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Stability and Randomness of Nonstationary D/M/1 Queue's GI/M/1 PSFFA Model with Ultra-Low Latency for Autonomous Driving

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Abstract

The current work reveals the fine-tuning between stability zones and randomness of the GI/M/1 Pointwise Stationary Fluid Flow Approximation (PSFFA) model of the nonstationary D/M/1 queueing system. More specifically, this provides more insights into developing a contemporary PSFFA theory that unifies nonstationary queueing theory with chaos theory and fields in theoretical physics and chaotic systems. This opens new grounds for stability analysis of nonstationary queueing systems. A notable application of the GI/M/1 queueing model to achieve ultra-low latency of autonomous driving service is highlighted. Concluding remarks are given on future avenues of research.

Keywords: State variable, Mean arrival rate, Time, Time-dependent root parameter, PSFFA, Ultra-low latency, Autonomous driving service.

1 | Introduction

Day-to-day queues include customers' time-varying arrival processes, which are interpreted by their variance-based nature based on the time of day. Factors like failure of network resources or nonstationary input loads can cause this. These bursty and nonstationary traffic in character networks as communication networks become more complicated with fluctuating data speeds and quality of service needs. Queuing theory deals with analyzing and understanding waiting times in various scenarios, such as waiting for service in banks or supermarkets, waiting for a response from computers, waiting for failures to occur, or waiting for public transport.

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Simulation techniques in the context of queueing systems involve tracking the system's behaviour through repeated simulation execution and averaging relevant quantities over different runs at specific time points. Collecting data at various time instants allows the system's Behavior to be evaluated over time [1].

In analytical transient investigations, transform techniques are commonly employed to solve differential/difference equation models that arise from an embedded Markov process/chain. These techniques help analyze the system's behaviour over time by transforming the equations into a more manageable form, facilitating the study of transient phenomena in queueing systems.

This paper's road map is as follows: PSFFA theory is overviewed in Section 2. Section 3 discusses the GI/M/1 Queueing Model in more detail. Section 4 reveals ϱ- threshold of the nonstationary D/M/1 queue's GI/M/1 PSFFA model of the. Section 5 presents typical numerical experiments to evidence the derived analytic results against the numerical portraits. A notable application of the GI/M/1 queueing model to achieve ultra-low latency of autonomous driving service is highlighted in Section 6. Closing remarks combined with the next research phase are highlighted in Section 7.

2 | PSFFA

The Pointwise of Stationary Fluid Flow Approximation Model (PSFFA) is a simulation technique that uses a single non-linear differential equation to estimate the queue's average number of users. An equation's form based on steady-state queueing relationships is obtained by this revolutionary approach to provide advantages in terms of generality, simplicity, and computational efficiency. Moreover, these methods have potential applications in developing dynamic network control mechanisms [1].

Think about a queueing system for a single server with a nonstationary arrival process. $\mu(t)$ and $\lambda(t)$ serve as the time-dependent average queue service and arrival rates, respectively. The system's ensemble average time-dependent state variable is referred to as $x(t)$, $x^{\cdot}(t) = \frac{dx(t)}{dt}$. Define $f_{in}(t)$ and $f_{out}(t)$ respectively, to be the system's time-dependent flow into and out. Notably, $x^{\cdot}(t)$, $f_{in}(t)$ and $f_{out}(t)$ are related by

$$x^{\cdot}(t) = -f_{out}(t) + f_{in}(t). \tag{1}$$

Consequently,

$$f_{out}(t) = \mu(t)\rho(t). \tag{2}$$

Here $\rho(t)$ defines the underlying queue's server utilization. For an infinite queue, waiting space is infinite,

$$f_{in}(t) = \lambda(t). \tag{3}$$

Eq. (1)'s fluid flow model becomes

$$x^{\cdot}(t) = -\mu(t)\rho(t) + \lambda(t), \qquad 1 > \rho(t) = \frac{\lambda(t)}{\mu(t)} > 0. \tag{4}$$

Setting $x^{\cdot}(t) = 0$, implies

$$x = G_1(\rho). \tag{5}$$

Additionally, we assume the numerical invertibility of $G_1(\rho)$, namely

$$\rho = G_1^{-1}(x). \tag{6}$$

Equationally, PSFFA rewrites to:

$$x^{\cdot}(t) = -\mu(t)\left(G_1^{-1}\big(x(t)\big)\right) + \lambda(t). \tag{7}$$

Notably, *Eq. (7)* is extremely general since the closed-form representation of G_1 can be computed for many queues. However, we can numerically or by data of an existing system's fitting curve calculate G_1.

