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An Introduction to Neutrosophic Possibility Theory: Modal Perspectives and Applications

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Abstract

Possibility theory focuses on quantifying the degree to which a statement or proposition is possible. It deals with the measurement of possibility and necessity in a similar manner to how probability theory deals with likelihood. While the concept of fuzzy logic has proven valuable in addressing uncertainties and imprecision, its main drawback arises when dealing with situations that involve not only uncertainty but also indeterminacy, as well as the coexistence of truth, falsity, and indeterminacy within a single statement. In this paper, we suggest using neutrosophic logic as a mathematical framework for reasoning with ambiguity and vagueness. We propose utilizing Kripke structures for neutrosophic propositions as conceptual abstract models, providing an alternative method to describe possibility theory in a neutrosophic environment. An illustrative scenario in the context of medical diagnosis is presented in order to demonstrate the efficacy and flexibility of our method. This novel approach not only enriches our knowledge of uncertainty, but it also provides pathways for more comprehensive and nuanced analysis in other domains such as in artificial intelligence, decision support systems, knowledge representation, cognitive computing etc., thus highlighting the potential benefit of merging neutrosophic logic with possibility and modal structures.

Keywords: Possibility theory, Neutrosophic logic, Kripke model, Possible world semantics, Uncertainty.

1 | Introduction

Possibility theory, first coined by Zadeh [1], is an uncertainty theory that deals with incomplete knowledge. It is closely related to probability theory as it is based on set functions. However, it differs from probability theory by using two dual set functions (possibility and necessity measurements) instead of just one. In his paper, Zadeh relates the notion of possibility to fuzzy sets by describing a possibility distribution as an elastic constraint on variable values. His aim was to provide a conceptual framework based on fuzzy sets that would

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properly deal with the meaning of information, rather than its measure. He concluded that this analysis required a possibilistic point of view, rather than a probabilistic one.

Fuzzy sets reflect ambiguous notions by allowing elements to have different degrees of belonging to a set. To achieve this, each element is assigned a membership degree to the set, ranging from 0 to 1, resulting in a membership function. While the concept of fuzzy logic has proved useful in resolving ambiguities and imprecision, its fundamental shortcoming appears when dealing with scenarios that contain not only uncertainty but also indeterminacy, as well as the coexistence of truth, falsehood, and indeterminacy inside a single assertion.

Smarandache [2] suggested Neutrosophy as a new field of philosophy on many-valued logics that integrated non-standard analysis with a tri-component logic/set/probability theory. Neutrosophy advocates that every idea/concept/thesis etc. possesses a degree of truth, as well as falsehood and indeterminacy, which must be considered individually. In other words, an indeterminacy assignment is explicitly defined, conjointly and independently with truth and falsity assignments. As a consequence, he introduced the theory of neutrosophic logic (NL) as a generalization of many-valued logics since fuzzy logic is thought to be incapable of demonstrating indeterminacy by itself. In a more formal definition, NL is a logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are called neutrosophic components which represent the truth, indeterminacy, and falsehood value respectively [3].

Modal logic was initially defined as the logic of necessary and possible facts. The possible world semantics provides an intuitive means for reasoning about situations. Within this framework, Kripke model [4] resembles a directed labelled graph whose graph's nodes represent potential worlds $\mathcal{s}$ from a set $\mathcal{S}$ labelled with truth assignments π.

