e \in C$

$$H(e_1) = \{< \{ß_1, ß_2, ß_3\}, \{ß_1, ß_3\}, \{ß_1, ß_2\} >,$$
$$H(e_3) = \{< \{ß_4, ß_5, ß_6\}, \{ß_5, ß_6\}, \{ß_1, ß_2, ß_3\} >,$$
$$H(e_5) = \{< \{ß_1, ß_3\}, \{ß_2, ß_3\}, \{ß_1, ß_2, ß_3\} >.$$

Definition 13. Let (Ꝑ, A) and (G, B) are two NCSS then "$(ꝑ, A)$ and (G, B)" denoted by $(ꝑ, A) \wedge (G, B)$ is defined by $(ꝑ, A) \wedge (G, B) = (H, A \times B)$; $H(ü, ς) = ꝑ(ü) \cap G(ς)$; for all $(ü, ς) \in A \times B$, which mean:

$$H(ü, ς) = ꝑ(ü) \cap G(ς)$$
$$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cup g^3(ς) >\} \text{ or}$$
$$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cup g^3(ς) >\}.$$

Definition 14. Let (Ꝑ, A) and (G, B) are two NCSS then "$(ꝑ, A)$ or $(G, \mathfrak{W})$" ($(ꝑ, A) \vee (G, \mathfrak{W})$); $(ꝑ, A) \vee (G, \mathfrak{W}) = (Q, A \times B)$; $Q(ü, ς) = ꝑ(ü) \cup G(ς)$; for all $(ü, ς) \in A \times B$, which mean:

$$Q(ü, ς) = ꝑ(ü) \cup G(ς)$$
$$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cap g^3(ς) >\} \text{ or}$$
$$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cap g^3(ς) >\}.$$

Proposition 1.

I. $((ꝑ, A) \wedge (G, \mathfrak{W}))^c = (ꝑ, A) \vee (G, \mathfrak{W})$.

II. $((ꝑ, A) \vee (G, \mathfrak{W}))^c = (ꝑ, A) \wedge (G, \mathfrak{W})$.

Proof:

I. $(ꝑ, A) \wedge (G, \mathfrak{W}) = (H, A \times \mathfrak{W})$

where $H(ü, ς) = ꝑ(ü) \cap G(ς)$

$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cup g^3(ς) >\}$ or
$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cup g^3(ς) >\}$.
therefore $((ꝑ, A) \wedge (G, \mathfrak{W}))^c =$
$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cup g^3(ς) >\}^c$ or
$= \{< f^1(ü) \cap g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cup g^3(ς) >\}^c$
$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cap g^3(ς) >\}$ or
$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cap g^3(ς) >\}$
$= (ꝑ, A) \vee (G, \mathfrak{W})$. Therefore
$((ꝑ, A) \wedge (G, \mathfrak{W}))^c = (ꝑ, A) \vee (G, \mathfrak{W})$.

II. $(ꝑ, A) \vee (G, \mathfrak{W}) = (H, A \times \mathfrak{W})$

where $H(ü, ς) = ꝑ(ü) \cup G(ς)$
$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cap g^3(ς) >\}$ or
$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cap g^2(ς), f^3(ü) \cap g^3(ς) >\}$.
therefore $((ꝑ, A) \vee (G, \mathfrak{W}))^c =$
$= \{< f^1(ü) \cup g^1(ς), f^2(ü) \cup g^2(ς), f^3(ü) \cap g^3(ς) >\}^c$ or

$= \{< f^1(\ddot{u}) \cup g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cap g^3(\varsigma) >\}^c$.

$= \{< f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cap g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) >\}$ or

$= \{< f^1(\ddot{u}) \cap g^1(\varsigma), f^2(\ddot{u}) \cup g^2(\varsigma), f^3(\ddot{u}) \cup g^3(\varsigma) >\}$.

$= (\text{p}, A) \wedge (G, \mathfrak{W})$. Therefore

$\left((\text{p}, A) \vee (G, \mathfrak{W})\right)^c = (\text{p}, A) \wedge (G, \mathfrak{W})$.

3.1 | Application of Neutrosophic Crisp Soft Sets in Decisions-Making

The following examples highlight the practical role of "NCSSs" in decision-making processes. A notable application is in the field of "home marketing", where these sets assist in evaluating and selecting optimal marketing strategies based on uncertain or multi-parameter data.

Example 5. Let $U = \{ա_1, ա_2, ա_3, ա_4, ա_5, ա_6\}$ be set consists of several houses, let the set $\ddot{E} = \{ꝛ_1, ꝛ_2, ꝛ_3, ꝛ_4, ꝛ_5, ꝛ_6, ꝛ_7\}$ give specifications to this houses (Ë is the set of parameters);

$ꝛ_1$ means the "expensive".

