Paper Type: Original Article

Soft Intersection-Difference Product of Groups

Aslıhan Sezgin[1],* iD **, Zeynep Ay[2]**

[1] Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Türkiye; aslihan.sezgin@amasya.edu.tr.

[2] Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Türkiye; ayzeynep875@gmail.com.

Abstract

Soft set theory constitutes a comprehensive mathematical apparatus for modeling and managing uncertainty. Central to this theory are soft set operations and product constructions, which facilitate novel methodologies for addressing problems characterized by parametric data. In the present study, we propose a new product structure for soft sets whose parameter sets possess a group structure, termed the soft intersection-difference product. A rigorous investigation of its fundamental algebraic properties is conducted, encompassing various soft subsets and notions of equality. The findings are anticipated to stimulate further scholarly inquiry, potentially laying the groundwork for a nascent soft group theory derived from this construction. Given that the development of soft algebraic structures fundamentally relies on well-defined soft set operations and products, the study offers a substantial contribution to the theoretical advancement of soft set theory.

Keywords: Soft sets, Soft subsets, Soft equalities, Soft intersection-difference product.

1|Introduction

Numerous researchers have proposed a variety of mathematical frameworks aimed at modeling and addressing complex problems characterized by uncertainty, vagueness, and ambiguity in domains such as engineering, economics, social sciences, and healthcare. Molodtsov [1] identified inherent limitations in existing frameworks. For instance, fuzzy set theory [2] often encounters challenges in the appropriate specification of membership functions, while probability theory relies on extensive trials to establish the existence of a mean value.

To overcome these limitations, Molodtsov [1] introduced soft set theory as a novel mathematical paradigm, demonstrating its potential applicability in diverse areas such as probability theory, game theory, and operations research. In contrast to classical approaches, soft set theory provides a more adaptable framework

by eliminating rigid requirements related to approximate descriptions. The seminal work by Maji et al. [3], which applied soft sets to decision-making, laid the groundwork for subsequent advancements. Building upon this foundation, numerous studies [4–10] introduced enhanced decision-making methodologies grounded in soft set theory. Notably, Çağman and Enginoğlu [11] proposed a soft set-based decision-making model, and further introduced the concept of soft matrices [12], formulating decision procedures based on their AND, OR, AND-NOT, and OR-NOT operations. These formulations proved effective in resolving real-world problems under uncertainty.

The adaptability and utility of soft set theory have led to its widespread adoption in decision-making contexts [13-24], where significant developments have emerged. These include bijective and exclusive disjunctive soft sets, generalized uni-int frameworks, soft approximations, operator-based decision processes, reduced and cardinality-inverse soft matrices, soft semantics, and a range of mean and generalized operators on fuzzy soft matrices, as well as soft set-valued mappings.

In recent years, scholarly interest has increasingly turned to the theoretical underpinnings of soft set theory. Maji et al. [25] conducted a foundational analysis involving concepts such as soft subsets and supersets, soft set equality, and basic operations including union, intersection, AND-product, and OR-product. Pei and Miao [26] refined these notions by exploring connections between soft sets and information systems and redefining intersection and subset relations. Further contributions by Ali et al. [27] introduced new operations such as restricted union, restricted intersection, restricted difference, and extended intersection. Subsequent investigations [28-40] have focused on the algebraic structure of soft set operations, correcting earlier conceptual inconsistencies and proposing new methodologies.

Significant progress has been made in the formalization of soft set operations, as evidenced by a wide array of newly defined and rigorously analyzed operations [41–48]. Central to the theory are soft equal relations and soft subsets. Maji et al. [25] initiated the formal definition of soft subsets, which was later extended by Pei and Miao [26] and Feng et al. [29]. Qin and Hong [49] introduced new notions of soft congruence and equality. To generalize Maji's distributive laws, Jun and Yang [50] incorporated broader soft subset classes and proposed J-soft equal relations. Inspired by their work, Liu et al. [51] explored soft L-subsets and soft L-equal relations, revealing that distributive laws do not universally apply across all soft equalities.

Building on this foundation, Feng et al. [52] investigated classifications of soft subsets and the properties of soft product operations introduced in [24], such as the AND- and OR-products, within the framework of soft L-subsets. Their comprehensive analysis addressed commutativity, associativity, and distributivity, resolving previously incomplete findings and demonstrating that soft L-equal relations constitute congruence relations in free soft algebras, where resulting quotient structures form commutative semigroups. For further developments in the theory of soft equalities (including generalized soft equality, soft lattices, relaxed parameter constraints, g-soft and gf-soft equality, and T-soft equality) see [53–57].

Çağman and Enginoğlu [11] revisited the definition of soft set by Maji et al. [25] and its operations to enhance practical applicability. They introduced four distinct product operations: AND-, OR-, AND-NOT-, and OR-NOT-products, alongside the uni-int decision function. These innovations were integrated into a unified decision-making framework, demonstrated through practical applications involving uncertainty. Sezgin et al. [58] subsequently analyzed the AND-product, a pivotal operation in soft set-based decision-making, within various equality frameworks, such as soft L-equality and J-equality. Their work systematically examined its algebraic properties, including idempotency, commutativity, and associativity, comparing these with results involving soft F-subsets, M-equality, L-equality, and J-equality.

The concept of the soft union product was first introduced for rings [59], semigroups [60], and groups [61], forming the basis for the development of soft union ring, semigroup, and group theories. Similarly, the soft intersection product was defined for groups [62], semigroups [63], and rings [64], with corresponding algebraic theories subsequently developed. Due to inherent differences among these algebraic structures, the definitions and properties of these products exhibit structural variations. In particular, the presence of a unit element and inverses in groups imparts unique characteristics to the group-based definitions.

47

Sezgin and Ay | Uncert. Disc. Appl. 2(1) (2025) 45-60

In this study, we propose a novel product for soft sets whose parameter sets form a group structure, referred to as the soft intersection-difference product, constructed within the definitional framework of Çağman and Enginoğlu [11]. A detailed analysis of its algebraic properties is undertaken, considering various soft subsets and equality relations, with the aim of inspiring the development of a new soft group theory rooted in this construct. The remainder of the paper is organized as follows: Section 2 revisits essential concepts in soft set theory; Section 3 introduces the soft intersection-difference product and presents a comprehensive algebraic analysis in relation to different types of soft subsets and equalities. The concluding section offers a summary of the results and outlines directions for future research.

2 | Preliminaries

This section is devoted to revisiting a selection of foundational definitions and structural properties that serve as a theoretical basis for the developments presented in the subsequent section. Although the notion of the soft set was initially proposed by Molodtsov [1], the conceptual framework, including key definitions and operational structures, was later substantially revised by Çağman and Enginoğlu [11] to enhance both theoretical rigor and applicability. Accordingly, the present study is grounded in this refined formulation, which will be employed consistently throughout the paper.

Definition 1 ([11]). Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $Y \subseteq E$. Then, the soft set $\int_Y$ over U is a function such that $\int_Y : E \to P(U)$, where for all $k \notin Y$, $\int_Y(k) = \emptyset$. That is,

$$\int_Y = \left\{ \left(k, \int_Y(k) \right) : k \in E \right\}.$$

From now on, soft set over U is abbreviated by SS.

Definition 2 ([11]). Let $\int_{\mathcal{H}}$ be an SS. If $\int_{\mathcal{H}}(k) = \emptyset$, for all $k \in E$, then $\int_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_E$, and if $\int_{\mathcal{H}}(k) = U$, for all $k \in E$, then $\int_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 3 ([11]). Let $\int_{\mathcal{H}}, \int_{\aleph}$ be two SSs. If $\int_{\mathcal{H}}(k) \subseteq \int_{\aleph}(k)$, for all $k \in E$, then $\int_{\mathcal{H}}$ is a soft subset of $\int_{\aleph}$ and indicated by $\int_{\mathcal{H}} \widetilde{\subseteq} \int_{\aleph}$. If $\int_{\mathcal{H}}(k) = \int_{\aleph}(k)$, for all $k \in E$, then $\int_{\mathcal{H}}$ is called soft equal to $\int_{\aleph}$, and denoted by $\int_{\mathcal{H}} = \int_{\aleph}$.

Definition 4 ([11]). Let $\int_{\mathcal{H}}, \int_{\aleph}$ be two SSs. The union of $\int_{\mathcal{H}}$ and $\int_{\aleph}$ is the SS $\int_{\mathcal{H}} \widetilde{\cup} \int_{\aleph}$, where $(\int_{\mathcal{H}} \widetilde{\cup} \int_{\aleph})(w) = \int_{\mathcal{H}}(w) \cup \int_{\aleph}(w)$, for all $w \in E$.