The aim of this article is twofold. First, we believe that it is high time to reframe and reinforce the statement made by Zadeh [1] regarding the importance of possibility theory and how it could be best examined and explained. This theory is crucial because a significant portion of the information used for human decision-making is possibilistic in nature, due to the inherent fuzziness of natural languages. It is widely acknowledged that natural languages frequently exhibit ambiguity, imprecision, and multiple interpretations, which makes them ideal subjects for a neutrosophic treatment. By incorporating neutrosophic elements, these models can better capture the multifaceted nature of language and enhance their ability to handle ambiguity and uncertainty in human communication. Secondly, our target is to extend the results presented in [1] by employing Kripke structures for neutrosophic propositions as building blocks for an extended possibility theory, thus forming a powerful and flexible mathematical framework designed to handle indeterminacy, inconsistency, and uncertainty simultaneously. This is achieved due to the concept of neutrosophy which posses no restriction on the sum of the of the neutrosophic components (T,I,F) other than they are subsets of $]^-0, 1^+[$, thus:

$$^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+.$$

This non-restriction allows room for paraconsistent, dialetheism, and incomplete information to be characterized in NL.

It is our belief that in this integrated framework, the ability of neutrosophic logic to express and manage indeterminate information supplements the ability of possibility theory to handle uncertainty. This fusion enables a more complex representation of knowledge, accounting for not only variable degrees of possibility but also the underlying ambiguity or imprecision in the information. The resultant system offers a richer and more expressive framework for reasoning in circumstances that involve uncertainty and indeterminacy. This integration could find applications in diverse fields such as artificial intelligence, decision support systems, and information retrieval, where a comprehensive treatment of uncertainty and indeterminacy is crucial for accurate and reliable analysis.

The structure of the article is as follows: Section 2 presents the basic definitions and concepts of possibility theory, neutrosophic logic, and Kripke modal logic that are necessary to build upon and construct our proposed conceptual framework. Subsequently, Section 3 will present and explain our methodology, while Section 4 will provide a brief discussion on the significance of our findings. Finally, in Section 5, concluding remarks will be stated.

2|Materials and Methods

In this section, we provide the basic terminology used throughout the article. While our goal is to make the current manuscript as self-contained as possible, it is important to note that there is a wealth of literature on possibility theory [5–13]. neutrosophic logic [14–19], and Kripke modal logics[20–26] that interested readers can refer to.

2.1|Possibility Theory

In his seminal paper [1], Zadeh suggested possibility theory as (fuzzy) set-based representation of incomplete information. The main concepts that were presented include the following:

Definition 1 ([1]). In possibility theory, a possibility distribution function assigns a probability to each potential occurrence or proposition. This function converts items from a sample space to values in the range [0, 1], where 0 denotes impossibility, 1 denotes certainty, and values in between reflect degrees of probability. Giving the formal definition now we have: Let X be a variable taking values in U, and let F act as a fuzzy restriction, R(X), associated with X. Then the proposition X is F, which translates into R(X) = F, associates a possibility distribution, Π_x, with X which is postulated to be equal to R (X), i.e.

$$\Pi_x = R(X). \quad (1)$$

The possibility distribution function for X (or Π_x) is given by π_x, which is numerically identical to the membership function of F, i.e.

$$\pi_x = \mu_F. \quad (2)$$

Definition 2 ([1]). The possibility measure, represented by μ, is a form of probability distribution associated with a proposition. It measures the probability that the proposition is true. The possibility measure has a value between 0 and 1, with higher values suggesting a greater degree of possibility. More formally, let A be a nonfuzzy subset of U and let Π_x be a possibility distribution associated with a variable X which takes values in U. Then, the possibility measure, $\pi(A)$, of A is defined as a number in [0, 1] given by

$$\pi(A) = \sup_{u\in A}\pi_x(u). \quad (3)$$

where $\pi_x(u)$ is the possibility distribution function of Π_x.

2.2|Neutrosophic Logic

Neutrosophic Logic (NL) is an extension of classical and fuzzy logic, introduced by Smarandache in the late 20th century. It provides a framework for dealing with indeterminate, imprecise, and inconsistent information by incorporating a third truth value called indeterminacy.

In neutrosophic logic, a concept A is T% true, I% indeterminate, and F% false, with $(T, I, F) \subset ||^{-}0, 1^{+}||^3$, where $||^{-}0, 1^{+}||$ is an interval of hyperreals.