3 | The GI/M/ 1 Queueing Model

This section discusses the GI/M/1 queueing model, in which the service time has an exponential distribution, and the inter-arrival process has an identical distribution with successive inter-arrival periods. Let $A(t)$ stand for the distribution of inter-arrival times. The GI/M/1 queue's steady state probability for the number of customers a new arrival finds in the system is a geometric distribution

$$\pi_n = (1-\sigma)\sigma^n. \tag{10}$$

$\sigma(1 > \sigma > 0)$ uniquely solves:

$$\sigma = f_a^*(s)|_{s=\mu(1-\sigma),} \tag{11}$$

where $f_a^*(s)$ is the Laplace-Stieltjes transform of the inter-arrival time distribution $A(t)$, that is

$$f_a^*(s) = \mathcal{L}^*\big(A(t)\big) = \int_0^\infty e^{-st}\, dA(t). \tag{12}$$

Notably, $\sigma = 1$ solves *Eq. (11)*, and the state variable, x, reads as

$$x = \frac{\lambda}{\mu(1-\sigma)} = \frac{\rho}{(1-\sigma)}. \tag{13}$$

In determining the PSFFA model, *Eq. (13)* rewrites to

$$\rho(t) = x(t)\mu\big(1-\sigma(t)\big). \tag{14}$$

We believe that the nonstationary load will exhibit sinusoidal mean behaviour, which will describe the cyclic load pattern over a specified time period (for example, day) in accordance with the prior research on nonstationary analysis of communication networks [2-6], namely $\lambda(t) = A + B\sin(wt + D)$, for more details see [7-9].

Thus, the required model reads as

$$x^{\cdot}(t) = \mu x(t)\big(1-\sigma(t)\big) + \lambda(t). \tag{15}$$

We can numerically solve *Eq. (15)* to visualize the queue's time-varying behaviour.

Depending on the inter-arrival distribution $A(t)$, the precise process for figuring out will vary, although it usually involves a root-finding approach like Laguerre's method. The time-varying D/M/1 queue's GI/M/1 PSFFA model reads

$$x^{\cdot}(t) = -\mu x(t)\big(1-\sigma(t)\big) + \lambda(t), \qquad \sigma(t) = e^{\frac{(\sigma(t)-1)}{\rho(t)}}. \tag{16}$$

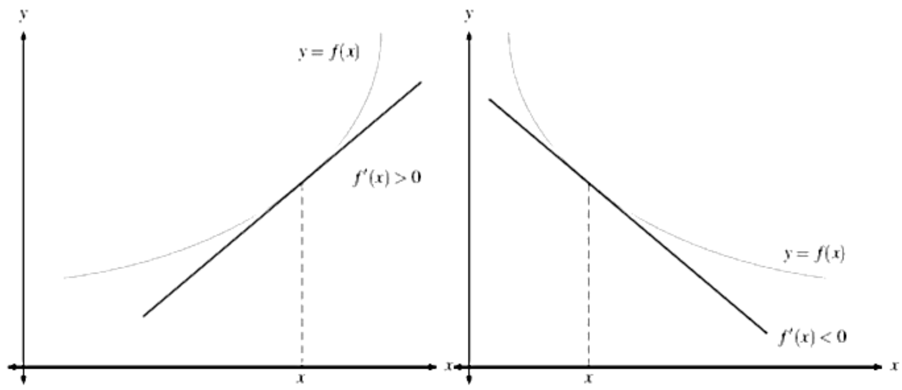
$$\rho(t) = \text{time} - \text{dependent server utilization} = \frac{\lambda(t)}{\mu}.$$

The D/M/1 case in *Eq. (16)* corresponds to a deterministic arrival process where the inter-arrival distribution $A(t)$ is a delta function (i.e., $dA(t) = f_a(t)dt$ and $f_a(t) = \delta(t - \frac{1}{\lambda})$).

Mastering the increase ability (decrease ability) for a function, $f(x)$, a shorthand note reads:

$$f' > 0 \Leftrightarrow f\uparrow.$$
$$f' < 0 \Leftrightarrow f\downarrow.$$

We can visualize *Fig. 1* (c.f., [10]) in a more tangible form.



1. If $f'(x) > 0$ on an open interval, then f is increasing on the interval.
2. If $f'(x) < 0$ on an open interval, then f is decreasing on the interval.

