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Effective Q–fuzzy Soft Expert Sets and Its Some Properties

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Abstract

In 2023, Alkhazaleh [1] introduced the concept of the Effective Fuzzy Soft Expert Set (EFSES) as a new mathematical tool to address uncertain problems in decision-making and medical diagnosis. The virtue of this concept is its adaptability to deal with uncertain problems involving external effects. However, some uncertain decision-making problems, especially those with external effects, must be judged by several experts. To this end, this paper extends the concept of EFSES to the concept of an Effective Q-Fuzzy Soft Expert Set (EQFSES). The concept of QFSES is further extended to include the operations of union, intersection AND, and OR using De Morgan's Law. Definitions and propositions on these operations are introduced.

Keywords: Fuzzy soft expert set, Q-fuzzy soft expert set, Effective Q-fuzzy soft expert set.

1 | Introduction

Many real-world problems are characterized by uncertainty, which complicates traditional decision-making methods in fields such as economics, engineering, and medicine. To address these challenges, Zadeh [2] introduced fuzzy set theory as a mathematical tool. He later expanded this idea to include interval-valued fuzzy sets [3], where the membership space consists of all closed subintervals between 0 and 1.

Further advancing these concepts, Molodtsov [4] introduced soft set theory as a new framework for tackling vague problems. Maji et al. [5] later generalized soft set theory into fuzzy soft set theory, enhancing its applicability to decision-making scenarios. Maji et al. [6] provided a precise definition of key operations within this framework, such as union, intersection, AND, and OR, establishing a foundational understanding of these concepts. Building on this, Roy and Maji [7] further investigated the application of fuzzy soft sets, particularly within decision-making processes, highlighting their practical utility and impact in this field.

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Yang et al. [8] took an innovative approach by integrating interval-valued fuzzy sets with soft sets. Lastly, Alkhazaleh and Salleh [9] presented soft expert set theory, which aggregates expert opinions into a cohesive model, detailing its properties and defining basic operations like union, intersection, AND, and OR.

Alkhazaleh and Salleh [10] defined a fuzzy soft expert set, which was later extended to a generalized fuzzy soft expert set [11], vague soft expert set [12], generalized vague soft expert set [13–15], and multi Q-fuzzy soft expert set [16]. Q-fuzzy soft sets [17–19] and multi-Q fuzzy soft sets were proposed by Adam and Hassan [20–22], effective fuzzy soft sets [23] and time-effective fuzzy soft sets [24]. Recently, Alkhazaleh [1] introduced the concept of Effective Fuzzy Soft Expert Set (EFSES) theory, defining fundamental operations such as complement, union, and intersection. This theory notably incorporates external effects in decision-making processes. A key advantage of this approach, compared to existing concepts like EFSS and fuzzy soft expert sets, is its ability to combine both internal and external influences while allowing up to two opinions from each expert for enhanced decision-making.

2 | Preliminaries

Definition 1 ([2]). The fuzzy sets defined on a non-empty V as objects having the form $A = \{< v: \mu_A(v), v \in V\}$ where the functions $\mu: V \to [0, 1]$ for $v \in V$.

Definition 2 ([21]). Let V be an initial universe set, and E be a set of parameters. Consider $A \subseteq E$. Let $P(V)$ denote the set of all fuzzy sets of V. The collection (F, A) is termed to be the soft fuzzy set over V, where F is a mapping given by $F: A \to P(V)$.

Definition 3 ([22]). V is an initial universe, E is a set of parameters, X is a set of experts (agents), and $O=\{agree=1, disagree=0\}$ a set of opinions. Let $Z=E\times X\times O$ and $A\subseteq Z$. A pair (F, A) is called a soft expert set over V, where F is the mapping given by

$$F: A \to P(V),$$

where $P(V)$ denotes the power set of V.

Definition 4 ([25]). Let I be a unit interval and k be a positive integer. A multi Q-fuzzy set $\widetilde{A}_Q$ in V and a non-empty set Q is a set of ordered sequences $\widetilde{A}_Q = \{(v.q), \mu_i(v, q): v \in V, q \in Q\}$ where

$$\mu_i: V \times Q \to I^k, \quad i = 1,2, \dots, k.$$

The function $(\mu_1(v, q), \mu_2(v, q), \dots, \mu_k(v, q))$ is called the membership function of multi Q-fuzzy set $\widetilde{A}_Q$: and $\mu_1(v, q) + \mu_2(v, q) + \dots + \mu_k(v, q) \leq 1, k$ is called the dimension of $\widetilde{A}_Q$. The set of all multi Q-fuzzy sets of dimension k in V and Q is denoted by $M^kQF(V)$.