$ꝛ_2$ means "beautiful".

$ꝛ_3$ means "wooden".

$ꝛ_4$ means "cheap".

$ꝛ_5$ means in "the green surroundings".

$ꝛ_6$ means "modern".

$ꝛ_7$ means "in good repair".

The "soft set" (F, Ë) describes the represents the appeal of various houses that Mr. X is considering for purchase. Where:

$H: A \to (\wp(U), \wp(U), \wp(U))$

$H(ꝛ_1) = \{< \emptyset, \emptyset, \emptyset >\}$,

$H(ꝛ_2) = H(ꝛ_4) = \{< U, \emptyset, \emptyset >\}$,

$H(ꝛ_3) = \{< \{ա_1, ա_2, ա_6\}, \emptyset, \emptyset >\}$,

$H(ꝛ_5) = \{< \{ա_1, ա_2, ա_3, ա_4, ա_6\}, \emptyset, \emptyset >\}$,

$H(ꝛ_6) = \{< \{ա_1, ա_2, ա_6\}, \emptyset, \emptyset >\}$,

$H(ꝛ_7) = \{< \{ա_1, ա_3, ա_6\}, \emptyset, \emptyset >\}$.

Let $A = \{ꝛ_2, ꝛ_3, ꝛ_4, ꝛ_5, ꝛ_7\}$ be the set of descending home by Mr.X.

We put $ա_{ij} = 1$ when $ա_i \in H(ꝛ_j)$. Otherwise, $ա_{ij} = 0$.

In the following diagram of the NCSS (the first part of this set), which contributes to the introduction of data to the computer, which allows the development of an algorithm to solve the issue and its programming.

U \ A	$ꝛ_2$	$ꝛ_3$	$ꝛ_4$	$ꝛ_5$	$ꝛ_7$	v_i^1
$ա_1$	1	1	1	1	1	5
$ա_2$	1	1	1	1	0	4
$ա_3$	1	1	0	1	1	4
$ա_4$	1	0	1	1	0	3
$ա_5$	1	0	1	0	0	2
$ա_6$	1	1	1	1	1	5

For every house ɰ_i , we give $v_i{}^1 = \sum_j \text{ɰ}_{ij}$, then $v_k{}^1 = \text{mak}\ (v_i{}^1)$ indicates the optimal option (choice) for the issue raised. In the previous example, Mr. X can choose ɰ_1 or ɰ_6. in this example, there is no need for diagram of the second and third part of this set, because it is empty.

Remark 1. The previous "*Example 5*" can be regarded as representing a soft set in the classical sense because, in this case, it aligns with the classical definition; $\text{ß}^2\left(\text{ꭒ}_j\right) = \text{ß}^3\left(\text{ꭒ}_j\right) = \emptyset;$ for all $j = 1, 2, 3, 4, 5, 6, 7.$

However, "*Example 5*" also highlights that a "NCSS" serves as a generalization of the classical soft set. In other words, the classical soft set can be viewed as a specific case of the NCSS. In "*Example 5*", the focus was narrowed to a single aspect of interest for a merchant seeking to purchase a house. This explains why only one parameter was considered. where $\text{ß}^2\left(\text{ꭒ}_j\right) = \text{ß}^3\left(\text{ꭒ}_j\right) = \emptyset;$ for all $j = 1, 2, 3, 4, 5, 6, 7.$

However, we could expand the analysis to incorporate all aspects relevant to the merchant's decision-making process, such as the property's description, location within the city, area, price, and other factors. From these, we could prioritize the three most significant aspects, as will be elaborated in the next example.

Example 6. Let $U = \{\text{ɰ}_1, \text{ɰ}_2, \text{ɰ}_3, \text{ɰ}_4, \text{ɰ}_5, \text{ɰ}_6\}$ be the set consists of several houses, and let the set $\ddot{E} = \{\text{ꭒ}_1, \text{ꭒ}_2, \text{ꭒ}_3, \text{ꭒ}_4, \text{ꭒ}_5, \text{ꭒ}_6, \text{ꭒ}_7\}$ give specifications to this houses (Ë is the set of parameters);

ꭒ_1 means the "expensive".

ꭒ_2 means "beautiful".

ꭒ_3 means "wooden".

ꭒ_4 means "cheap".

ꭒ_5 means in "the green surroundings".

ꭒ_6 means "modern".

ꭒ_7 means "in good repair".