Fig. 1. Increasing/decreasing test.

4 | The ρ −Theshold Theory of the Non-stationary the D/M/1 Queue GI/M/1 PSFFA model.

Theorem 1. The time-dependent server utilization, $\rho(t)$ (c.f., *Eq. (16)*), is forever increasing in $\sigma(t)\left(\sigma(t) \in (0,1)\right)$.

Proof. Let the time-dependent root parameter, $\sigma(t)$ be such that $1 > \sigma(t) > 0$. By *Eq. (16)*, it follows that:

$$\rho(t) = \frac{\lambda(t)}{\mu} = \frac{(\sigma(t) - 1)}{\ln\left(\sigma(t)\right)}. \tag{17}$$

We have

$$\frac{\partial\rho}{\partial\sigma} = \frac{\ln\sigma - 1 + \frac{1}{\sigma}}{(\ln\sigma)^2}. \tag{18}$$

Following mathematical analysis (c.f., [11]),

$$1 - \frac{1}{\sigma} < \ln\sigma < \sigma - 1 \tag{19}$$

The result of communicating *Eq. (18)* and *Eq. (19)* is as follows: We can see that

$$\lim_{\sigma(t)\to 1}\rho(t) = \lim_{\sigma(t)\to 1}\frac{(\sigma(t)-1)}{\ln(\sigma(t))} = \lim_{\sigma(t)\to 1}\frac{1}{\frac{1}{(\sigma(t))}} = 1. \ \text{(L'Hopital's rule)} \tag{20}$$

and

$$\lim_{\sigma(t)\to \infty}\rho(t) = \lim_{\sigma(t)\to \infty}\frac{(\sigma(t)-1)}{\ln(\sigma(t))} = \lim_{\sigma(t)\to \infty}\frac{1}{\frac{1}{(\sigma(t))}} = \infty. \ \text{(L'Hopital's rule)} \tag{21}$$

5 | Typical Numerical Experiments

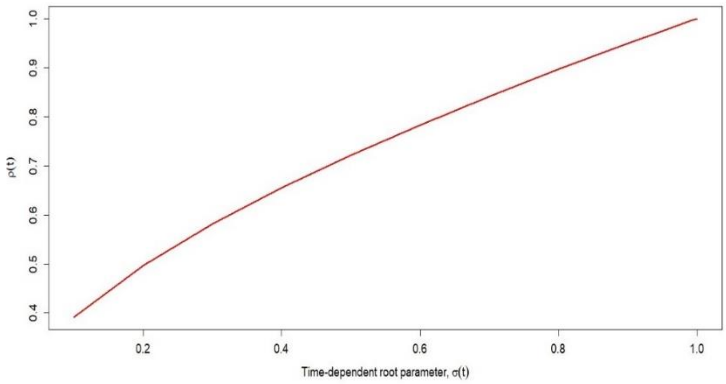


Fig. 2. Stability and approachability to high traffic intensity zone.

R code for Fig. 2

sigma <-c(0.1, 0.2, 0.3,0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99, 1)

rho (-c(0.3908650337, 0.4970679476, 0.5814084816, 0.6548140008, 0.7213475204, 07830460756, 0.8411019756, 0.8962840235, 0.9491221581, 0.9747862873, 0.99499162471, 1)

plot (sigma, rho

type="1"

col="red"

xlab=expression(paste("Time-dependent root parameter," ,sigma(t))),

ylab-expression (rho (t)),

main="Stability and Approachability to High Traffic Intensity Zone",

cex.lab=1.2,

cex.axis=1.2

)

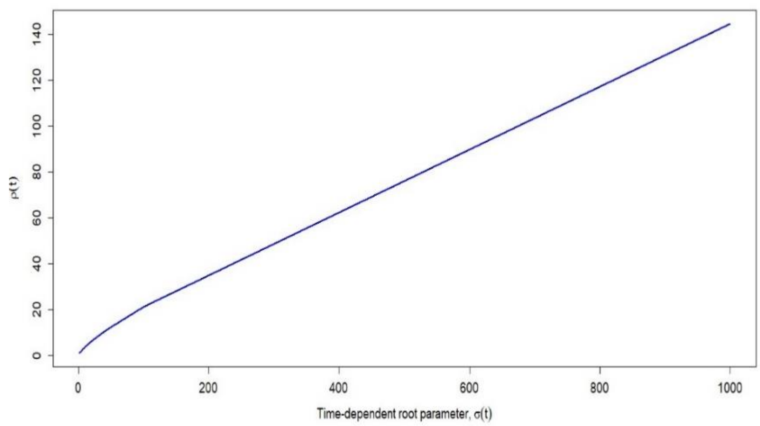


Fig. 3. Beyond high traffic intensity zone.