Definition 5 ([1]). A functional, effective set Λ in a universe of discourse A is a fuzzy set defined as $\Lambda: U \text{ x } X \to I^A$ and the set of effective parameters is represented by C, such that the membership values perhaps modified after the implementation effective set and have positive (or no) effect on membership values given by

$$\Lambda = \{(u.x).\delta_\Lambda(a): a \in C\}.$$

Definition 6 ([1]). A pair $(F, S)_\Lambda$ is called an ENSES over U, provided that $F(U)$ denotes the set of all fuzzy subsets of U and the set of effective parameters is represented by C such that Λ be the effective set over C. Then F is a mapping given by $F: Z \to F(U)$ and defined as follows:

$$F(s)_\Lambda = \left\{\frac{u_j}{T_{U(u_j)_\Lambda}}: u_j \in U, s \in S\right\}.$$

For all $s \in S$ and for all $a_k \in C$, we have:

$$T_{U(u_j)_\Lambda} = \begin{cases} T_U(u_j) + \dfrac{(1-T_U(u_j))\sum_k \delta_{\Lambda_{Xj}}(a_k)}{|A|}, & \text{if } T_U(u_j) \in (0,1), \\ T_u(u_j), & \text{O.W.} \end{cases}$$

3 | Effective Q− Fuzzy Soft Expert Sets

We will now propose the definition of Effective Q-Fuzzy Soft Expert Set (EQFSES) and propose some of its properties. Throughout the discussion, V is the initial universe, E is the set of parameters, Q be a set of supplies, Λ is the set of effective parameters, X is the set of experts, and $O = \{\text{agree} = 1, \text{disagree} = 0\}$ a set of suggestions. Let $A \subseteq Z$ where $Z = E \times X \times O$ and $S \subseteq Z$.

Definition 7. (F_Q, A) is a QFSES over V, where F_Q is the mapping $F_Q: A \to QFSES$ such that $QFSES$ is the set of all QFSES over V.

$$F_{Q_\Lambda}(S) = \bigwedge_{i=1}^{m\times k} F_Q(e_t, p_j, 1)_{\Lambda_i}, \bigwedge_{i=1}^{m\times k} F_Q(e_t, p_j, 0)_{\Lambda_i}, \ t = 1,2, \dots, m, j = 1,2, \dots, k.$$

$$F_Q(e_t. p_j. 1)_{\Lambda_i} = \left\{ \frac{u_j}{v_{U(u_j)_\Lambda}} : u_j \in U, s \in S \right\}.$$

$$F_Q(e_t. p_j. 0)_{\Lambda_i} = \left\{ \frac{u_j}{v_{U(u_j)_\Lambda}} : u_j \in U, s \in S \right\}.$$

For all $s \in S$ and for all $a_k \in C$, we have

$$T_{U(u_j)_\Lambda} = \begin{cases} v_U(u_j) + \dfrac{(1-v_U(u_j))\sum_k \delta_{\Lambda_{Xj}}(a_k)}{|\Lambda|}, & \text{if } v_U(u_j) \in (0,1), \\ v_u(u_j), & \text{O.W.} \end{cases}$$

for all $(a_k) \in \Lambda$, and $|\Lambda|$ is a cardinality Λ.

Example 1. Suppose a customer who wants to build a new house wants to get feedback from several experts. Let $V = \{v_1, v_2\}$ be the set of houses, $Q = \{k_1, k_2\}$ be the set of construction companies, $E = \{e_1, e_2\}$ be the set of decision parameters, and the set of effective parameters is represented by $\Lambda = \{l_1, l_2\}$. Let $X = \{p_1, p_2\}$ be the set of experts. Assume that:

$$\Lambda^1(v_1 . k_1, p_1) = \left\{ \frac{l_1}{0,5} . \frac{l_2}{0,4} \right\}.$$

$$\Lambda^2(v_1 . k_2, p_1) = \left\{ \frac{l_1}{0,3} . \frac{l_2}{0,8} \right\}.$$