Definition 5 ([11]). Let $\int_{\mathcal{H}}$ be an SS. Then, the complement of $\int_{\mathcal{H}}$, denoted by $(\int_{\mathcal{H}})^c$, is defined by the SS $\int_{\mathcal{H}}^c : E \to P(U)$ such that $\int_{\mathcal{H}}^c(e) = U \backslash \int_{\mathcal{H}}(e) = (\int_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 6 ([65]). Let $\int_K$ and q_L be two SSs. Then, $\int_K$ is called a soft S-subset of q_L, denoted by $\int_K \widetilde{\subseteq}_S q_L$, if for all $e \in E$, $\int_K(e) = \bar{A}$ and $q_K(e) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} \subseteq B$. Moreover, two SSs $\int_K$ and q_L are said to be soft S-equal, denoted by $\int_K =_S q_L$, if $\int_K \widetilde{\subseteq}_S q_L$ and $q_L \widetilde{\subseteq}_S \int_K$.

It is obvious that if $\int_K =_S q_L$, then $\int_K$ and q_L are the same constant functions, that is, for all $e \in E$, $\int_K(e) = q_K(e) = \bar{A}$, where $\bar{A}$ is a fixed set.

Definition 7 (65]). Let $\int_K$ and q_L be two SSs. Then, $\int_K$ is called a soft A-subset of q_L, denoted by $\int_K \widetilde{\subseteq}_A q_L$, if, for each $g, h \in E$, $\int_K(g) \subseteq q_L(h)$.

Definition 8 ([65]). Let $\int_K$ and q_L be two SSs. Then, $\int_K$ is called a soft S-complement of q_L, denoted by $\int_K =_S (q_L)^c$, if, for all $e \in E$, $\int_K(e) = \bar{A}$ and $q_L(e) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} = B'$. Here, $B' = U \backslash B$.

For additional information on SSs, we refer to [66-82].

Definition 9 ([61]). Let $\int_G$ and q_G be two SSs, where G is a group. Then, the soft intersection-union product $\int_G \otimes_{i/u} q_G$ is defined by

$$\left(\int_G \otimes_{i/u} \mathfrak{q}_G\right)(\Bbbk) = \bigcap_{\Bbbk=\hbar\mathfrak{u}} \left(\int_G (\hbar) \cup \mathfrak{q}_G(\mathfrak{u})\right), \quad \hbar, \mathfrak{u} \in G,$$

for all $\Bbbk \in G$.

Definition 10 ([83]). Let $\int_G$ and $\mathfrak{q}_G$ be two SSs where G is a group. Then, the soft intersection-symmetric difference product $\int_G \otimes_{i/s} \mathfrak{q}_G$ is defined by

$$\left(\int_G \otimes_{i/s} \mathfrak{q}_G\right)(\Bbbk) = \bigcap_{\Bbbk=\hbar\mathfrak{u}} \left(\int_G (\hbar) \Delta \mathfrak{q}_G(\mathfrak{u})\right), \quad \hbar, \mathfrak{u} \in G,$$

for all $\Bbbk \in G$.

Definition 11 ([83]). Let $\int_G$ and $\mathfrak{q}_G$ be two SSs, where G is a group. Then, the soft union-difference product $\int_G \otimes_{u/d} \mathfrak{q}_G$ is defined by

$$\left(\int_G \otimes_{u/d} \mathfrak{q}_G\right)(\Bbbk) = \bigcup_{\Bbbk=\hbar\mathfrak{u}} \left(\int_G (\hbar) \setminus \mathfrak{q}_G(\mathfrak{u})\right), \quad \hbar, \mathfrak{u} \in G,$$

for all $\Bbbk \in G$.

3 | Soft Intersection-Difference Product of Groups

In this section, we introduce a novel binary operation for soft sets, referred to as the soft intersection-difference product, defined in the context of soft sets whose parameter set is a group structure. A rigorous algebraic analysis of this product is carried out, with particular attention devoted to its characteristics under diverse notions of soft equalities and various classifications of soft subsets. Theoretical findings are further elucidated through representative examples that illustrate the structural characteristics of the proposed operation.

From now on, G denotes a group, $S_G(U)$ is the collections of SSs, whose parameter set is G, and all the SSs are the elements of $S_G(U)$.

Definition 12. Let $\int_G$ and $\mathfrak{q}_G$ be two SSs. Then, the soft intersection-difference product $\int_G \otimes_{i/d} \mathfrak{q}_G$ is defined by

$$\left(\int_G \otimes_{i/d} \mathfrak{q}_G\right)(\Bbbk) = \bigcap_{\Bbbk=\hbar\mathfrak{u}} \left(\int_G(\hbar) \setminus \mathfrak{q}_G(\mathfrak{u})\right), \quad \hbar, \mathfrak{u} \in G,$$

for all $\Bbbk \in G$.

Note here that since G is a group, there always exists $\hbar, \mathfrak{u} \in G$ such that $\Bbbk = \hbar\mathfrak{u}$, for all $\Bbbk \in G$. Let the order of the group be n, that is, $|G| = n$. Then, it is obvious that there exist n different combinations of writing styles for each $\Bbbk \in G$ such that $\Bbbk = \hbar\mathfrak{u}$, where $\hbar, \mathfrak{u} \in G$.

Note 1: The soft intersection-difference product is well-defined in $S_G(U)$. In fact, let $\int_G, \mathfrak{q}_G, \mathfrak{b}_G, \zeta_G \in S_G(U)$ such that $(\int_G, \mathfrak{q}_G) = (\mathfrak{b}_G, \zeta_G)$. Then, $\int_G = \mathfrak{b}_G$ and $\mathfrak{q}_G = \zeta_G$, implying that $\int_G(\Bbbk) = \mathfrak{b}_G(\Bbbk)$ and $\mathfrak{q}_G(\Bbbk) = \zeta_G(\Bbbk)$, for all $\Bbbk \in G$. Thereby, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} \mathfrak{q}_G\right)(\Bbbk) = \bigcap_{\Bbbk=\hbar\mathfrak{u}} \left(\int_G (\hbar) \setminus \mathfrak{q}_G(\mathfrak{u})\right) = \bigcap_{\Bbbk=\hbar\mathfrak{u}} \left(\mathfrak{b}_G (\hbar) \setminus \zeta_G(\mathfrak{u})\right) = \left(\mathfrak{b}_G \otimes_{i/d} \zeta_G\right)(\Bbbk).$$

Hence, $\int_G \otimes_{i/d} \mathfrak{q}_G = \mathfrak{b}_G \otimes_{i/d} \zeta_G$.

Example 1. Consider the group $G = \{\mathfrak{a}, \mathfrak{b}\}$ with the following operation:

·	ϱ	Ƅ
ϱ	ϱ	Ƅ
Ƅ	Ƅ	ϱ

Let $\int_G$ and q_G be two SSs over $U = D_2 = \{<x, y>: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$$\int_G = \{(ϱ, \{e, x, yx\}), (Ƅ, \{x, yx\})\}, q_G = \{(ϱ, \{e, y, yx\}), (Ƅ, \{e, y\})\}.$$

Since $ϱ = ϱϱ = ƄƄ$, $(\int_G \otimes_{i/d} q_G)(ϱ) = (\int_G(ϱ) \setminus q_G(ϱ)) \cap (\int_G(Ƅ) \setminus q_G(Ƅ)) = \{x\}$, and since $Ƅ = ϱƄ = Ƅϱ$, $(\int_G \otimes_{i/d} q_G)(Ƅ) = (\int_G(ϱ) \setminus q_G(Ƅ)) \cap (\int_G(Ƅ) \setminus q_G(ϱ)) = \{x\}$ is obtained. Hence,

$$\int_G \otimes_{i/d} q_G = \{(ϱ, \{x\}), (Ƅ, \{x\})\}.$$

Proposition 1. The set $S_G(U)$ is closed under the soft intersection-difference product. That is, if $\int_G$ and q_G are two SSs, then so is $\int_G \otimes_{i/d} q_G$.

Proof: It is obvious that the soft intersection-difference product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft intersection-difference product.

Proposition 2. The soft intersection-difference product is not associative in $S_G(U)$.