(F, Ë) describes the represents the appeal of various houses, which Mr. X (say) is going to buy. Where:

$$
\begin{aligned}
&H: A \rightarrow (\wp(U), \wp(U), \wp(U));\\
&H(\text{ꭒ}_1) = \{< U, \{\text{ɰ}_1, \text{ɰ}_2, \text{ɰ}_6\}, \emptyset >\}\\
&H(\text{ꭒ}_2) = H(\text{ꭒ}_4) = \{< \{\text{ɰ}_2, \text{ɰ}_4, \text{ɰ}_6\}, \{\text{ɰ}_2, \text{ɰ}_6\}, \{\text{ɰ}_1, \text{ɰ}_6\} >\},\\
&H(\text{ꭒ}_3) = \{< \{\text{ɰ}_3, \text{ɰ}_4, \text{ɰ}_6\}, \{\text{ɰ}_1, \text{ɰ}_2\}, \{\text{ɰ}_1, \text{ɰ}_4\} >\},\\
&H(\text{ꭒ}_5) = \{< \{\text{ɰ}_3, \text{ɰ}_4, \text{ɰ}_5\}, \{\text{ɰ}_1, \text{ɰ}_2, \text{ɰ}_6\}, \{\text{ɰ}_1, \text{ɰ}_2, \text{ɰ}_6\} >\}\\
&H(\text{ꭒ}_6) = \{< \{\text{ɰ}_1, \text{ɰ}_2, \text{ɰ}_3, \text{ɰ}_4, \text{ɰ}_5\}, \emptyset, \{\text{ɰ}_4\} >\}\\
&H(\text{ꭒ}_7) = \{< \{\text{ɰ}_2, \text{ɰ}_3, \text{ɰ}_4\}, \{\text{ɰ}_1, \text{ɰ}_6\}, \{\text{ɰ}_5, \text{ɰ}_6\} >\}
\end{aligned}
$$

Let $A = \{\text{ꭒ}_2, \text{ꭒ}_3, \text{ꭒ}_4, \text{ꭒ}_5, \text{ꭒ}_7\}$ be the set of descending home by Mr.X.

We put $\text{ɰ}_{ij} = 1$ or 2 or 3 if ɰ_i belong to one or two or three subset of $\{\text{ß}^1\left(\text{ꭒ}_j\right), \text{ß}^2\left(\text{ꭒ}_j\right), \text{ß}^3\left(\text{ꭒ}_j\right)\}$. Otherwise, $\text{ɰ}_{ij} = 0$.

In the following diagram of the NCSS (the first, second, third part of this set), which contributes to the introduction of data to the computer, which allows the development of an algorithm to solve the issue and its programming.

A \ U	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$	$\mathcal{U}_5$	$\mathcal{U}_7$	v_i
$ɥ_1$	1	2	1	2	1	7
$ɥ_2$	2	1	2	2	1	8
$ɥ_3$	0	1	0	1	1	3
$ɥ_4$	1	2	1	1	1	7
$ɥ_5$	0	0	0	1	1	2
$ɥ_6$	3	1	3	2	2	11

For every house $ɥ_i$, we give $v_i = \sum_j ɥ_{ij}$, then $v_k = \text{mak}(v_i)$ indicates the optimal option (choice) for the issue raised. In the previous example, Mr. X can choose $ɥ_6$.

4 | Neutrosophic Crisp Soft Point

In this part we introduced the concept of NCSPs.

Definition 15. Let x belong to χ, A special type of NCSS, such as (F, A={x}) is called a "NCSP" related to x type i (NCSP(i)) over χ, where F: {x}→(℘(χ), ℘(χ), ℘(χ)) is a mapping, where $(F,\{x\})^i = \{< f^1(e), f^2(e), f^3(e) >: e \in A\} \subseteq < ℘(χ), ℘(χ), ℘(χ) >$; $f^i(x) \neq \emptyset$ and $f^j(e) = \emptyset$, if $i \neq j$; i, j =1, 2, 3.

Example 7. Let A = $\{\mathcal{U}_1\} \subseteq$E, and B = $\{\mathcal{U}_2\} \subseteq$E.

$(F,A)^1$ and $(G,B)^1$ is two "NCSPs" over $U = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ type 1;

$$ꝑ(\mathcal{U}_1) = \{< \{ß_1\}, \emptyset, \emptyset >\},$$

$$G(\mathcal{U}_2) = \{< \{ß_2\}, \emptyset, \emptyset >\},$$

(H, A) is "NCSPs" over $U = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ type 2;

$$H(\mathcal{U}_1) = \{< \emptyset, \{ß_1\}, \emptyset >\},$$

(T, A) is "NCSPs" over $U = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ type 3;

$$t^1(\mathcal{U}_1) = \emptyset, \quad t^2(\mathcal{U}_1) = \emptyset, \quad t^3(\mathcal{U}_1) = \{ß_1\}.$$

Definition 16. Let (F, A) be a NCSS over U and $(G, \{ü\})^i$ be a NCSP type i over U. We say that $(G,\{ü\})^i \in (F, A)$ whenever $ü \in A$ and $g^i(ü) \subseteq f^i(ü)$. Note that $(G,\{ü\})^i \notin (F,A)$ if $ü \notin A$ or $g^i(ü) \not\subset f^i(ü)$; i =1, 2, 3.