$$\Lambda^3(v_1 . k_1, p_2) = \left\{ \frac{l_1}{0,7} . \frac{l_2}{0,3} \right\}.$$

$$\Lambda^4(v_1 . k_2 . p_2) = \left\{ \frac{l_1}{0,2} . \frac{l_2}{0,6} \right\}.$$

$$\Lambda^5(v_2 . k_1 . p_1) = \left\{ \frac{l_1}{0,1} . \frac{l_2}{0,4} \right\}.$$

$$\Lambda^6(v_2 . k_1 . p_2) = \left\{ \frac{l_1}{0,9} . \frac{l_2}{0,4} \right\}.$$

$$\Lambda^{7}(v_2 . k_2 . p_1) = \left\{\frac{l_1}{0,1} . \frac{l_2}{0,2}\right\}.$$

$$\Lambda^{8}(v_2 . k_2 . p_2) = \left\{\frac{l_1}{0,9} . \frac{l_2}{0,7}\right\}.$$

Let F be the QFSES defined as follows:

$$F_Q(e_1, p_1, 1) = \left\{\left(\frac{(v_1, k_1)}{0.6}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.4}, \frac{(v_2, k_2)}{0.2}\right)\right\}.$$

$$F_Q(e_1, p_2, 1) = \left\{\left(\frac{(v_1, k_1)}{0.4}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.8}\right)\right\}.$$

$$F_Q(e_2, p_1, 1) = \left\{\left(\frac{(v_1, k_1)}{0.5}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.5}\right)\right\}.$$

$$F_Q(e_2, p_2, 1) = \left\{\left(\frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.4}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.6}\right)\right\}.$$

$$F_Q(e_1, p_1, 0) = \left\{\left(\frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.3}\right)\right\}.$$

$$F_Q(e_1, p_2, 0) = \left\{\left(\frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}\right)\right\}.$$

$$F_Q(e_2, p_1, 0) = \left\{\left(\frac{(v_1, k_1)}{0.2}, \frac{(v_1, k_2)}{0.5}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.1}\right)\right\}.$$

$$F_Q(e_2, p_2, 0) = \left\{\left(\frac{(v_1, k_1)}{0.5}, \frac{(v_1, k_2)}{0.4}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}\right)\right\}.$$

Then, by applying *Definition 6*, we get

$$F_Q(e_1, p_1, 1)_{\Lambda^1} = \left\{\begin{array}{c} \dfrac{(v_1, k_1)}{0.6 + \left[(1 - 0.6)\dfrac{0.5 + 0.4}{2}\right]} \\ \dfrac{(v_1, k_2)}{0.7 + \left[(1 - 0.7)\dfrac{0.5 + 0.4}{2}\right]} \\ \dfrac{(v_2, k_1)}{0.4 + \left[(1 - 0.4)\dfrac{0.5 + 0.4}{2}\right]} \\ \dfrac{(v_2, k_2)}{0.2 + \left[(1 - 0.2)\dfrac{0.5 + 0.4}{2}\right]} \end{array}\right\} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}.$$

Similarly, when the calculations are continued, the EQFSES is found as follows

$$F_Q(e_1, p_2, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.9}.$$

$$F_Q(e_2, p_1, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.7}.$$

$$F_Q(e_2, p_2, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$F_Q(e_1, p_1, 0)_{\Lambda^1} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.6}, \frac{(v_2, k_2)}{0.6}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.6,},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^2}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^2}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^2}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^2}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^3}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^3}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^3}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^3}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.5},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^4}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^4}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^4} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^4} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^4} = \frac{(v_1,k_1)}{0.4},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^4} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.4,}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.5},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^5} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^5} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^5} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.3}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^5} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^6} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^6} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^6} = \frac{(v_1,k_1)}{0.3},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0,7}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^6}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0,9}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^7}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.3}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^7}=\frac{(v_1,k_1)}{0.5},\frac{(v_1,k_2)}{0.4},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^7}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^7}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.5},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^7}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.4},\frac{(v_2,k_2)}{0.4}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^7}=\frac{(v_1,k_1)}{1.0},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^7}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.4},\frac{(v_2,k_2)}{0.2}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^7}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.5},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^8}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^8}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{1.0}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^8}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^8}=\frac{(v_1,k_1)}{1.0},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{1.0},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^8}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^8}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^8}=\frac{(v_1,k_1)}{0.2},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^8}=\frac{(v_1,k_1)}{0.5},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