In this paradigm, truth, falsehood, and indeterminacy may coexist, allowing for a more comprehensive representation of complex and ambiguous information. Sets containing neutrosophic components are employed in neutrosophic logic, with constituents having degrees of truth, falsehood, and indeterminacy. Its capacity to deal with ambiguity and uncertainty makes it useful in circumstances where standard logic systems may fail to offer correct representations.

In this framework, a formula φ is characterized by a triplet of truth-values, called the neutrosophical value defined as [27]:

$$NL(\varphi) = (T(\varphi), I(\varphi), F(\varphi)),$$

$$\text{where } (T(\varphi), I(\varphi), F(\varphi)) \subset ||-0, 1+||^3. \quad (4)$$

Now we are ready to give the definition of possibility measure in the context of neutrosophic logic which is analogous to the one that Zadeh presented in [27].

Assume that p is represented as neutrosophic proposition of the form X is P where X takes values in a space U and P is a neutrosophic set in U with a specified truth membership function μ_T, indeterminacy membership function μ_I and falsity membership function μ_F. Similarly assume that F is represented as a neutrosophic proposition of the form X is F where F is a neutrosophic set in U, with a specified truth membership function ν_T, indeterminacy membership function ν_I and falsity membership function ν_F. Let u be a generic value of X. Denote the neutrosophic possibility that X=u as $Poss_N$ (X=u).

Definition 3. $Poss_N$ (X=u) is defined as the grade of (t, i, f)-membership of u in P, i.e.

$$Poss_N (X=u) = (\mu_T (u), \mu_I (u), \mu_F (u)). \quad (5)$$

Definition 4. The neutrosophic possibility measure of P given F, $Poss_N$ (P|F) is defined as

$Poss_N$ (P|F) = $\sup_u$ (F ∩ P) or, more concretely,

$$Poss_N (P|F) = \sup_u ((\nu_T (u), \nu_I (u), \nu_F (u)) \text{ and } (\mu_T (u), \mu_I (u), \mu_F (u)). \quad (6)$$

where and = conjunction, the latter given, in neutrosophic environment, by the following equation:

Given two sentences a_1, a_2 and a neutrosophic valuation υ such that υ (a_1) = (t_1, i_1, f_1) and υ (a_2) = (t_2 ,i_2, f_2) the truth value of the conjuction $\alpha_1 \wedge \alpha_2$ can be defined as [28]

$$Poss_N (P|F) = \sup_u ((\nu_T (u), \nu_I (u), \nu_F (u)) \text{ and } (\mu_T (u), \mu_I (u), \mu_F (u)). \quad (7)$$

$$(\alpha_1 \wedge \alpha_2) = (\min (t_1, t_2), \max (i_1, i_2), \max (f_1, f_2)).$$

2.3 | Kripke Model

A Kripke model [4] is a mathematical framework used in modal logic to depict several worlds and their interactions. Kripke models, named after the framework's creator Saul Kripke, are used to offer semantics for various modal logics, such as modal propositional logic and modal predicate logic.

A Kripke model has several worlds or states. Each world represents a possible state of circumstances. An accessibility link exists between worlds. This relation specifies which worlds are accessible from other worlds. It expresses the concept of "possible transitions" between states. More formally,

Definition 5 ([29]). A Kripke model is a triple structure S_K of the form $\langle \mathcal{S}, \mathcal{R}, \pi \rangle$ where:

$\mathcal{S}$ is a non-empty set (the set of possible worlds).

$\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ is the accessibility relation $\pi : (\mathcal{S} \rightarrow P) \rightarrow \{0; 1\}$ is a truth assignment to the propositions per possible world.

where $P = \{p_1, \ldots, p_n\}$ is a set of propositional variables, and {0; 1} stands for {True; False}.

A world s is regarded possible concerning another world s' if there is an edge connecting the two. This connection is specified by an arbitrary binary relation, referred to as the accessibility relation.