Proof: Consider the group G and the SSs $\int_G$ and q_G in *Example 1*. Let $ħ_G$ be an SS over $U = \{e, x, y, yx\}$ such that $ħ_G = \{(ϱ, \{e, y\}), (Ƅ, \{y, yx\})\}$.

Since $\int_G \otimes_{i/d} q_G = \{(ϱ, \{x\}), (Ƅ, \{x\})\}$, then

$$(\int_G \otimes_{i/d} q_G) \otimes_{i/d} ħ_G = \{(ϱ, \{x\}), (Ƅ, \{x\})\}.$$

Moreover, since $q_G \otimes_{i/d} ħ_G = \{(ϱ, \emptyset), (Ƅ, \emptyset)\}$, then

$$\int_G \otimes_{i/d} (q_G \otimes_{i/d} ħ_G) = \{(ϱ, \{x, yx\}), (Ƅ, \{x, yx\})\}.$$

Thereby, $(\int_G \otimes_{i/d} q_G) \otimes_{i/d} ħ_G \neq \int_G \otimes_{i/d} (q_G \otimes_{i/d} ħ_G)$.

Proposition 3. The soft intersection-difference product is not commutative in $S_G(U)$.

Proof: Consider the SSs $\int_G$ and q_G in *Example 1*. Then,

$$\int_G \otimes_{i/d} q_G = \{(ϱ, \{x\}), (Ƅ, \{x\})\},$$

and

$$q_G \otimes_{i/d} \int_G = \{(ϱ, \{y\}), (Ƅ, \{y\})\},$$

implying that $\int_G \otimes_{i/d} q_G \neq q_G \otimes_{i/d} \int_G$.

Proposition 4. The soft intersection-difference product is not idempotent in $S_G(U)$.

Proof: Consider the SS $\int_G = \{(ϱ, \{e, x, yx\}), (Ƅ, \{x, yx\})\}$ in *Example 1*. Then,

$$\int_G \otimes_{i/d} \int_G = \{(ϱ, \emptyset), (Ƅ, \emptyset)\},$$

implying that $\int_G \otimes_{i/d} \int_G \neq \int_G$.

Proposition 5. Let $\int_G$ be a constant SS. Then, $\int_G \otimes_{i/d} \int_G = \emptyset_G$.

Proof: Let $\int_G$ be a constant SS such that, for all $k \in G$, $\int_G(k) = \bar{A}$, where $\bar{A}$ is a fixed set. Then, for all $k \in G$,

$$\left(\int_G \otimes_{i/d} \int_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\int_G(\dagger) \setminus \int_G(\mathrm{u})\right) = \emptyset_G(\Bbbk).$$

Thereby, $\int_G \otimes_{i/d} \int_G = \emptyset_G$.

Remark 1. Let $S_G{}^*(U)$ be the collection of all constant SSs. Then, the soft intersection-difference product is not idempotent in $S_G{}^*(U)$ either.

Proposition 6. $\emptyset_G$ is the left absorbing element of the soft intersection-difference product in $S_G(U)$.

Proof: Let $x \in G$. Then,

$$\left(\emptyset_G \otimes_{i/d} \int_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\emptyset_G(\dagger) \setminus \int_G(\mathrm{u})\right) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\emptyset \setminus \int_G(\mathrm{u})\right) = \emptyset_G(\Bbbk).$$

Thus, $\emptyset_G \otimes_{i/d} \int_G = \emptyset_G$.

Proposition 7. $\emptyset_G$ is not the right absorbing element of the soft intersection-difference product in $S_G(U)$.

Proof: Consider the SS $\int_G = \{(\mathfrak{Q}, \{e, x, yx\}), (\mathfrak{b}, \{x, yx\})\}$ in *Example 1*. Then,

$$\int_G \otimes_{i/d} \emptyset_G = \{(\mathfrak{Q}, \{x, yx\}), (\mathfrak{b}, \{x, yx\})\},$$

implying that $\int_G \otimes_{i/d} \emptyset_G \neq \emptyset_G$.

Remark 2. $\emptyset_G$ is not the absorbing element of soft intersection-difference product in $S_G(U)$.

Proposition 8. Let $\int_G$ be a constant SS. Then, $\int_G \otimes_{i/d} \emptyset_G = \int_G$.

Proof: Let $\int_G$ be a constant SS such that, for all $\Bbbk \in G$, $\int_G(\Bbbk) = \bar{A}$, where $\bar{A}$ is a fixed set. Then, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} \emptyset_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\int_G(\dagger) \setminus \emptyset_G(\mathrm{u})\right) = \int_G(\Bbbk).$$

Thereby, $\int_G \otimes_{i/d} \emptyset_G = \int_G$.

Remark 3. $\emptyset_G$ is not the absorbing element of soft intersection-difference product in $S_G{}^*(U)$ either. Moreover, $\emptyset_G$ is the right identity element of soft intersection-difference product in $S_G{}^*(U)$.

Proposition 9. Let $\int_G$ be an SS. Then, $\int_G \otimes_{i/d} U_G = \emptyset_G$.

Proof: Let $\int_G$ be an SS. Then, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} U_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\int_G(\dagger) \setminus U_G(\mathrm{u})\right) = \emptyset_G(\Bbbk).$$

Thereby, $\int_G \otimes_{i/d} U_G = \emptyset_G$.

Proposition 10. Let $\int_G$ be a constant SS. Then, $U_G \otimes_{i/d} \int_G = \int_G{}^c$.

Proof: Let $\int_G$ be a constant SS such that, for all $\Bbbk \in G$, $\int_G(\Bbbk) = \bar{A}$, where $\bar{A}$ is a fixed set. Then, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} U_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(U_G(\dagger) \setminus \int_G(\mathrm{u})\right) = \int_G{}^c(\Bbbk).$$

Thereby, $U_G \otimes_{i/d} \int_G = \int_G{}^c$.

Proposition 11. Let $\int_G$ be a constant SS. Then, $\int_G{}^c \otimes_{i/d} \int_G = \int_G{}^c$.

Proof: Let $\int_G$ be a constant SS such that, for all $\Bbbk \in G$, $\int_G(\Bbbk) = \bar{A}$, where $\bar{A}$ is a fixed set. Then, for all $\Bbbk \in G$,

$$\left(\int_G{}^c \otimes_{i/d} \int_G\right)(\Bbbk) = \bigcap_{\Bbbk = \dagger \mathrm{u}} \left(\int_G{}^c(\dagger) \setminus \int_G(\mathrm{u})\right) = \int_G{}^c(\Bbbk).$$

Thereby, $\int_G{}^c \otimes_{i/d} \int_G = \int_G{}^c$.

Proposition 12. Let $\int_G$ be a constant SS. Then, $\int_G \otimes_{i/d} \int_G{}^c = \int_G$.

Proof: Let $\int_G$ be a constant SS such that, for all $k \in G$, $\int_G(k) = \bar{A}$, where $\bar{A}$ is a fixed set. Then, for all $k \in G$,

$$\left(\int_G \otimes_{i/d} \int_G{}^c\right)(k) = \bigcap_{k=\daleth\mathrm{u}}\left(\int_G(\daleth) \setminus \int_G{}^c(\mathrm{u})\right) = \int_G(k).$$

Thereby, $\int_G \otimes_{i/d} \int_G{}^c = \int_G$.

Proposition 13. Let $\int_G$, q_G and $\hbar_G$ be three SSs. If $\int_G \cong \mathrm{q}_G$, then $\int_G \otimes_{i/d} \hbar_G \cong \mathrm{q}_G \otimes_{i/d} \hbar_G$ and $\hbar_G \otimes_{i/d} \mathrm{q}_G \cong \hbar_G \otimes_{i/d} \int_G$.