Definition 8. For two EQFSES $(F_Q,A)_{\Lambda^i}$ and $(G_Q,B)_{\Lambda^i}$ over V, $(F_Q,A)_{\Lambda^i}$ is called an EQFSE subset of $(G_Q,B)_{\Lambda^i}$ if

I. $B\subseteq A$.

II. $G_Q(\varepsilon)_{\Lambda^i}$ is an effective Q-fuzzy soft expert subset $F_Q(\varepsilon)_{\Lambda^i}$ for all $\varepsilon\in B$.

Example 2. Consider *Example 1*, where

$A=\{(e_1,p_1,1),(e_2,p_1,1)\}$.

and

$B = \{(e_1, p_1, 1)\}$.

B is an effective Q-fuzzy soft expert subset of A, hence $B \subseteq A$. Define $(G_Q, B)_{\Lambda^3}$ and $(F_Q, A)_{\Lambda^3}$ as follows:

$$(F_Q, A)_{\Lambda^3} = \left\{(e_1, p_1, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}.\right.$$

$$\left.(e_2, p_1, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}\right\}.$$

$$(G_Q, B)_{\Lambda^3} = \left\{(e_1, p_1, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}\right\}.$$

Therefore

$$(G_Q, B)_{\Lambda^3} \tilde{\subseteq} (F_Q, A)_{\Lambda^3}.$$

Definition 9. Two EQFSES $(F_Q, A)_{\Lambda^i}$ and $(G_Q, B)_{\Lambda^i}$ over V are equal if $(F_Q, A)_{\Lambda^i}$ is an EQFSES subset of $(G_Q, B)_{\Lambda^i}$ and $(G_Q, B)_{\Lambda^i}$ is an EQFSES subset of $(F_Q, A)_{\Lambda^i}$ for all i.

Definition 10. Agree-EQFSES $(F_Q, A)_{\Lambda}$ over V is an EQFSES subset of $(F_Q, A)_{\Lambda}$ defined as

$$(F_Q, A)_{\Lambda}{}^1 = \left\{F_Q{}^1(\propto)_{\Lambda} : \propto \in E \times X \times \{1\}\right\}.$$

Example 3. Using our previous *Example 1*, the agree-EQFSES $(F_Q, Z)_{\Lambda}{}^1$ over V is

$$(F_Q, Z)_{\Lambda}{}^1 = \left\{(e_1, p_1, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}.\right.$$

$$(e_1, p_2, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.9}.$$

$$(e_2, p_1, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.7}.$$

$$(e_2, p_2, 1)_{\Lambda^1} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$(e_1, p_1, 1)_{\Lambda^2} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}.$$

$$(e_1, p_2, 1)_{\Lambda^2} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.9}.$$

$$(e_2, p_1, 1)_{\Lambda^2} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$(e_2, p_2, 1)_{\Lambda^2} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$(e_1, p_1, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.6}.$$

$$(e_1, p_2, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.9}.$$

$$F_Q(e_2, p_1, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$F_Q(e_2, p_2, 1)_{\Lambda^3} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^4} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.4,}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.5},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^5} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^6} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^6} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^7} = \frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.3}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^7} = \frac{(v_1,k_1)}{0.5},\frac{(v_1,k_2)}{0.4},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^7} = \frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^7} = \frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.5},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,1)_{\Lambda^8} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_2,1)_{\Lambda^8} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{1.0}.$$

$$F_Q(e_2,p_1,1)_{\Lambda^8} = \frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$$F_Q(e_2,p_2,1)_{\Lambda^8} = \frac{(v_1,k_1)}{1.0},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{1.0},\left.\frac{(v_2,k_2)}{0.9}\right\}.$$

Definition 11. A disagree-EQFSES $(F_Q,Z)_\Lambda{}^0$ over V is an EQFSES subset of $(F_Q,Z)_\Lambda$ defined as

$$(F_Q,Z)_\Lambda{}^0 = \left\{F_Q{}^0(\propto)_\Lambda : \propto \in E \times X \times \{0\}\right\}.$$