R code for Fig. 3

Sigma <-c(1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100, 1000)

```
rho <-c(1.049205869, 1.09696299, 1.143448406, 1.188805365, 1.233151731, 1.276585887, 1.319190975, 1.361038022, 1.402188285, 1.442695041, 1.820478453, 2.164042561, 2.485339738, 3.908650337, 6.342355813, 8.52640901, 10.5723162, 12.52548871, 21.49757685, 144.6200625)

plot (sigma, rho,

type="1",

col="blue",

xlab=expression(paste("Time-dependent root parameter,", sigma(t))),

ylab=expression( rho(t)),

main="Beyond High Traffic Intensity Zone"

lwd=1.2,

cex.lab=1.2,

cex.axis-12

)
```

It is observed from *Fig. 2* and *Fig. 3* that the time-dependent root parameter, $\sigma(t)$ has a significant impact on the underlying queue's stability by directly impacting the time-dependent server utilization, $\rho(t)$. Touching upon stability, it can be seen that $\sigma(t)$ acts as a cutting-edge fine tuning to either approaching a high traffic intensity zone, corresponding to $\rho(t) = 1$.

Moreover, the progressive increase of $\sigma(t)$, will steer the whole system into a randomness zone corresponding to $\rho(t) > 1$.

It can be easily verified that the numerical setup validates the obtained analytic results of *Theorem 1*.

6 | GI/M/1 - Based Ultra-low Latency of Autonomous Driving Service

The investigation on how 5G networks are anticipated to accommodate different network services with varying performance criteria, such as high-rate traffic, low latency, and high reliability, was conducted by the authors of [12]. New technologies, including Network Functions Virtualization (NFV), network slicing, and Software-Defined Networking (SDN), are being implemented to satisfy these expectations. By combining fog, edge, and cloud computing with network slicing to map autonomous driving functionalities into service slices, these technologies will help build a distributed and scalable SDN core network architecture that will ultimately improve transmission efficiency and meet low latency constraints. The goal is to improve the quality of service for autonomous driving applications.

The global system manager in an autonomous driving system [12] plays a crucial role by overseeing three main functions: driving mode management, which includes manual, run, and pause modes; fault management system, which monitors module statuses for safe driving; and emergency response to system faults or operator interventions to ensure the safety and efficiency of the autonomous vehicle operations. These functions are essential for maintaining the operational integrity and safety of the autonomous driving system by managing driving modes, monitoring module statuses, and responding to emergencies effectively. Moreover, this describes a 5G-enabled scalable SDN core network architecture for autonomous driving systems [12], emphasizing the importance of ultra-low latency and high reliability. It introduces the concept of a global system management service implemented as a service slice to meet the stringent requirements of autonomous driving modes, ensuring efficient operation and safety through features like fail-safe mechanisms and human-machine interfaces displaying critical vehicle information, as shown in *Fig. 4* (c.f., [12]).