Example 8. Let $A = \{\mathcal{U}_1, \mathcal{U}_3, \mathcal{U}_4, \mathcal{U}_5\} \subseteq$E, and $B = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_5\} \subseteq$E.

If $(ꝑ, A)$ and $(G, \mathfrak{B})$ be two "NCSSs" over $U = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$;

$$ꝑ(\mathcal{U}_1) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_4, ß_2\} >,$$

$$ꝑ(\mathcal{U}_3) = \{< \{ß_2, ß_3, ß_6\}, \{ß_3, ß_4\}, \{ß_1, ß_3\} >,$$

$$ꝑ(\mathcal{U}_5) = \{< \{ß_2, ß_3\}, \{ß_2, ß_4\}, \{ß_1, ß_3\} >,$$

$$ꝑ(\mathcal{U}_4) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_2, ß_3\} >,$$

$$G(\mathcal{U}_1) = \{< \{ß_2, ß_3, ß_4\}, \{ß_2, ß_3, ß_5\}, \{ß_2\} >,$$

$$G(\mathcal{U}_3) = \{< \{ß_4, ß_6\}, \{ß_5, ß_6\}, \{ß_3\} >,$$

$$G(\mathcal{U}_5) = \{< \{ß_2, ß_3\}, \{ß_2, ß_3\}, \{ß_2, ß_3\} >,$$

$$G(\mathcal{U}_2) = \{< \{ß_1, ß_3\}, \{ß_1, ß_2\}, \{ß_2, ß_3\} >.$$

Let $A_1 = \{\mathfrak{r}_1\} \subseteq E$, and $A_2 = \{\mathfrak{r}_2\} \subseteq E$.

$(\mathrm{p}_1, A_1)^1$ and $(\mathrm{p}_2, A_2)^1$ is two NCSPs over the same universe U type 1;

$\mathrm{p}(\mathfrak{r}_1) = \{< \{ß_1\}, \emptyset, \emptyset >\}$,
$G(\mathfrak{r}_2) = \{< \{ß_2\}, \emptyset, \emptyset >\}$,
$(\mathrm{p}_1, A_1)^1 \in (\mathrm{p}, A),\ (\mathrm{p}_1, A_1)^1 \notin (G, \mathfrak{W}), (\mathrm{p}_2, A_2)^1 \notin (\mathrm{p}, A), (\mathrm{p}_2, A_2)^1 \notin (G, \mathfrak{W})$

$(\mathrm{p}_3, A_1)^2$ is NCSP over the same universe U type 2;

$\mathrm{p}_3(\mathfrak{r}_1) = \{< \emptyset, \{ß_1\}, \emptyset >\}$,
$(\mathrm{p}_3, A_1)^2 \in (F, A),\ (\mathrm{p}_3, A_1)^2 \notin (G, B)$

$(T, A_1)^3$ is NCSP over the same universe U type 3;

$t^1(\mathfrak{r}_1) = \emptyset, \quad t^2(\mathfrak{r}_1) = \emptyset, \quad t^3(\mathfrak{r}_1) = \{ß_1\}$.
$(T, A_1)^3 \notin (\mathrm{p}, A),\ (T, A_1)^3 \notin (G, \mathfrak{W})$

5 | Separation Axioms in a "Neutrosophic Crisp Soft Topological Space"

Here, we will define the concept "neutrosophic crisp STS" and studied some concepts connected to this new space, and finally, we studied separation axioms in this space.

Definition 17. A Neutrosophic Crisp Soft Topology (NCST) on a set $\chi \neq \emptyset$ is a family $\mathfrak{T}$ of "neutrosophic crisp soft subsets" in χ satisfying the following conditions:

I. $\widetilde{\emptyset_N}, \widetilde{\chi_N} \in \mathfrak{T}$

II. If $\mathfrak{T}$,B belong to $\mathfrak{T}$ then $A \cap B \in \mathfrak{T}$.

III. The union of any number of sets in $\mathfrak{T}$ belongs to $\mathfrak{T}$.

Then $(\chi, \mathfrak{T})$is said to be a "NCSTS".