Example 4. Using our previous *Example 1*, the disagree-EQFSES $(F_Q,Z)_\Lambda{}^0$ over V is

$$\left(F_Q,Z\right)_{\Lambda}{}^{0}=\left\{F_Q(e_1,p_1,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.6}.\right.$$

$$F_Q(e_1,p_2,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.6,},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^1}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^2}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.5},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.7},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^3}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^4}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.6}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^4}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^4}=\frac{(v_1,k_1)}{0.4},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^4}=\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^5}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.5}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^5}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.8},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_2,p_1,0)_{\Lambda^5}=\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.3}.$$

$$F_Q(e_2,p_2,0)_{\Lambda^5}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.7}.$$

$$F_Q(e_1,p_1,0)_{\Lambda^6}=\frac{(v_1,k_1)}{0.9},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.8},\frac{(v_2,k_2)}{0.8}.$$

$$F_Q(e_1,p_2,0)_{\Lambda^6}=\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.9},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.9}.$$

$F_Q(e_2, p_1, 0)_{\wedge^6} = \frac{(v_1, k_1)}{0.3}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0{,}7}.$

$F_Q(e_2, p_2, 0)_{\wedge^6} = \frac{(v_1, k_1)}{0.6}, \frac{(v_1, k_2)}{0.8}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0{,}9}.$

$F_Q(e_1, p_1, 0)_{\wedge^7} = \frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.4}, \frac{(v_2, k_2)}{0.4}.$

$F_Q(e_1, p_2, 0)_{\wedge^7} = \frac{(v_1, k_1)}{1.0}, \frac{(v_1, k_2)}{0.7}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.7}.$

$F_Q(e_2, p_1, 0)_{\wedge^7} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.4}, \frac{(v_2, k_2)}{0.2}.$

$F_Q(e_2, p_2, 0)_{\wedge^7} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.5}, \frac{(v_2, k_1)}{0.7}, \frac{(v_2, k_2)}{0.7}.$

$F_Q(e_1, p_1, 0)_{\wedge^8} = \frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.9}.$

$F_Q(e_1, p_2, 0)_{\wedge^8} = \frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.9}, \frac{(v_2, k_1)}{0.9}, \frac{(v_2, k_2)}{0.9}\Big\}.$

Definition 12. The complement of an EQFSES $(F_Q, A)_\wedge$ is

$$(F_Q, A)_\wedge{}^c = (F_Q{}^c, \neg A)_\wedge.$$

Such that $F_Q{}^{(c)}{}_\wedge : \neg A \rightarrow EQFSE(V)$ a mapping

$$F_Q{}^{(c)}(\propto)_\wedge = \left\{\bar{1} - \mu_{F_Q(\propto)_\wedge}\right\}.$$

For each $\propto \in E$. It is clear that it is $\left((F_Q, A)_\wedge{}^c\right)^c = (F_Q, A)_\wedge$.

Example 5. Using our previous *Example 1*, the complement of the EQFSES $F_{Q_\wedge}$ denoted by $F_Q{}^{(c)}{}_\wedge$ is given as follows:

$$(F_Q, Z)_\wedge{}^c = \left\{\neg(e_1, p_2, 1)_{\wedge^1} = \frac{(v_1, k_1)}{0.3}, \frac{(v_1, k_2)}{0.4}, \frac{(v_2, k_1)}{0.2}, \frac{(v_2, k_2)}{0.1}.\right.$$

$$\neg(e_2, p_1, 1)_{\wedge^1} = \frac{(v_1, k_1)}{0.3}, \frac{(v_1, k_2)}{0.2}, \frac{(v_2, k_1)}{0.2}, \frac{(v_2, k_2)}{0.3}.$$

And so on.