2.3.1 | Kripke model for neutrosophic propositions

A Kripke structure for neutrosophic propositions can be modified to reflect the indeterminacy inherent in neutrosophic logic [27]. In a classic Kripke model, the worlds represent potential states, while the accessibility relation denotes possible state transitions. In the framework of neutrosophic logic, we'll broaden this to include degrees of truth, falsehood, and indeterminacy connected with propositions.

Let us first define the main components of a Kripke structure for neutrosophic propositions.

Worlds/states

Each world in the Kripke structure represents a possible interpretation or situation.

Accessibility relation

The accessibility relation between worlds indicates the possibility of transitioning from one interpretation to another. It reflects the idea that certain worlds are accessible from others.

Neutrosophic propositions

At each world, there are neutrosophic propositions, each associated with degrees of truth, falsity, and indeterminacy. For example, a proposition p might have associated values (π_T, π_F, π_I) where π_T, is the truth assignment, π_F is the falsity assignment and π_I is the indeterminacy assignment.

Assignment function

The assignment function specifies the degrees of truth, falsity, and indeterminacy associated with each neutrosophic proposition at each world.

Validity in a world

A neutrosophic proposition is considered valid in a specific world if its associated degrees (truth, falsity, indeterminacy) meet certain criteria based on the assignment function.

Validity across worlds

A neutrosophic proposition is considered universally valid in the Kripke structure if it is true, false, or indeterminate in all accessible worlds.

Hence, we have:

Definition 4 ([27]). A Kripke model for neutrosophic propositions is a triple structure S_K^{NL} of the form $\langle \mathcal{S}, \mathcal{R}, \vec{\pi} \rangle$ where $\mathcal{S}$ is a non-empty set (the set of possible worlds).

$\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ is the accessibility relation.

$\vec{\pi} = (\pi_T, \pi_F, \pi_I)$ is a neutrosophic assignment to the propositions per possible world, i.e.

$\pi : (\mathcal{S} \rightarrow P) \rightarrow ||\ {}^{-}0, 1^{+}||$ with π being either π_T, or π_I or π_F.

where $P = \{p_1, \ldots, p_n\}$ is a set of propositional variables.

3 | Results

In this section, we will present an illustrative hypothetical scenario from the field of medical diagnosis to showcase our proposed methodology.

3.1 | Scenario

Consider the context of a medical diagnosis system where a neutrosophic Kripke model is employed to represent possibilities associated with a patient's health condition. In this scenario, the three main components of neutrosophic logic—truth, indeterminacy, and falsity—are used to capture the uncertainty inherent in medical assessments. The neutrosophic Kripke model extends this by incorporating Kripke structures to account for the dynamic nature of the diagnosis process.

Imagine a patient exhibiting symptoms that could be indicative of various medical conditions, and a medical expert assessing the patient's case. The neutrosophic Kripke model represents the possibility of the patient having a specific medical condition, such as a rare disease. The model consists of possible worlds, each representing a different diagnostic outcome based on available information.

Consider a set of possible worlds W corresponding to different diagnostic outcomes. Each possible world w ∈W represents a distinct state of the diagnostic process based on available information. Possibilistic measures allow us to quantify the degree of possibility associated with different states of the diagnostic process. We will denote the possibilistic measures for each possible world as Pw, where Pw is a value in the interval [0,1]representing the degree of possibility associated with the diagnostic outcome at world *w*.

3.1.1 | Modeling of the scenario

Our proposed conceptual framework utilizing neutrosophic Kripke model in possibility theory is then defined as follows:

Let W be the set of possible worlds, each representing a distinct diagnostic state.

S is the set of stages representing different points in the diagnostic process.

Pw represents the possibilistic measure associated with each possible world w ∈ W, where Pw ∈ [0,1].

For each possible world w∈W, we have the neutrosophic triplet $\{T_w,I_w,F_w\}$, where T_w, I_w, and F_w are in the interval [0,1].