Definition 18. If $(\chi, \mathfrak{T})$ is a (NCSTS) in χ, The elements in $\mathfrak{T}$ are said to be "Neutrosophic Crisp Soft Open Sets" (NCSOS), a "NCSS" F is closed NCSCS if F^c (its complement) is an NCSOS.

Example 9. Let $A = \{\mathfrak{r}_1, \mathfrak{r}_3\} \subseteq E$, and $B = \{\mathfrak{r}_1, \mathfrak{r}_2\} \subseteq E$.

Let (p, A) and $(G, \mathfrak{W})$ be two NCSS over $\chi = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$;

$\mathrm{p}(\mathfrak{r}_1) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_4, ß_2\} >$,

$\mathrm{p}(\mathfrak{r}_3) = \{< \{ß_2, ß_3, ß_6\}, \{ß_3, ß_4\}, \{ß_1, ß_3\} >$,

$G(\mathfrak{r}_1) = \{< \{ß_2, ß_3, ß_4\}, \{ß_2, ß_3, ß_5\}, \{ß_2\} >$,

$G(\mathfrak{r}_2) = \{< \{ß_1, ß_3\}, \{ß_1, ß_2\}, \{ß_2, ß_3\} >$.

$(\mathrm{p}, A) \cup (G, \mathfrak{W}) = (H, C)$

where C = AU$\mathfrak{W}$, and

H: $A \cup \mathfrak{W} \to (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $\mathfrak{r} \in C$,

$H(ꭒ_1) = ꝑ(ꭒ_1) \cup G(ꭒ_1) = \{< \{ß_1, ß_2, ß_3, ß_4\}, \{ß_1, ß_2, ß_3, ß_4, ß_5\}, \{ß_2\} >,$

$H(ꭒ_3) = ꝑ(ꭒ_3) = \{< \{ß_2, ß_3, ß_6\}, \{ß_3, ß_4\}, \{ß_1, ß_3\} >,$

$H(ꭒ_2) = G(ꭒ_2) = \{< \{ß_1, ß_3\}, \{ß_1, ß_2\}, \{ß_2, ß_3\} >.$

$(ꝑ, A) \cap (G, \mathfrak{B}) = (H, C)$

where $C = A \cap \mathfrak{B}$, and

$K: A \cap B \to (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $ꭒ \in C$,

$K(ꭒ_1) = \{< \{ß_3\}, \emptyset, \{ß_2, ß_4\} >,$

$\mathfrak{T} = \{\emptyset_N, \chi_N, (A, ꝑ), (\mathfrak{B}, G), (A \cap \mathfrak{B}, K), (A \cup \mathfrak{B}, H)\}$

Then $(\chi, \mathfrak{T})$ is a NCSTS.

Example 10. Let $A = \{ꭒ_1, ꭒ_3, ꭒ_4, ꭒ_5\} \subseteq E$, and $\mathfrak{B} = \{ꭒ_1, ꭒ_2, ꭒ_3, ꭒ_5\} \subseteq E$.

Let (F, A) and $(G, \mathfrak{B})$ be two NCSS over $\chi = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ such that

$ꝑ(ꭒ_1) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_4, ß_2\} >,$

$ꝑ(ꭒ_3) = \{< \{ß_2, ß_3, ß_6\}, \{ß_3, ß_4\}, \{ß_1, ß_3\} >,$

$ꝑ(ꭒ_5) = \{< \{ß_2, ß_3\}, \{ß_2, ß_4\}, \{ß_1, ß_3\} >,$

$ꝑ(ꭒ_4) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_2, ß_3\} >,$

$G(ꭒ_1) = \{< \{ß_2, ß_3, ß_4\}, \{ß_2, ß_3, ß_5\}, \{ß_2\} >,$

$G(ꭒ_3) = \{< \{ß_4, ß_6\}, \{ß_5, ß_6\}, \{ß_3\} >,$

$G(ꭒ_5) = \{< \{ß_2, ß_3\}, \{ß_2, ß_3\}, \{ß_2, ß_3\} >,$

$G(ꭒ_2) = \{< \{ß_1, ß_3\}, \{ß_1, ß_2\}, \{ß_2, ß_3\} >.$

$(ꝑ, A) \cup (G, \mathfrak{B}) = (H, C)$

where $C = A \cup \mathfrak{B}$, and

$H: A \cup \mathfrak{B} \to (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $ꭒ \in C$,

$H(ꭒ_1) = \{< \{ß_1, ß_2, ß_3, ß_4\}, \{ß_1, ß_2, ß_3, ß_4, ß_5\}, \{ß_2\} >,$

$H(ꭒ_3) = \{< \{ß_6\}, \emptyset, \{ß_5, ß_6\} >,$

$H(ꭒ_4) = ꝑ(ꭒ_4) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_2, ß_3\} >,$