Proof: Let $\int_G, \mathrm{q}_G$ and $\hbar_G$ be three SSs such that $\int_G \cong \mathrm{q}_G$. Then, for all $k \in G$, $\int_G(k) \subseteq \mathrm{q}_G(k)$ and $(\mathrm{q}_G(k))' \subseteq (\int_G(k))'$. Thus, for all $k \in G$,

$$\left(\int_G \otimes_{i/d} \hbar_G\right)(k) = \bigcap_{k=\daleth\mathrm{u}}\left(\int_G(\daleth) \setminus \hbar_G(\mathrm{u})\right) \subseteq \bigcap_{k=\daleth\mathrm{u}}\left(\mathrm{q}_G(\daleth) \setminus \hbar_G(\mathrm{u})\right) = \left(\mathrm{q}_G \otimes_{i/d} \hbar_G\right)(k),$$

for all $k \in G$, implying that $\int_G \otimes_{i/d} \hbar_G \cong \mathrm{q}_G \otimes_{i/d} \hbar_G$. Similarly, for all $k \in G$,

$$\left(\hbar_G \otimes_{i/d} \mathrm{q}_G\right)(k) = \bigcap_{k=\daleth\mathrm{u}}\left(\hbar_G(\daleth) \setminus \mathrm{q}_G(\mathrm{u})\right) \subseteq \bigcap_{k=\daleth\mathrm{u}}\left(\hbar_G(\daleth) \setminus \int_G(\mathrm{u})\right) = \left(\hbar_G \otimes_{i/d} \int_G\right)(k),$$

for all $k \in G$, implying that $\hbar_G \otimes_{i/d} \mathrm{q}_G \cong \hbar_G \otimes_{i/d} \int_G$. $\square$

Proposition 14. Let $\int_G, \mathrm{q}_G, \mathrm{b}_G,$ and ζ_G be four SSs. If $\mathrm{b}_G \cong \int_G$ and $\zeta_G \cong \mathrm{q}_G$, then $\zeta_G \otimes_{i/d} \int_G \cong \mathrm{q}_G \otimes_{i/d} \mathrm{b}_G$ and $\mathrm{b}_G \otimes_{i/d} \mathrm{q}_G \cong \int_G \otimes_{i/d} \zeta_G$.

Proof: Let $\int_G, \mathrm{q}_G, \mathrm{b}_G$ and ζ_G be four SSs such that $\mathrm{b}_G \cong \int_G$ and $\zeta_G \cong \mathrm{q}_G$. Then, for all $k \in G$, $\mathrm{b}_G(k) \subseteq \int_G(k)$ and $\zeta_G(k) \subseteq \mathrm{q}_G(k)$. Thus, $\left(\int_G(k)\right)' \subseteq \left(\mathrm{b}_G(k)\right)'$ and $\left(\mathrm{q}_G(k)\right)' \subseteq \left(\zeta_G(k)\right)'$, for all $k \in G$. Hence, for all $k \in G$,

$$\left(\zeta_G \otimes_{i/d} \int_G\right)(k) = \bigcap_{k=\daleth\mathrm{u}}\left(\zeta_G(\daleth) \setminus \int_G(\mathrm{u})\right) \subseteq \bigcap_{k=\daleth\mathrm{u}}\left(\mathrm{q}_G(\daleth) \setminus \mathrm{b}_G(\mathrm{u})\right) = \left(\mathrm{q}_G \otimes_{i/d} \mathrm{b}_G\right)(k),$$

implying that $\zeta_G \otimes_{i/d} \mathrm{b}_G \cong \mathrm{q}_G \otimes_{i/d} \int_G$. Similarly, for all $k \in G$,

$$\left(\mathrm{b}_G \otimes_{i/d} \mathrm{q}_G\right)(k) = \bigcap_{k=\daleth\mathrm{u}}\left(\mathrm{b}_G(\daleth) \setminus \mathrm{q}_G(\mathrm{u})\right) \subseteq \bigcap_{k=\daleth\mathrm{u}}\left(\int_G(\daleth) \setminus \zeta_G(\mathrm{u})\right) = \left(\int_G \otimes_{i/d} \zeta_G\right)(k),$$

is obtained. Thereby, $\mathrm{b}_G \otimes_{i/d} \mathrm{q}_G \cong \int_G \otimes_{i/d} \zeta_G$. $\square$

Theorem 1. Let $\int_G$ and q_G be two SSs. Then, $\int_G \otimes_{i/d} \mathrm{q}_G = U_G$ if and only if $\int_G = U_G$ and $\mathrm{q}_G = \emptyset_G$.

Proof: Let $\int_G$ and q_G be two SSs. Suppose that $\int_G = U_G$ and $\mathrm{q}_G = \emptyset_G$. Hence, for all $k \in G$, $\int_G(k) = U_G(k) = U$ and $\mathrm{q}_G(k) = \emptyset_G(k) = \emptyset$. Thus, for all $k \in G$,

$$\left(\int_G \otimes_{i/d} \mathrm{q}_G\right)(k) = \cap_{k=\daleth\mathrm{u}}\left(\int_G(\daleth) \setminus \mathrm{q}_G(\mathrm{u})\right) = \cap_{k=\daleth\mathrm{u}}\left(U_G(\daleth) \setminus \emptyset_G(\mathrm{u})\right) = U_G(k).$$

Thereby, $\int_G \otimes_{i/d} \mathrm{q}_G = U_G$.

Conversely, suppose that $\int_G \otimes_{i/d} \mathrm{q}_G = U_G$. Then, $\left(\int_G \otimes_{i/d} \mathrm{q}_G\right)(k) = U_G(k) = U$, for all $k \in G$. Thus, for all $k \in G$,

$$U_G(\Bbbk) = U = \int_G \otimes_{i/d} q_G(\Bbbk) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right).$$

This implies that $\int_G(\dagger) \setminus q_G(w) = U$, for all $\dagger, w \in G$. Thus, $\int_G(\dagger) = U$ and $q_G(w) = \emptyset$, for all $\dagger, w \in G$. Thereby, $\int_G = U_G$ and $q_G = \emptyset_G$.

Proposition 15. Let $\int_G$ and q_G be two SSs. If $\int_G \widetilde{\subseteq}_A q_G$, then $\int_G \otimes_{i/d} q_G = \emptyset_G$.

Proof: Let $\int_G$ and q_G be two SSs and $\int_G \widetilde{\subseteq}_A q_G$. Thus, $\int_G(\mathfrak{Q}) \subseteq q_G(\mathfrak{b})$ for each $\mathfrak{Q}, \mathfrak{b} \in G$. Hence, for all $\Bbbk \in G$,

$$\int_G \otimes_{i/d} q_G(\Bbbk) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right) = \emptyset = \emptyset_G(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} q_G = \emptyset_G$ is obtained.

Proposition 16. Let $\int_G$ and q_G be two SSs. If $\int_G \widetilde{\subseteq}_S (q_G)^c$, then $\int_G \otimes_{i/d} q_G = \int_G$.

Proof: Let $\int_G$ and q_G be two SSs and $\int_G \widetilde{\subseteq}_S (q_G)^c$. Hence, for all $\mathfrak{Q} \in G, \int_G(\mathfrak{Q}) = \bar{A}$ and $q_G(\mathfrak{Q}) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} \subseteq B'$. Thus, for all $\Bbbk \in G$,

$$\int_G \otimes_{i/d} q_G(\Bbbk) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \cap q_G{}^c(w) \right) = \int_G(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} q_G = \int_G$.

Proposition 17. Let $\int_G$ and q_G be two SSs. If $(q_G{}^c) \widetilde{\subseteq}_S \int_G$, then $\int_G \otimes_{i/d} q_G = q_G{}^c$.

Proof: Let $\int_G$ and q_G be two SSs and $q_G{}^c \widetilde{\subseteq}_S \int_G$. Hence, for all $\mathfrak{Q} \in G, \int_G(\mathfrak{Q}) = \bar{A}$ and $q_G(\mathfrak{Q}) = B$, where $\bar{A}$ and B are two fixed sets and $B' \subseteq \bar{A}$. Thus, for all $\Bbbk \in G$,

$$\int_G \otimes_{i/d} q_G(\Bbbk) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \cap q_G{}^c(w) \right) = q_G{}^c(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} q_G = q_G{}^c$.

Proposition 18. Let $\int_G$ and q_G be two SSs. Then, $\left(\int_G \otimes_{i/d} q_G \right)^c = \int_G \otimes_{u/p} q_G$.

Proof: Let $\int_G$ and q_G be two SSs. Then, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} q_G \right)^c (\Bbbk) = \left(\bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right) \right)' = \bigcup_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right)'$$

$$= \bigcup_{\Bbbk = \dagger w} \left(\int_G(\dagger) \cap q_G{}^c(w) \right)' = \bigcup_{\Bbbk = \dagger w} \left(\int_G{}^c(\dagger) \cup q_G(w) \right) = \left(\int_G \otimes_{u/p} q_G \right)(\Bbbk).$$

Thereby, $\left(\int_G \otimes_{i/d} q_G \right)^c = \int_G \otimes_{u/p} q_G$.

For more on soft union-plus product of groups, we refer to [85].

Proposition 19. Let $\int_G$ and q_G be two SSs. Then, $\int_G \otimes_{i/d} q_G \widetilde{\subseteq} \int_G \otimes_{i/u} q_G$.