Definition 13. The union of two EQFSES $(F_Q, A)_\wedge$ and $(G_Q, B)_\wedge$ over V, denoted by

$$(F_Q, A)_\wedge \tilde{\cup} (G_Q, B)_\wedge,$$

is the EQFSES $(H_Q, C)_\wedge$ such that $C = A \cup B$ as follows:

$$\mu_{H_Q(e)} = \begin{cases} \mu_{F_Q(e)}(m), & \text{if } e \in A - B, \\ \mu_{G_Q(e)}(m), & \text{if } e \in B - A, \\ \max\left(\mu_{F_Q(e)}(m), \mu_{G_Q(e)}(m)\right), & \text{if } e \in A \cap B. \end{cases}$$

Example 6. Suppose that $(F_Q, A)_\wedge$ and $(G_Q, B)_\wedge$ are two EQFSES over V, such that

$$(F_Q, A)_\wedge = \left\{\left[(e_1, q, 0)\left(\frac{(v_1, k_1)}{0.4}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.4}\right)\right].\right.$$

$$\left.\left[(e_2, q, 0), \left(\frac{(v_1, k_1)}{0.8}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.5}, \frac{(v_2, k_2)}{0.1}\right)\right]\right\}.$$

$$\left(G_Q,B\right)_\Lambda = \left\{\left[(e_1,q,0)\left(\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.4},\frac{(v_2,k_2)}{0.2}\right)\right].\right.$$
$$\left.\left[(e_2,q,1),\left(\frac{(v_1,k_1)}{0.4},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.5}\right)\right]\right\}.$$

Then $\left(F_Q,A\right)_\Lambda \tilde{\cup} \left(G_Q,B\right)_\Lambda = \left(H_Q,C\right)_\Lambda$ where

$$\left(H_Q,C\right)_\Lambda = \left\{\left[(e_1,q,0),\left(\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.4},\frac{(v_2,k_2)}{0.4}\right)\right].\right.$$
$$\left[(e_2,q,0),\left(\frac{(v_1,k_1)}{0.8},\frac{(v_1,k_2)}{0.3},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.1}\right)\right].$$
$$\left.\left[(e_2,q,1),\left(\frac{(v_1,k_1)}{0.4},\frac{(v_1,k_2)}{0.7},\frac{(v_2,k_1)}{0.9},\frac{(v_2,k_2)}{0.5}\right)\right]\right\}.$$

Proposition 1. If $\left(F_Q,A\right)_\Lambda$, $\left(G_Q,B\right)_\Lambda$ and $\left(H_Q,C\right)_\Lambda$ are three EQFSES over V, then

I. $\left(\left(F_Q,A\right)_\Lambda \tilde{\cup} \left(G_Q,B\right)_\Lambda\right) \tilde{\cup} \left(H_Q,C\right)_\Lambda = \left(F_Q,A\right)_\Lambda \tilde{\cup} \left(\left(G_Q,B\right)_\Lambda \tilde{\cup} \left(H_Q,C\right)_\Lambda\right)$.

II. $\left(F_Q,A\right)_\Lambda \tilde{\cup} \left(F_Q,A\right)_\Lambda \cong \left(F_Q,A\right)_\Lambda$.

Definition 14. Suppose $\left(F_Q,A\right)_\Lambda$ and $\left(G_Q,B\right)_\Lambda$ are two EQFSES over the common universe V. The intersection of $\left(F_Q,A\right)_\Lambda$ and $\left(G_Q,B\right)_\Lambda$ is $\left(F_Q,A\right)_\Lambda \tilde{\cap} \left(G_Q,B\right)_\Lambda = \left(K_Q,C\right)_\Lambda$ as follows:

$$\mu_{K_Q(e)} = \begin{cases} \mu_{F_Q(e)}(m), & \text{if } e \in A - B, \\ \mu_{G_Q(e)}\ (m), & \text{if } e \in B - A, \\ \min\left(\mu_{F_Q(e)}(m), \mu_{G_Q(e)}(m)\right), & \text{if } e \in A \cap B. \end{cases}$$

Example 7. Suppose that $\left(F_Q,A\right)_\Lambda$ and $\left(G_Q,B\right)_\Lambda$ are two EQFSES over V, such that

$$\left(F_Q,A\right)_\Lambda = \left\{\left[(e_1,p,1),\left(\frac{(v_1,k_1)}{0.3},\frac{(v_2,k_1)}{0.9}\right)\right].\right.$$
$$\left[(e_2,q,1),\left(\frac{(v_1,k_1)}{0.8},\frac{(v_2,k_1)}{0.3}\right)\right].$$
$$\left.\left[(e_2,q,0),\left(\frac{(v_1,k_1)}{0.7},\frac{(v_2,k_1)}{0.8}\right)\right]\right\}.$$
$$\left(G_Q,B\right)_\Lambda = \left\{\left[(e_1,p,1),\left(\frac{(v_1,k_1)}{0.4},\frac{(v_2,k_1)}{0.7}\right)\right]\right\}.$$