$ꝑ(ꭒ_5) = \{< \{ß_2, ß_3\}, \{ß_2, ß_3, ß_4\}, \{ß_3\} >,$

$(ꝑ, A) \cap (G, \mathfrak{B}) = (H, C)$

where $C = A \cap \mathfrak{B}$, and

$K: A \cap \mathfrak{B} \to (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $ꭒ \in C$,

$K(\mathcal{U}_1) = \{< \{ß_3\}, \emptyset, \{ß_2, ß_4\} >,$

$K(\mathcal{U}_3) = \{< \{ß_6\}, \emptyset, \{ß_1, ß_3\} >,$

$K(\mathcal{U}_5) = \{< \{ß_2, ß_3\}, \{ß_2\}, \{ß_3\} >.$

$\mathfrak{T} = \{\emptyset_N, \chi_N, (A, p), (\mathfrak{W}, G), (A \cap \mathfrak{W}, K), (A \cup \mathfrak{W}, H)\}$

Then $(\chi, \mathfrak{T})$ is a "NCSTS".

Definition 19. If $(\chi, \mathfrak{T})$ be an NCSTS. $(G, \{x\})^i$ is a NCSP(i), and $(H, B) \in \mathfrak{T}$ is a NCSOS, (H, B) is defined to be neutrosophic crisp soft open nhd of $(G, \{x\})^i$ in $(\chi, \mathfrak{T})$ if $(G, \{x\})^i \in (H, B)$

Definition 20. Let $(\chi, \mathfrak{T})$ be an NCSTS. $(G, \{x\})^i$ is NCSP(i), and (H, B) is a NCSS, (H, B) is defined to be neutrosophic crisp soft nhd of $(G, \{x\})^i$ in $(\chi, \mathfrak{T})$ if there is a NCSOS (F, A) containing $(G, \{x\})^i$ such that $(F, A) \subseteq (H, B)$.

Remark 2. Every neutrosophic crisp soft open nhd of $(G, \{x\})^i$ is be neutrosophic crisp soft nhd of $(G, \{x\})^i$, but the opposite is not true (in general).

Now, we study "Separation axioms" in a NCSTS.

Definition 21. A NCSTS $(\chi, \mathfrak{T})$ is called

I. N_1S-$\mathfrak{T}_o$-space if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi$ $\exists$ a NCSOS $(F, \mathrm{Ŋ})$ in χ such that $(G, \{x\})^1 \in (F, \mathrm{Ŋ})$, $(H, \{y\})^1 \notin (F, \mathrm{Ŋ})$ or $(G, \{x\})^1 \notin (F, \mathrm{Ŋ})$, $(H, \{y\})^1 \in (F, \mathrm{Ŋ})$.

II. N_2S-$\mathfrak{T}_o$-space if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi$ $\exists$ a NCSOS $(F, \mathrm{Ŋ})$ in χ such that $(G, \{x\})^2 \in (F, \mathrm{Ŋ})$, $(H, \{y\})^2 \notin (F, \mathrm{Ŋ})$ or $(G, \{x\})^2 \notin (F, \mathrm{Ŋ})$, $(H, \{y\})^2 \in (F, \mathrm{Ŋ})$.

III. N_3S-$\mathfrak{T}_o$-space if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi$ $\exists$ a NCSOS $(F, \mathrm{Ŋ})$ in χ such that $(G, \{x\})^3 \in (F, \mathrm{Ŋ})$, $(H, \{y\})^3 \notin (F, \mathrm{Ŋ})$ or $(G, \{x\})^3 \notin (F, \mathrm{Ŋ})$, $(H, \{y\})^3 \in (F, \mathrm{Ŋ})$.

IV. N_1S-$\mathfrak{T}_1$-space if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^1 \in (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^1 \notin (F_2, \mathrm{Ŋ}_2)$ and $(G, \{x\})^1 \notin (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^1 \in (F_2, \mathrm{Ŋ}_2)$.

V. N_2S-$\mathfrak{T}_1$-space if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^2 \in (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^2 \notin (F_2, \mathrm{Ŋ}_2)$ and $(G, \{x\})^2 \notin (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^2 \in (F_2, \mathrm{Ŋ}_2)$.

VI. N_3S-$\mathfrak{T}_1$-space if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^3 \in (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^3 \notin (F_2, \mathrm{Ŋ}_2)$ and $(G, \{x\})^3 \notin (F_1, \mathrm{Ŋ}_1)$, $(H, \{y\})^3 \in (F_2, \mathrm{Ŋ}_2)$.