Proof: Let $\int_G$ and q_G be two SSs. Then, for all $\Bbbk \in G$,

$$\left(\int_G \otimes_{i/d} q_G \right)(\Bbbk) = \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \setminus q_G(w) \right) \subseteq \bigcap_{\Bbbk = \dagger w} \left(\int_G(\dagger) \cup q_G(w) \right) = \left(\int_G \otimes_{i/u} q_G \right)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} q_G \widetilde{\subseteq} \int_G \otimes_{i/u} q_G$. $\square$

Proposition 20. Let $\int_G$ and $\mathfrak{q}_G$ be two SSs. If one of the assertions following is satisfied, then $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{i/u} \mathfrak{q}_G$:

I. $\mathfrak{q}_G = \emptyset_G$

II. $\int_G = \emptyset_G$ and $\mathfrak{q}_G(\Bbbk) \cap \mathfrak{q}_G(\dagger) = \emptyset$ for all $\Bbbk, \dagger \in G$

Proof: Let $\int_G$ and $\mathfrak{q}_G$ be two SSs

    I. Let $\mathfrak{q}_G = \emptyset_G$. Then, for all $\Bbbk \in G$, $\mathfrak{q}_G(\Bbbk) = \emptyset_G(\Bbbk) = \emptyset$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} \mathfrak{q}_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \setminus \mathfrak{q}_G(\mathfrak{w})\big) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \setminus \emptyset_G(\mathfrak{w})\big)$$

$$= \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \cup \emptyset_G(\mathfrak{w})\big) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \cup \mathfrak{q}_G(\mathfrak{w})\big) = (\int_G \otimes_{i/u} \mathfrak{q}_G)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{i/u} \mathfrak{q}_G$.

    II. Let $\int_G = \emptyset_G$ and $\mathfrak{q}_G(\Bbbk) \cap \mathfrak{q}_G(\dagger) = \emptyset$, for all $\Bbbk, \dagger \in G$. Then, for all $\Bbbk \in G$, $\int_G(\Bbbk) = \emptyset_G(\Bbbk) = \emptyset$. Hence, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} \mathfrak{q}_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \setminus \mathfrak{q}_G(\mathfrak{w})\big) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\emptyset_G(\dagger) \setminus \mathfrak{q}_G(\mathfrak{w})\big)$$

$$= \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \cup \mathfrak{q}_G(\mathfrak{w})\big) = \emptyset = (\int_G \otimes_{i/u} \mathfrak{q}_G)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{i/u} \mathfrak{q}_G$.

Proposition 21. Let $\int_G$ and $\mathfrak{q}_G$ be two SSs. Then, $\int_G \otimes_{i/d} \mathfrak{q}_G \cong \int_G \otimes_{i/s} \mathfrak{q}_G$.

Proof: Let $\int_G$ and $\mathfrak{q}_G$ be two SSs. Then, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} \mathfrak{q}_G)(\Bbbk) = \cap_{\Bbbk=\dagger\mathfrak{w}}\big(\int_G(\dagger) \setminus \mathfrak{q}_G(\mathfrak{w})\big) \subseteq \cap_{\Bbbk=\dagger\mathfrak{w}}\big(\int_G(\dagger) \Delta \mathfrak{q}_G(\mathfrak{w})\big) = (\int_G \otimes_{i/s} \mathfrak{q}_G)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} \mathfrak{q}_G \cong \int_G \otimes_{i/s} \mathfrak{q}_G$. $\square$

Proposition 22. Let $\int_G$ and $\mathfrak{q}_G$ be two SSs. If one of the assertions following is satisfied, then $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{i/s} \mathfrak{q}_G$:

    I. $\mathfrak{q}_G \cong_s \int_G$

    II. $\int_G =_s \mathfrak{q}_G$

    III. $\int_G = \emptyset_G$ and $\mathfrak{q}_G(\Bbbk) \cap \mathfrak{q}_G(\dagger) = \emptyset$ for all $\Bbbk, \dagger \in G$

    IV. $\mathfrak{q}_G = \emptyset_G$

Proof: Let $\int_G$ and $\mathfrak{q}_G$ be two SSs.

    I. Suppose that $\mathfrak{q}_G \cong_s \int_G$. Hence, for all $\mathfrak{Q} \in G$, $\mathfrak{q}_G(\mathfrak{Q}) = \bar{A}$ and $\int_G(\mathfrak{Q}) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} \subseteq B$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} \mathfrak{q}_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \setminus \mathfrak{q}_G(\mathfrak{w})\big) = \bigcap_{\Bbbk=\dagger\mathfrak{w}} \big(\int_G(\dagger) \Delta \mathfrak{q}_G(\mathfrak{w})\big) = (\int_G \otimes_{i/s} \mathfrak{q}_G)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{i/s} \mathfrak{q}_G$.

    II.   It follows from *Proposition 22 (I)*.

III. Let $\int_G = \emptyset_G$ and $q_G(\Bbbk) \cap q_G(\dagger) = \emptyset$ for all $\Bbbk, \dagger \in G$. Then, for all $\Bbbk \in G, \int_G(\Bbbk) = \emptyset_G(\Bbbk) = \emptyset$. Hence, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\emptyset_G(\dagger) \setminus q_G(\mathrm{w})\big) = \emptyset_G(\Bbbk).$$

Similarly, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger)\Delta q_G(\mathrm{w})\big) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\emptyset_G(\dagger)\Delta q_G(\mathrm{w})\big) = \emptyset_G(\Bbbk).$$

Hence, $\int_G \otimes_{i/d} q_G = \int_G \otimes_{i/s} q_G$.

IV. Let $q_G = \emptyset_G$. Then, for all $\Bbbk \in G$, $q_G(\Bbbk) = \emptyset_G(\Bbbk) = \emptyset$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus \emptyset_G(\mathrm{w})\big) =$$
$$= \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger)\Delta q_G(\mathrm{w})\big) = (\int_G \otimes_{i/s} q_G)(\Bbbk).$$

Thereby, $\int_G \otimes_{i/d} q_G = \int_G \otimes_{i/s} q_G$.

Proposition 23. Let $\int_G$ and q_G be two SSs. Then, $\int_G \otimes_{i/d} q_G \cong \int_G \otimes_{u/d} q_G$.

Proof: Let $\int_G$ and q_G be two SSs. Then, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q(\mathrm{w})\big) \subseteq \bigcup_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q(\mathrm{w})\big) = (\int_G \otimes_{u/d} q_G)(\Bbbk).$$

Thus, $\int_G \otimes_{i/d} q_G \cong \int_G \otimes_{u/d} q_G$. $\square$

Proposition 24. Let $\int_G$ and q_G be two SSs. If one of the assertions following is satisfied, then $\int_G \otimes_{i/d} q_G = \int_G \otimes_{u/d} q_G$.

   I. $q_G \tilde{\subseteq}_S \int_G$

   II. $\int_G \tilde{\subseteq}_A q_G$

   III. $\int_G =_S q_G$

   IV. $\int_G =_S (q_G)^c$

Proof: Let $\int_G$ and q_G be two SSs.

   I. Suppose that $q_G \tilde{\subseteq}_S \int_G$. Hence, for all $\mathfrak{Q} \in G, q_G(\mathfrak{Q}) = \bar{A}$, $\int_G(\mathfrak{Q}) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} \subseteq B$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big) = \bigcup_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big) = (\int_G \otimes_{u/d} q_G)(\Bbbk).$$

Hence, $\int_G \otimes_{i/d} q_G = \int_G \otimes_{u/d} q_G$.

   II. Suppose that $\int_G \tilde{\subseteq}_A q_G$. Then, $\int_G(\mathfrak{Q}) \subseteq q_G(\mathfrak{b})$, for each $\mathfrak{Q}, \mathfrak{b} \in G$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} q_G)(\Bbbk) = \bigcap_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big) = \emptyset = \bigcup_{\Bbbk=\dagger\mathrm{w}}\big(\int_G(\dagger) \setminus q_G(\mathrm{w})\big)$$
$$= (\int_G \otimes_{u/d} q_G)(\Bbbk).$$

Hence, $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{u/d} \mathfrak{q}_G$.