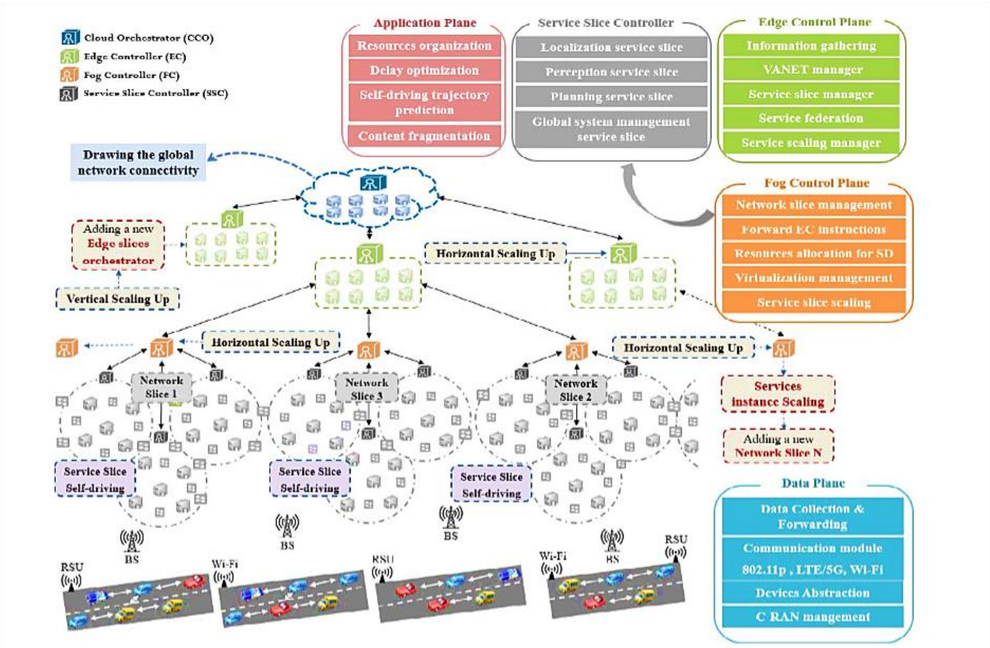
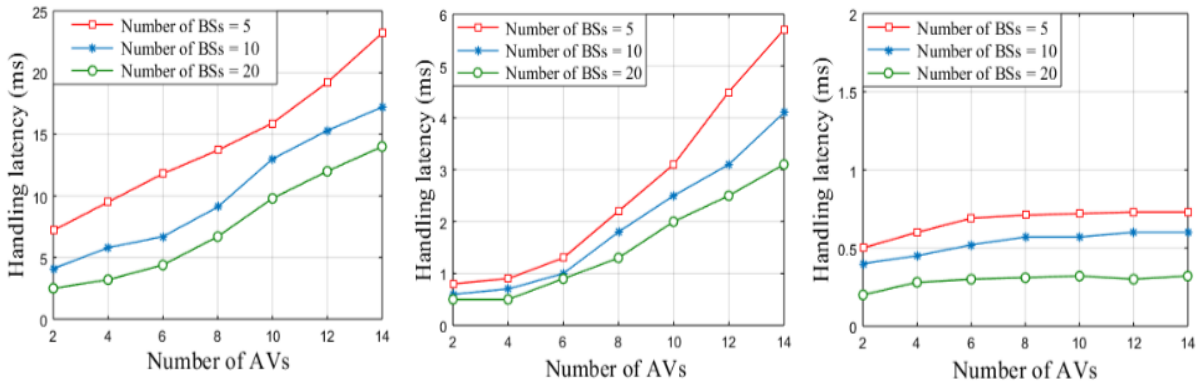


Fig. 4. A 5G-enabled scalable SDN core network architecture for autonomous driving systems.

Accordingly, the authors [12] provided a simulation-based assessment of their system with a particular emphasis on SliceScal and the 5G slicing model for driverless vehicles. They evaluate the system's performance in a realistic urban setting using the network simulator NS-3, the cars traffic simulator Veins-SUMO, and the OpenFlow SDN controller, considering vehicle dispersion, radio propagation, and latency management in autonomous driving resource slicing. The evaluation platform and algorithm developed aim to optimize resource allocation and latency handling for autonomous driving services in a dynamic urban environment.

Fig. 5 (c.f., [12]) offers a visual description for handling the latency of autonomous vehicle service requests in different scenarios involving service slicing and a slice management algorithm. It shows that as Autonomous Vehicle (AV) density increases, the handling latency also increases. Service slicing reduces the handling latency by 60%, and the SliceMan algorithm further reduces it by 90% compared to scenarios without service slicing, demonstrating the effectiveness of managing resources for service slices in improving latency.



a. **b.** **c.**

Fig. 5. A visual description for handling the latency of autonomous vehicle service requests; a. with service slicing, b. with service slicing, c. with sliceman algorithm.

7 | Conclusion

An exposition is undertaken to reveal the threshold theory of the time-dependent server utilization of the $D/M/1$ queueing system's $GI/M/1$ PSFFA closed-form expression. Moreover, some numerical experiments are provided to validate the analytic results. The influential impact of the $GI/M/1$ queueing model on ultra-low latency of autonomous driving service is investigated. Future work involves further investigation of similar threshold theorems of the $G/M/1$ PSFFA model of the nonstationary $E_k/M/1$ and $IPP/M/1$ queueing systems.

Author Contributions

The provided work was majorly done by Ismail A Mageed, where Amina Becheroul has undertaken the plotting of Figure 2, and the R coding.

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Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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