VII. N_1S-$\mathfrak{T}_2$-space if for all $(G, \{x\})^1 \neq (H, \{y\})^1 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^1 \in (F_1, \mathrm{Ŋ}_1)$ and $(H, \{y\})^1 \in (F_2, \mathrm{Ŋ}_2)$ with $(F_1, \mathrm{Ŋ}_1) \cap (F_1, \mathrm{Ŋ}_1) = \widetilde{\emptyset_N}$.

VIII. N_2S-$\mathfrak{T}_2$-space if for all $(G, \{x\})^2 \neq (H, \{y\})^2 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^2 \in (F_1, \mathrm{Ŋ}_1)$ and $(H, \{y\})^2 \in (F_2, \mathrm{Ŋ}_2)$ with $(F_1, \mathrm{Ŋ}_1) \cap (F_1, \mathrm{Ŋ}_1) = \widetilde{\emptyset_N}$.

IX. N_3S-$\mathfrak{T}_2$-space if for all $(G, \{x\})^3 \neq (H, \{y\})^3 \in \chi$ $\exists$ a two NCSOS $(F_1, \mathrm{Ŋ}_1), (F_1, \mathrm{Ŋ}_1)$ in χ such that $(G, \{x\})^3 \in (F_1, \mathrm{Ŋ}_1)$ and $(H, \{y\})^3 \in (F_2, \mathrm{Ŋ}_2)$ with $(F_1, \mathrm{Ŋ}_1) \cap (F_1, \mathrm{Ŋ}_1) = \widetilde{\emptyset_N}$.

Example 11. Let $\mathrm{Ŋ} = \{\mathcal{U}_1, \mathcal{U}_3\} \subseteq E$, and $B = \{\mathcal{U}_1, \mathcal{U}_2\} \subseteq E$.

Let $(F, \mathrm{Ŋ})$ and (G, B) be two NCSS over the same universe $\chi = \{ß_1, ß_2, ß_3, ß_4, ß_5, ß_6\}$ such that

$p(\mathcal{U}_1) = \{< \{ß_1, ß_3\}, \{ß_1, ß_4\}, \{ß_4, ß_2\} >,$

$p(\mathcal{U}_3) = \{< \{ß_2, ß_3, ß_6\}, \{ß_3, ß_4\}, \{ß_1, ß_3\} >,$

$G(\mathcal{U}_1) = \{< \{ß_2, ß_3, ß_4\}, \{ß_2, ß_3, ß_5\}, \{ß_2\} >,$

$G(\mathfrak{u}_2) = \{< \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} >$.

$(\mathfrak{p}, \mathfrak{N}) \cup (G, B) = (H, C)$

where C = ÐU B, and

H: $\mathfrak{N} \cup B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $\mathfrak{u} \in C$,

$H(\mathfrak{u}_1) = \mathfrak{p}(\mathfrak{u}_1) \cup G(\mathfrak{u}_1) = \{< \{\beta_1, \beta_2, \beta_3, \beta_4\}, \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\}, \{\beta_2\} >$,

$H(\mathfrak{u}_3) = \mathfrak{p}(\mathfrak{u}_3) = \{< \{\beta_2, \beta_3, \beta_6\}, \{\beta_3, \beta_4\}, \{\beta_1, \beta_3\} >$,

$H(\mathfrak{u}_2) = G(\mathfrak{u}_2) = \{< \{\beta_1, \beta_3\}, \{\beta_1, \beta_2\}, \{\beta_2, \beta_3\} >$.

$(\mathfrak{p}, \mathfrak{N}) \cap (G, B) = (H, C)$

where C = Ð ∩ B, and

K: $Ð \cap B \rightarrow (\wp(\chi), \wp(\chi), \wp(\chi))$

and for all $\mathfrak{u} \in C$,

$K(\mathfrak{u}_1) = \{< \{\beta_3\}, \emptyset, \{\beta_2, \beta_4\} >$,

$\mathfrak{T} = \{\emptyset_N, \chi_N, (\mathfrak{N}, \mathfrak{p}), (\mathfrak{W}, G), (\mathfrak{N} \cap \mathfrak{W}, K), (\mathfrak{N} \cup \mathfrak{W}, H)\}$

Then $(\chi, \mathfrak{T})$ is a NCSTS.

Then $(\chi, \mathfrak{T})$ is not N_1S-$\mathfrak{T}_o$-space, because for $(K, \{\mathfrak{u}_1\})^1 \neq (H, \{\mathfrak{u}_3\})^1 \in \chi$

Where

$K(\mathfrak{u}_1) = \{< \{\beta_5\}, \emptyset, \emptyset >, H(\mathfrak{u}_3) = \{< \{\beta_5\}, \emptyset, \emptyset >$,

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, $(\chi, \mathfrak{T})$ is not N_1S-$\mathfrak{T}_i$-space, i=1, 2.