III. It follows from *Proposition 24 (I)*.

IV. Let $\int_G =_S (\mathfrak{q}_G)^c$. Then, for all $\Bbbk \in G$, $\int_G(\Bbbk) = \bar{A}$, $\mathfrak{q}_G(\Bbbk) = B$, where $\bar{A}$ and B are two fixed sets and $\bar{A} = B'$. Thus, for all $\Bbbk \in G$,

$$(\int_G \otimes_{i/d} \mathfrak{q}_G)(\Bbbk) = \bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} \left(\int_G(\mathfrak{t}) \setminus \mathfrak{q}_G(\mathfrak{u}) \right) = \bigcup_{\Bbbk = \mathfrak{t}\mathfrak{u}} \left(\int_G(\mathfrak{t}) \setminus \mathfrak{q}_G(\mathfrak{u}) \right) = (\int_G \otimes_{u/d} \mathfrak{q}_G)(\Bbbk).$$

Hence, $\int_G \otimes_{i/d} \mathfrak{q}_G = \int_G \otimes_{u/d} \mathfrak{q}_G$.

Proposition 25. The soft intersection-difference product distributes over soft intersection operation from the right side.

Proof: Let $\int_G, \mathfrak{q}_G$ and $\hbar_G$ be three SSs. Then, for all $\Bbbk \in G$,

$$((\int_G \tilde{\cap} \mathfrak{q}_G) \otimes_{i/d} \hbar_G)(\Bbbk) = \bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} \left((\int_G \tilde{\cap} \mathfrak{q}_G)(\mathfrak{t}) \setminus \hbar_G(\mathfrak{u}) \right)$$

$$= \bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} \left((\int_G(\mathfrak{t}) \cap \mathfrak{q}_G(\mathfrak{t})) \setminus \hbar_G(\mathfrak{u}) \right)$$

$$= \bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} \left[(\int_G(\mathfrak{t}) \setminus \hbar_G(\mathfrak{u})) \cap (\mathfrak{q}_G(\mathfrak{t}) \setminus \hbar_G(\mathfrak{u})) \right]$$

$$= \left[\bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} (\int_G(\mathfrak{t}) \setminus \hbar_G(\mathfrak{u})) \right] \cap \left[\bigcap_{\Bbbk = \mathfrak{t}\mathfrak{u}} (\mathfrak{q}_G(\mathfrak{t}) \setminus \hbar_G(\mathfrak{u})) \right]$$

$$= \left(\int_G \bigotimes_{\frac{i}{d}} \hbar_G \right)(\Bbbk) \cap \left(\mathfrak{q}_G \bigotimes_{\frac{i}{d}} \hbar_G \right)(\Bbbk)$$

$$= \left(\left(\int_G \bigotimes_{\frac{i}{d}} \hbar_G \right) \tilde{\cap} \left(\mathfrak{q}_G \bigotimes_{\frac{i}{d}} \hbar_G \right) \right)(\Bbbk).$$

Thus, $(\int_G \tilde{\cap} \mathfrak{q}_G) \otimes_{i/d} \hbar_G = (\int_G \otimes_{i/d} \hbar_G) \tilde{\cap} (\mathfrak{q}_G \otimes_{i/d} \hbar_G)$.

Example 2. Consider the SSs $\int_G$ and $\mathfrak{q}_G$ in *Example 1*. Let $\hbar_G$ be an SS as follows: $\hbar_G = \{(\mathfrak{2}, \{e, yx\}), (\mathfrak{b}, \{x, y\})\}$. Since $\int_G \otimes_{i/d} \hbar_G = \{(\mathfrak{2}, \emptyset), (\mathfrak{b}, \emptyset)\}$ and $\mathfrak{q}_G \otimes_{i/d} \hbar_G = \{(\mathfrak{2}, \emptyset), (\mathfrak{b}, \emptyset)\}$, then

$$(\int_G \otimes_{i/d} \hbar_G) \tilde{\cap} (\mathfrak{q}_G \otimes_{i/d} \hbar_G) = \{(\mathfrak{2}, \emptyset), (\mathfrak{b}, \emptyset)\}.$$

Moreover, since $\int_G \tilde{\cap} \mathfrak{q}_G = \{(\mathfrak{2}, \{e, yx\}), (\mathfrak{b}, \emptyset)\}$,

$$(\int_G \tilde{\cap} \mathfrak{q}_G) \otimes_{i/d} \hbar_G = \{(\mathfrak{2}, \emptyset), (\mathfrak{b}, \emptyset)\}.$$

Thus, $(\int_G \tilde{\cap} \mathfrak{q}_G) \otimes_{i/d} \hbar_G = (\int_G \otimes_{i/d} \hbar_G) \tilde{\cap} (\mathfrak{q}_G \otimes_{i/d} \hbar_G)$. $\square$

4 | Conclusion

This study introduces a novel binary operation for soft sets whose parameter sets possess a group structure, termed the soft intersection-difference product. A comprehensive examination of its foundational algebraic properties is undertaken, with particular emphasis on its interaction with various classes of soft subsets and equality relations. The theoretical framework proposed herein not only addresses existing gaps but also offers a potential pathway for the development of a new branch of soft group theory grounded in this product. Prospective research directions may include the formulation of additional soft product operations and a

deeper exploration of soft equality structures, both of which are expected to contribute significantly to the advancement of the theoretical and applied dimensions of soft set theory.

Author Contributions

All authors contributed to the study's conception and design. AS supervised the study, performed material preparation. ZA performed data collection, and analysis, and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

Consent for Publication

The author has approved the manuscript for publication.

Ethics Approval and Consent to Participate

This study does not involve any research conducted on human participants or animals.

Funding

No funding was received to conduct this study.

Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author (Aslıhan Sezgin, aslihan.sezgin@amasya.edu.tr) on reasonable request.

Conflict of Interest

The authors stated that there are no conflicts of interest regarding the publication of this article.

References

[1]   Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[2]   Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[3]   Maji, P. K., Roy, A. R., & Biswas, R. (2002). An application of soft sets in a decision making problem. *Computers and mathematics with applications*, *44*(8–9), 1077–1083. https://doi.org/10.1016/S0898-1221(02)00216-X

[4]   Chen, D. G., Tsang, E. C. C., & Yeung, D. S. (2003). Some notes on the parameterization reduction of soft sets. *Proceedings of the 2003 international conference on machine learning and cybernetics (IEEE cat. no. 03ex693)* (Vol. 3, pp. 1442–1445). IEEE. https://doi.org/10.1109/ICMLC.2003.1259720

[5]   Chen, D., Tsang, E. C. C., Yeung, D. S., & Wang, X. (2005). The parameterization reduction of soft sets and its applications. *Computers and mathematics with applications*, *49*(5–6), 757–763. https://doi.org/10.1016/j.camwa.2004.10.036

[6]   Xiao, Z., Chen, L., Zhong, B., & Ye, S. (2005). Recognition for soft information based on the theory of soft sets. *2005 international conference on services systems and services management, proceedings of icsssm'05* (Vol. 2, pp. 1104–1106). IEEE. https://doi.org/10.1109/ICSSSM.2005.1500166

[7]   Mushrif, M. M., Sengupta, S., & Ray, A. K. (2006). Texture classification using a novel, soft-set theory based classification algorithm. *Lecture notes in computer science (including subseries lecture notes in artificial intelligence and lecture notes in bioinformatics)* (Vol. 3851 LNCS, pp. 246–254). Springer. https://doi.org/10.1007/11612032_26

[8] Herawan, T., & Deris, M. M. (2009). A direct proof of every rough set is a soft set. *Proceedings - 2009 3rd asia international conference on modelling and simulation, AMS 2009* (pp. 119–124). IEEE. https://doi.org/10.1109/AMS.2009.148

[9] Herawan, T., & Deris, M. M. (2010). Soft decision making for patients suspected influenza. *Lecture notes in computer science (including subseries lecture notes in artificial intelligence and lecture notes in bioinformatics)* (Vol. 6018 LNCS, pp. 405–418). Springer. https://doi.org/10.1007/978-3-642-12179-1_34

[10] Herawan, T. (2012). Soft set-based decision making for patients suspected influenza-like illness. *International journal of modern physics: Conference series* (Vol. 9, pp. 259-270). World Scientific Publishing Company. https://doi.org/10.1142/s2010194512005302

[11] Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni–int decision making. *European journal of operational research*, 207(2), 848-855. https://doi.org/10.1016/j.ejor.2010.05.004

[12] Çağman, N., & Enginoğlu, S. (2010). Soft matrix theory and its decision making. *Computers and mathematics with applications*, 59(10), 3308–3314. https://doi.org/10.1016/j.camwa.2010.03.015