Then $\left(F_Q,A\right)_\Lambda \tilde{\cap} \left(G_Q,B\right)_\Lambda = \left(K_Q,C\right)_\Lambda$ where

$$\left(K_Q,C\right)_\Lambda = \left\{\left[(e_1,p,1),\left(\frac{(v_1,k_1)}{0.3},\frac{(v_2,k_1)}{0.7}\right)\right]\right\}.$$

Proposition 2. If $\left(F_Q,A\right)_\Lambda$, $\left(G_Q,B\right)_\Lambda$ and $\left(K_Q,C\right)_\Lambda$ are three EQFSES over V, the following properties hold true.

I. $\left(\left(F_Q,A\right)_\Lambda \tilde{\cap} \left(G_Q,B\right)_\Lambda\right) \tilde{\cap} \left(K_Q,C\right)_\Lambda = \left(F_Q,A\right)_\Lambda \tilde{\cap} \left(\left(G_Q,B\right)_\Lambda \tilde{\cap} \left(K_Q,C\right)_\Lambda\right)$.

II. $\left(F_Q,A\right)_\Lambda \tilde{\cap} \left(F_Q,A\right)_\Lambda \subseteq \left(F_Q,A\right)_\Lambda$.

Proposition 3. If $(F_Q, A)_\Lambda$, $(G_Q, B)_\Lambda$ and $(K_Q, C)_\Lambda$ are three EQFSES over V, then

I. $\left((F_Q, A)_\Lambda \tilde{\cup} (G_Q, B)_\Lambda\right) \tilde{\cap} (K_Q, C)_\Lambda = \left((F_Q, A)_\Lambda \tilde{\cap} (K_Q, C)_\Lambda\right) \tilde{\cup} \left((G_Q, B)_\Lambda \tilde{\cap} (K_Q, C)_\Lambda\right)$.

II. $\left((F_Q, A)_\Lambda \tilde{\cap} (G_Q, B)_\Lambda\right) \tilde{\cup} (K_Q, C)_\Lambda = \left((F_Q, A)_\Lambda \tilde{\cup} (K_Q, C)_\Lambda\right) \tilde{\cap} \left((G_Q, B)_\Lambda \tilde{\cup} (K_Q, C)_\Lambda\right)$.

Definition 15. If $(F_Q, A)_\Lambda$ and $(G_Q, B)_\Lambda$ are two EQFSES over V, then $(F_Q, A)_\Lambda$ and $(G_Q, B)_\Lambda$ is

$$(F_Q, A)_\Lambda \wedge (G_Q, B)_\Lambda = (H_Q, A \times B)_\Lambda.$$

Such that $H_Q(\alpha, \beta) = F_Q(\alpha) \cap G_Q(\beta)$ and memberships of $(H_Q, A \times B)_\Lambda$ is as follows:

$$\mu_{H_Q(\alpha,\beta)}(m) = \min\left(\mu_{F_Q(\alpha)}(m), \mu_{G_Q(\beta)}(m)\right),$$

where for all $\alpha \in A$,for all $\beta \in B$.

Example 8. Suppose that $(F_Q, A)_\Lambda$ and $(G_Q, B)_\Lambda$ are two EQFSES over V, such that

$$(F_Q, A)_\Lambda = \left\{\left[(e_1, p, 1), \left(\frac{(v_1, k_1)}{0.6}, \frac{(v_1, k_2)}{0.4}, \frac{(v_2, k_1)}{0.4,}, \frac{(v_2, k_2)}{0.8}\right)\right]\right\}.$$

$$(G_Q, B)_\Lambda = \left\{\left[(e_1, p, 1), \left(\frac{(v_1, k_1)}{0.5}, \frac{(v_1, k_2)}{0.9,}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.1}\right)\right].\right.$$

$$\left.\left[(e_2, q, 0), \left(\frac{(v_1, k_1)}{0.9}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.8}, \frac{(v_2, k_2)}{0.2}\right)\right]\right\}.$$