Then $(\chi, \mathfrak{T})$ is not N_2S-$\mathfrak{T}_o$-space, because for $(K, \{\mathfrak{u}_1\})^2 \neq (H, \{\mathfrak{u}_3\})^2 \in \chi$

Where

$K(\mathfrak{u}_1) = \{< \emptyset, \{\beta_6\}, \emptyset >, H(\mathfrak{u}_3) = \{< \emptyset, \{\beta_6\}, \emptyset >$,

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, $(\chi, \mathfrak{T})$ is not N_2S-$\mathfrak{T}_i$-space, i=1, 2.

Then $(\chi, \mathfrak{T})$ is not N_3S-$\mathfrak{T}_o$-space, because for $(K, \{\mathfrak{u}_1\})^3 \neq (H, \{\mathfrak{u}_3\})^3 \in \chi$

Where

$K(\mathfrak{u}_1) = \{< \emptyset, \emptyset, \{\beta_5\} >, H(\mathfrak{u}_3) = \{< \emptyset, \emptyset, \{\beta_5\} >$,

there are no a NCSOS (K, O) in χ containing one of them but not the other. So, $(\chi, \mathfrak{T})$ is not N_3S-$\mathfrak{T}_i$-space, i=1, 2.

Example 12. Let $A = \{\mathfrak{u}_1, \mathfrak{u}_2\} \subseteq E = \{\mathfrak{u}_1, \mathfrak{u}_2\}$.

Let $(F, \mathfrak{N})$ be a NCSS on universe $\chi = \{\beta_1, \beta_2, \beta_3\}$ such that

$ꝑ(\mathcal{U}_1) = \{< \{ß_1, ß_2\}, \{ß_2, ß_3\}, \{ß_3\} >,$

$ꝑ(\mathcal{U}_2) = \{< \{ß_1, ß_3\}, \{ß_1, ß_2\}, \{ß_1, ß_2\} >,$

$J = \{\emptyset_N, \chi_N, (N, ꝑ)\}$

Then $(\chi, \mathfrak{I})$ is a NCSTS

I. Then $(\chi,\mathfrak{I})$ is N_1-$\mathfrak{I}_o$-space, but $(\chi,\mathfrak{I})$ is not N_1-$\mathfrak{I}_i$-space, i=1, 2.

II. Then $(\chi,\mathfrak{I})$ is N_2-$\mathfrak{I}_o$-space, but $(\chi,\mathfrak{I})$ is not N_2-$\mathfrak{I}_i$-space, i=1, 2.

III. Then $(\chi,\mathfrak{I})$ is N_3-$\mathfrak{I}_o$-space, but $(\chi,\mathfrak{I})$ is not N_3-$\mathfrak{I}_i$-space, i=1, 2.

Remark 3. For a NCSTS $(\chi, \mathfrak{I})$

I. Every N_iS-$\mathfrak{I}_1$-space is N_iS-$\mathfrak{I}_o$-space, i =1, 2, 3.

II. Every N_iS-$\mathfrak{I}_2$-space is N_iS-$\mathfrak{I}_1$-space, i=1, 2, 3.

Proof: the proof follows directly from defintions.

The converse of *Remark 3* does not hold, as demonstrated in the *Example 13*:

Example 13. In *Example 12*.

I. Then $(\chi,\mathfrak{I})$ is N_1S-$\mathfrak{I}_o$-space, $(\chi,\mathfrak{I})$ is not N_1S-$\mathfrak{I}_i$-space, i=1, 2.

II. Also, $(\chi,\mathfrak{I})$ is N_2S-$\mathfrak{I}_o$-space, $(\chi,\mathfrak{I})$ is not N_2S-$\mathfrak{I}_i$-space, i=1, 2.

III. Also, $(\chi,\mathfrak{I})$ is N_3S-$\mathfrak{I}_o$-space, $(\chi,\mathfrak{I})$ is not N_3S-$\mathfrak{I}_i$-space, i=1, 2.

6 | Conclusion

This study has successfully established a ground-breaking connection between "NCSs" and "soft sets", introducing innovative mathematical tools and concepts that enhance our understanding of these theories. Through the definition of "NCSTSs", "NCSSs", "NCSPs", and "neutrosophic crisp soft separation axioms", we have developed a cohesive framework that bridges these areas of neutrosophic crisp soft theory. This framework paves the way for novel interdisciplinary research opportunities across various scientific domains.

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