[13] Gong, K., Xiao, Z., & Zhang, X. (2010). The bijective soft set with its operations. *Computers and mathematics with applications*, 60(8), 2270–2278. https://doi.org/10.1016/j.camwa.2010.08.017

[14] Xiao, Z., Gong, K., Xia, S., & Zou, Y. (2010). Exclusive disjunctive soft sets. *Computers and mathematics with applications*, 59(6), 2128–2137. https://doi.org/10.1016/j.camwa.2009.12.018

[15] Feng, F., Li, Y., & Çağman, N. (2012). Generalized uni-int decision making schemes based on choice value soft sets. *European journal of operational research*, 220(1), 162–170. https://doi.org/10.1016/j.ejor.2012.01.015

[16] Feng, Q., & Zhou, Y. (2014). Soft discernibility matrix and its applications in decision making. *Applied soft computing journal*, 24, 749–756. https://doi.org/10.1016/j.asoc.2014.08.042

[17] Kharal, A. (2014). Soft approximations and uni-int decision making. *The scientific world journal*, 2014(1), 327408. https://doi.org/10.1155/2014/327408

[18] Dauda, M. K., Mamat, M., & Waziri, M. Y. (2015). An application of soft set in decision making. *Jurnal teknologi (sciences & engineering)*, 77(13), 119–122. https://journals.utm.my/jurnalteknologi/issue/view/226

[19] Inthumathi, V., Chitra, V., & Jayasree, S. (2017). The role of operators on soft sets in decision making problems. *International journal of computational and applied mathematics*, 12(3), 899–910. http://www.ripublication.com

[20] Atagün, A. O., Kamacı, H., & Oktay, O. (2018). Reduced soft matrices and generalized products with applications in decision making. *Neural computing and applications*, 29(9), 445–456. https://doi.org/10.1007/s00521-016-2542-y

[21] Kamaci, H., Saltik, K., Fulya Akiz, H., & Osman Atagün, A. (2018). Cardinality inverse soft matrix theory and its applications in multicriteria group decision making. *Journal of intelligent and fuzzy systems*, 34(3), 2031–2049. https://doi.org/10.3233/JIFS-17876

[22] Yang, J., & Yao, Y. (2020). Semantics of soft sets and three-way decision with soft sets. *Knowledge-based systems*, 194, 105538. https://doi.org/10.1016/j.knosys.2020.105538

[23] Petchimuthu, S., Garg, H., Kamacı, H., & Atagün, A. O. (2020). The mean operators and generalized products of fuzzy soft matrices and their applications in MCGDM. *Computational and applied mathematics*, 39(2), 1–32. https://doi.org/10.1007/s40314-020-1083-2

[24] Zorlutuna, İ. (2021). Soft set-valued mappings and their application in decision making problems. *Filomat*, 35(5), 1725–1733. https://doi.org/10.2298/FIL2105725Z

[25] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and mathematics with applications*, 45(4–5), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6

[26] Pei, D., & Miao, D. (2005). From soft sets to information systems. *2005 IEEE international conference on granular computing* (Vol. 2005, pp. 617–621). IEEE. https://doi.org/10.1109/GRC.2005.1547365

[27] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & mathematics with applications*, 57(9), 1547–1553. https://doi.org/10.1016/j.camwa.2008.11.009

[28] Yang, C. F. (2008). A note on "soft set theory" [comput. math. appl. 45 (4--5)(2003) 555--562]. *Computers & mathematics with applications*, 56(7), 1899–1900. https://doi.org/10.1016/j.camwa.2008.03.019

[29] Feng, F., Li, C., Davvaz, B., & Ali, M. I. (2010). Soft sets combined with fuzzy sets and rough sets: A tentative approach. *Soft computing*, 14(9), 899–911. https://doi.org/10.1007/s00500-009-0465-6

[30] Jiang, Y., Tang, Y., Chen, Q., Wang, J., & Tang, S. (2010). Extending soft sets with description logics. *Computers & mathematics with applications*, *59*(6), 2087–2096. https://doi.org/10.1016/j.camwa.2009.12.014

[31] Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers and mathematics with applications*, *61*(9), 2647–2654. https://doi.org/10.1016/j.camwa.2011.03.011

[32] Neog, T. J., & Sut, D. K. (2011). A new approach to the theory of soft sets. *International journal of computer applications*, *32*(2), 1–6. https://d1wqtxts1xzle7.cloudfront.net/53823653/2011-new_approach_soft_set-libre.pdf?1499755677=&response-contentdisposition=inline%3B+filename%3DA_New_Approach_to_the_Theory_of_Soft_Set.pdf&Expires=1740299428&Signature=abUw8rHu4pPoWo9GGYn8lFITB64pDE61N9d

[33] Li, F. (2011). Notes on the soft operations. *ARPN journal of systems and software*, *1*(6), 205–208. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=5e028fe2a1df00303f8012ac465fa114611788d5

[34] Ge, X., & Yang, S. (2011). Investigations on some operations of soft sets. *World academy of science, engineering and technology*, *51*, 1112–1115. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=0f185b915f3223c39847c632a2ea34d1e64a0083

[35] Singh, D., & Onyeozili, I. A. (2012). Some conceptual misunderstandings of the fundamentals of soft set theory. *ARPN journal of systems and software*, *2*(9), 251–254. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=92f0b823a431a365680bc6c0f1b12dd6bb4f8d30

[36] Singh, D., & Onyeozili, I. A. (2012). Some results on distributive and absorption properties on soft operations. *IOSR journal of mathematics*, *4*(2), 18–30. https://www.iosrjournals.org/iosr-jm/papers/Vol4-issue2/C0421830.pdf

[37] Singh, D., & A. Onyeozili, I. (2012). On some new properties of soft set operations. *International journal of computer applications*, *59*(4), 39–44. https://doi.org/10.5120/9538-3975

[38] Singh, D., & Onyeozili, I. A. (2012). Notes on soft matrices operations. *ARPN journal of science and technology*, *2*(9), 861–869. http://www.ejournalofscience.org

[39] Zhu, P., & Wen, Q. (2013). Operations on soft sets revisited. *Journal of applied mathematics*, *2013*(1), 105752. https://doi.org/10.1155/2013/105752

[40] Sen, J. (2014). On algebraic structure of soft sets. *Annals of fuzzy mathematics and informatics*, *7*(6), 1013–1020. https://B2n.ir/ws4845

[41] Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. *Computers & mathematics with applications*, *61*(5), 1457–1467. https://doi.org/10.1016/j.camwa.2011.01.018

[42] Stojanović, N. (2021). A new operation on soft sets: extended symmetric difference of soft sets. *Vojnotehnicki glasnik*, *69*(4), 779–791. https://doi.org/10.5937/vojtehg69-33655

[43] Sezgin, A., Çağman, N., Atagün, A. O., & Aybek, F. N. (2023). Complemental binary operations of sets and their application to group theory. *Matrix science mathematic*, *7*(2), 114–121. https://doi.org/10.26480/msmk.02.2023.114.121

[44] Sezgin, A., & Dagtoros, K. (2023). Complementary soft binary piecewise symmetric difference operation: A novel soft set operation. *Scientific journal of mehmet akif ersoy university*, *6*(2), 31–45. https://dergipark.org.tr/en/pub/sjmakeu/issue/82332/1365021

[45] Sezgin, A., & Çalışıcı, H. (2024). A comprehensive study on soft binary piecewise difference operation. *Eskişehir teknik üniversitesi bilim ve teknoloji dergisi b-teorik bilimler*, *12*(1), 32-54. https://doi.org/10.20290/estubtdb.1356881

[46] Sezgin, A., & Yavuz, E. (2024). Soft binary piecewise plus operation: A new type of operation for soft sets. *Uncertainty discourse and applications*, *1*(1), 79–100. https://doi.org/10.48313/uda.v1i1.26

[47] Sezgin, A., & Şenyiğit, E. (2025). A new product for soft sets with its decision-making: Soft star-product. *Big data and computing visions*, *5*(1), 52–73. https://doi.org/10.22105/bdcv.2024.492834.1221

[48] Sezgin, A., & Demirci, A. M. (2023). A new soft set operation: Complementary soft binary piecewise star (*) operation. *Ikonion journal of mathematics*, *5*(2), 24–52. https://doi.org/10.54286/ikjm.1304566

[49] Qin, K., & Hong, Z. (2010). On soft equality. *Journal of computational and applied mathematics*, *234*(5), 1347–1355. https://doi.org/10.1016/j.cam.2010.02.028