Define an accessibility relation *R* on W such that w_1 R w_2 indicates that the information available at stage/world w_2 includes or extends that of stage/world w_1.

For each stage s ∈ S, let Ws be the set of possible worlds at stage s.

Then, the neutrosophic Kripke model is characterized by a set of functions {T,I,F,P,R}.

T:W×S→[0,1] assigns truth values to possible worlds at different stages.

I:W×S→[0,1] assigns indeterminacy values to possible worlds at different stages.

F:W×S→[0,1] assigns falsity values to possible worlds at different stages.

P:W→[0,1] assigns possibilistic measures to possible worlds.

R:W×W→{0,1} defines the accessibility relation.

3.1.2 | Algorithmic approach

Next, we give a step by step theoretical approach to the above scenario in order to better understand our proposed conceptual framework.

Step 1. Define possible worlds: each possible world in the neutrosophic Kripke model reflects a unique diagnostic result or scenario. Define potential worlds depending on the patient's medical circumstances, including both common and unusual disorders.

Step 2. Assign neutrosophic values: for each conceivable scenario, assign neutrosophic values that indicate the degree of truth, indeterminacy, and falsehood about the patient's medical state. Neutrosophic values vary from 0 to 1 and represent the degree of membership, indeterminacy, or non-membership.

Step 3. Incorporate available information: use the relevant clinical information, test findings, and symptoms to update the neutrosophic values in all potential worlds. Adjust the settings to reflect the information's relevance and dependability.

Step 4. Evaluate neutrosophic propositions: formulate neutrosophic propositions that describe diagnostic assertions, such as the patient has disease X. Use neutrosophic values to determine if these assertions are true, indeterminate, or false in each possible universe. Also in this step we calculate the possibility measure of the likelihood of each possible world being true.

3.1.3 | Scenario revisited

A patient is exhibiting symptoms that could be indicative of various medical conditions, including rare diseases. The neutrosophic Kripke model is used to represent the possibility of the patient having a specific medical condition based on available information.

Step 1. Define possible worlds.

I. World A: common disease.

II. World B: rare disease.

Step 2. Assign neutrosophic values.

I. Initial assignments: world initial: unknown/indeterminate disease

This world is a representation of the scenario where the available data does not decisively point toward either a common or rare disease. This world provides a space within the neutrosophic Kripke model to account for evolving understanding and emerging information about the patient's health.

$T_{IN} = 0.15$ (15% certainty).

$I_{IN} = 0.6$ (60% indeterminacy).

$F_{IN} = 0.25$ (25% falsity).

I. World A: common disease

$T_A = 0.7$ (70% certainty).

$I_A = 0.1$ (10% indeterminacy).

$F_A = 0.2$ (20% falsity).

II. World B: rare disease

$T_B = 0.3$ (30% certainty).

$I_B = 0.5$ (50% indeterminacy).

$F_B = 0.2$ (20% falsity).

Step 3. Incorporate available information.

III. Hypothetical update

New information increases certainty in World A: updated $T_A = 0.8$, $I_A = 0.05$, $F_A = 0.15$.

New information reduces indeterminacy in World B: updated $T_B = 0.3$, $I_B = 0.4$, $F_B = 0.3$.

Step 4. Neutrosophic propositions

Proposition for World A (common disease): the patient has disease X in World A.

Neutrosophic values: $T_A = 0.8$, $I_A = 0.05$, $F_A = 0.15$.

Then according to eq. 6, we get Neutrosophic possibility measure $\mathrm{Poss_N}\,(W_A|W_{IN})$ = (min (0.8, 0.15), max (0.05, 0.6), max (0.15, 0.25)) = (0.15, 0.6, 0.25).

Proposition for World B (rare disease): the patient has Disease X in World B.

Neutrosophic values: $T_B = 0.3$, $I_B = 0.4$, $F_B = 0.3$.