Then $(F_Q, A)_\Lambda \wedge (G_Q, B)_\Lambda = (H_Q, A \times B)_\Lambda$ where

$$(H_Q, A \times B)_\Lambda = \left\{\left[(e_1, p, 1), (e_1, p, 1), \left(\frac{(v_1, k_1)}{0.5}, \frac{(v_1, k_2)}{0.4}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.1}\right)\right].\right.$$

$$\left.\left[(e_1, p, 1), (e_2, q, 0), \left(\frac{(v_1, k_1)}{0.6}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.6}, \frac{(v_2, k_2)}{0.2}\right)\right]\right\}.$$

Definition 16. If $(F_Q, A)_\Lambda$ and $(G_Q, B)_\Lambda$ are two EQFSES over V, then $(F_Q, A)_\Lambda$ OR $(G_Q, B)_\Lambda$ is

$$(F_Q, A)_\Lambda \vee (G_Q, B)_\Lambda = (K_Q, A \times B)_\Lambda.$$

Such that $K_Q(\alpha, \beta) = F_Q(\alpha) \cup G_Q(\beta)$ and the memberships of truth, indeterminacy, and falsity of $(K_Q, A \times B)_\Lambda$ are as follows:

$$\mu_{K_Q(\alpha,\beta)}(m) = \max\left(\mu_{F_Q(\alpha)}(m), \mu_{G_Q(\beta)}(m)\right),$$

where for all $\alpha \in A$, for all $\beta \in B$.

Example 9. Suppose that $(F_Q, A)_\Lambda$ and $(G_Q, B)_\Lambda$ are two EQNSES over V, such that

$$(F_Q, A)_\Lambda = \left\{\left[(e_1, p, 1), \left(\frac{(v_1, k_1)}{0.6}, \frac{(v_1, k_2)}{0.3}, \frac{(v_2, k_1)}{0.5}, \frac{(v_2, k_2)}{0.1}\right)\right]\right\}.$$

$$(G_Q, B)_\Lambda = \left\{\left[(e_1, p, 1), \left(\frac{(v_1, k_1)}{0.7}, \frac{(v_1, k_2)}{0.6}, \frac{(v_2, k_1)}{0.6}, \frac{(v_2, k_2)}{0.2}\right)\right].\right.$$

$$\left.\left[(e_2, q, 0), \left(\frac{(v_1, k_1)}{0.3}, \frac{(v_1, k_2)}{0.2}, \frac{(v_2, k_1)}{0.3}, \frac{(v_2, k_2)}{0.9}\right)\right]\right\}.$$

Then $(F_Q, A)_\Lambda \vee (G_Q, B)_\Lambda = (K_Q, A \times B)_\Lambda$ where

$$\left(K_Q, A\times B\right)_\Lambda = \left\{\left[(e_1,p,1),(e_1,p,1)\left(\frac{(v_1,k_1)}{0.7},\frac{(v_1,k_2)}{0.6},\frac{(v_2,k_1)}{0.6},\frac{(v_2,k_2)}{0.2}\right)\right].\right.$$

$$\left.\left[(e_1,p,1),(e_2,q,0),\left(\frac{(v_1,k_1)}{0.6},\frac{(v_1,k_2)}{0.3},\frac{(v_2,k_1)}{0.5},\frac{(v_2,k_2)}{0.9}\right)\right]\right\}.$$

Proposition 4. If $(F_Q,A)_\Lambda$ and $(G_Q,B)_\Lambda$ are EQNSES over V, then

I. $\left((F_Q,A)_\Lambda \wedge (G_Q,B)_\Lambda\right)^c = (F_Q,A)_\Lambda^{\ c} \vee (G_Q,B)_\Lambda^{\ c}$.

II. $\left((F_Q,A)_\Lambda \vee (G_Q,B)_\Lambda\right)^c = (F_Q,A)_\Lambda^{\ c} \wedge (G_Q,B)_\Lambda^{\ c}$.

4 | Conclusion

We introduced the concept of EQFSES theory as a new mathematical tool to deal with uncertainty. Furthermore, we presented some of its properties and defined its basic operations as complement, union, intersection, AND, and OR. As a future direction, researchers can develop this concept into an effective neutrosophic vague soft expert set.

Author Contributaion

Conceptualization, Z. B. and V. U. Methodology, Z. B. writing-reviewing and editing, Z. B. and V. U. All authors have read and agreed to the published version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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