[50] Jun, Y. B., & Yang, X. (2011). A note on the paper "combination of interval-valued fuzzy set and soft set" [Comput. Math. Appl. 58 (2009) 521527]. *Computers and mathematics with applications*, *61*(5), 1468–1470. https://doi.org/10.1016/j.camwa.2010.12.077

[51] Liu, X., Feng, F., & Jun, Y. B. (2012). A note on generalized soft equal relations. *Computers and mathematics with applications*, *64*(4), 572–578. https://doi.org/10.1016/j.camwa.2011.12.052

[52] Feng, F., & Li, Y. (2013). Soft subsets and soft product operations. *Information sciences*, *232*, 44–57. https://doi.org/10.1016/j.ins.2013.01.001

[53] Abbas, M., Ali, B., & Romaguera, S. (2014). On generalized soft equality and soft lattice structure. *Filomat*, *28*(6), 1191–1203. https://doi.org/10.2298/FIL1406191A

[54] Abbas, M., Ali, M. I., & Romaguera, S. (2017). Generalized operations in soft set theory via relaxed conditions on parameters. *Filomat*, *31*(19), 5955–5964. https://doi.org/10.2298/FIL1719955A

[55] Al-Shami, T. M. (2019). Investigation and corrigendum to some results related to g-soft equality and g f-soft equality relations. *Filomat*, *33*(11), 3375–3383. https://doi.org/10.2298/FIL1911375A

[56] Alshami, T., & El-Shafei, M. (2020). T-soft equality relation. *Turkish journal of mathematics*, *44*(4), 1427–1441. https://doi.org/10.3906/mat-2005-117

[57] Ali, B., Saleem, N., Sundus, N., Khaleeq, S., Saeed, M., & George, R. (2022). A contribution to the theory of soft sets via generalized relaxed operations. *Mathematics*, *10*(15), 2636. https://doi.org/10.3390/math10152636

[58] Sezgin, A., Atagün, A. O., & Çağman, N. (2025). A complete study on and-product of soft sets. *Sigma journal of engineering and natural sciences*, *43*(1), 1–14. https://doi.org/10.14744/sigma.2025.00002

[59] Sezer, A. S. (2012). A new view to ring theory via soft union rings, ideals and bi-ideals. *Knowledge-based systems*, *36*, 300–314. https://doi.org/10.1016/j.knosys.2012.04.031

[60] Sezgin, A. (2016). A new approach to semigroup theory I: Soft union semigroups, ideals and bi-ideals. *Algebra letters*, *2016*(3), 1–46. https://scik.org/index.php/abl/article/view/2989

[61] Muştuoğlu, E., Sezgin, A., & Türk, Z. K. (2016). Some characterizations on soft uni-groups and normal soft uni-groups. *International journal of computer applications*, *155*(10), 1–8. https://doi.org/10.5120/ijca2016912412

[62] Kaygisiz, K. (2012). On soft int-groups. *Annals of fuzzy mathematics and informatics*, *4*(2), 365–375. http://www.afmi.or.kr/fmihttp//www.kyungmoon.com

[63] Sezer, A. S., Çagman, N., Atagün, A. O., Ali, M. I., & Türkmen, E. (2015). Soft intersection semigroups, ideals and bi-ideals; A new application on semigroup theory I. *Filomat*, *29*(5), 917–946. https://doi.org/10.2298/FIL1505917S

[64] Sezgin, A., Çağman, N., & Atagün, A. O. (2017). A completely new view to soft intersection rings via soft uni-int product. *Applied soft computing journal*, *54*, 366–392. https://doi.org/10.1016/j.asoc.2016.10.004

[65] Sezgin, A., Durak, İ., & Ay, Z. (2025). Some new classifications of soft subsets and soft equalities with soft symmetric difference-difference product of groups. *Amesia*, *6*(1), 16-32. https://doi.org/10.54559/amesia.1730014

[66] Khan, A., Izhar, M., & Sezgin, A. (2017). Characterizations of abel grassmann's groupoids by the properties of their double-framed soft ideals. *International journal of analysis and applications*, *15*(1), 62–74. http://www.etamaths.com

[67] Atagün, A. O., & Sezer, A. S. (2015). Soft sets, soft semimodules and soft substructures of semimodules. *Mathematical sciences letters*, *4*(3), 235–242. https://B2n.ir/n85398

[68] Sezer, A. S., Atagün, A. O., & Çagman, N. (2014). N-group SI-action and its applications to N-Group Theory. *Fasciculi mathematici*, *54*, 139–153. https://B2n.ir/n19479

[69] Atagün, A. O., & Sezgin, A. (2017). Int-soft substructures of groups and semirings with applications. *Applied mathematics and information sciences*, *11*(1), 105–113. https://doi.org/10.54559/jauist.158924210.18576/amis/110113

[70] Gulistan, M., Feng, F., Khan, M., & Sezgin, A. (2018). Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. *Mathematics*, *6*(12), 293. https://doi.org/10.54559/jauist.158924210.3390/math6120293

[71] Sezer, A. S., Atagün, A. O., & Çağman, N. (2013). A new view to n-group theory: Soft N-groups. *Fasciculi mathematici*, *51*(51), 123–140. https://B2n.ir/m81412

[72] Jana, C., Pal, M., Karaaslan, F., & Sezgin, A. (2019). (α, β)-soft intersectional rings and ideals with their applications. *New mathematics and natural computation*, *15*(2), 333–350. https://doi.org/10.1142/S1793005719500182

[73] Atagün, A. O., Kamacı, H., Tatekin, İ., & Sezgin, A. (2019). P-properties in Near-rings. *Journal of mathematical and fundamental sciences*, *51*(2), 152–167. https://dx.doi.org/10.5614/j.math.fund.sci.2019.51.2.5

[74] Sezgin, A., & Orbay, M. (2022). Analysis of semigroups with soft intersection ideals. *Acta universitatis sapientiae, mathematica*, *14*(1), 166–210. https://doi.org/10.2478/ausm-2022-0012

[75] Atagün, A. O., & Sezgin, A. (2018). A new view to near-ring theory: Soft near-rings. *South east Asian journal of mathematics & mathematical sciences*, *14*(3), 19-32. https://rsmams.org/journals/articleinfo.php?articleid=313&tag=seajmams

[76] Manikantan, T., Ramasamy, P., & Sezgin, A. (2023). Soft Quasi-ideals of soft near-rings. *Sigma*, *41*(3), 565–574. https://doi.org/10.14744/sigma.2023.00062

[77] Naeem, K. (2017). *Soft set theory & soft sigma algebras*. LAP LAMBERT Academic Publishing. https://www.amazon.com/Soft-Set-Theory-Sigma-Algebras/dp/3330073055

[78] Riaz, M., Naeem, K., & Ahmad, M. O. (2017). Novel concepts of soft sets with applications. *Annals of fuzzy mathematics and informatics*, *13*(2), 239-251. https://doi.org/10.30948/afmi.2017.13.2.239

[79] Sezgin, A., Yavuz, E., & Özlü, Ş. (2024). Insight into soft binary piecewise lambda operation: A new operation for soft sets. *Journal of umm al-qura university for applied sciences*, 1-15. https://doi.org/10.1007/s43994-024-00187-1

[80] Memiş, S. (2022). Another view on picture fuzzy soft sets and their product operations with soft decision-making. *Journal of new theory*, (38), 1–13. https://doi.org/10.53570/jnt.1037280

[81] Naeem, K., & Memiş, S. (2023). Picture fuzzy soft σ -algebra and picture fuzzy soft measure and their applications to multi-criteria decision-making. *Granular computing*, *8*(2), 397–410. https://doi.org/10.1007/s41066-022-00333-2

[82] Sezgin, A., Aybek, F., & Güngör, N. B. (2023). A new soft set operation: Complementary soft binary piecewise union operation. *Acta informatica malaysia*, *7*(1), 38–53. https://actainformaticamalaysia.com/archives/AIM/1aim2023/1aim2023-38-53.pdf

[83] Sezgin, A., & Durak, İ. (2025). Soft intersetion-symmetric difference product of groups. *Matrix science mathematic*, *9*(2), 54-60. https://www.researchgate.net/publication/393509804_Soft_Intersection-symmetric_Difference_Product_of_Groups

[84] Ay, Z., & Sezgin, A. (2025). Soft union-plus product of groups. *International journal of mathematics, statistics, and computer science*, *3*, 365-376. https://doi.org/10.59543/ijmscs.v3i.14961