Again, according to eq. 6, we get Neutrosophic possibility measure $Poss_N$ ($W_B|W_{IN}$)= (min (0.3, 0.15), max (0.4, 0.6), max (0.3, 0.25)) = (0.15, 0.6, 0.3).

Neutrosophic possibility measure represents the quantitative measure of the likelihood or possibility of a diagnostic proposition being true within a specific possible world. In our propose methodology we go step further in the sense that we calculate the conditional possibility of a diagnosis being T% true, I% indeterminate and F% false within a specific possible world given an initial possible world, thus adding a more realistic approach. This could serve as a valuable tool in guiding decision-making and treatment considerations within the framework of neutrosophic logic.

The findings of the neutrosophic Kripke model, when applied to the scenario of medical diagnosis, demonstrate the model's significance in navigating the complexity and ambiguity inherent in the diagnostic process. By considering several alternative universes, each with varying degrees of truth, indeterminacy, and falsehood, the model allows for a more nuanced representation of diagnostic results. The model's flexibility is evident in the hypothetical adjustments to neutrosophic values based on new data, reflecting the dynamic nature of clinical evaluations.

The initial world, labeled as Unknown/Indeterminate, emerges as a crucial component, highlighting the recognition of ambiguity and the importance of making cautious decisions in situations where clear diagnostic approaches are lacking. This approach simplifies the creation of diagnostic hypotheses by providing related possibility measurements and a quantitative measure for the confidence level of each outcome.

4 | Applications

The findings of this study demonstrate great promise for a variety of applications across multiple fields. The integration of neutrosophic logic with Kripke structures, as outlined in this study, enhances the ability of possibility theory to address uncertainty and indeterminacy. This approach shows particular potential in areas such as artificial intelligence, decision support systems, and information retrieval, where effectively managing ambiguity is essential. The results extend previous studies in both possibility theory, pioneered by Zadeh [1], [27], and neutrosophic logic, introduced by Smarandache [2] ,[3]. While Zadeh's possibility theory effectively handles uncertain information, it does not fully address indeterminacy. Neutrosophic logic, on the other hand, excels in managing this indeterminacy but lacks robust frameworks for reasoning about possibilities. The combination of these two approaches, enhanced by Kripke models, creates a more versatile and comprehensive framework for handling complex information. Finally, the suggested paradigm enhances the theoretical understanding of possibility and indeterminacy, while also opening up new opportunities for practical applications across various domains.

In the broader context, the suggested approach has implications across various fields. For example, in artificial intelligence, this hybrid framework can improve decision-making algorithms that need to operate under conditions of uncertainty and incomplete data. It can enhance the capacity of decision support systems to provide more accurate recommendations by incorporating not just degrees of uncertainty, but also the indeterminacy present in real-world data. Within the realm of medical diagnosis, this framework can be utilized to represent complex patient data, enabling healthcare providers to navigate unclear or incomplete information more effectively when diagnosing disorders. For instance, it can assist in refining diagnostic hypotheses by assigning degrees of truth, indeterminacy, and falsehood to different medical conditions, as demonstrated in the medical scenario outlined in this study.

5 | Conclusion

In this paper, we analyzed the multifaceted environment of neutrosophic Kripke structures as formalism for modeling possibility theory, providing a novel and adaptable way to deal with uncertainty and ambiguity, primarily in the context of natural language. The combination of neutrosophic logic with Kripke structures has resulted in a robust framework capable of reflecting the complex interplay between possibility, necessity, and indeterminacy.

Our investigation began with a concise yet comprehensive study of key features that characterize possibility theory and neutrosophic logic, emphasizing its potential to describe triadic links between truth, indeterminacy, and falsity. Next, we integrated Kripke structures, a conceptual tool normally associated with modal logic, into the neutrosophic paradigm. The synergy between these two formalisms not only extended the expressive power of neutrosophic logic but also facilitated the representation of dynamic and evolving systems, enriching the scope of possibility theory.

By providing examples, we aimed to demonstrate the potential influence and practical usefulness of this innovative formalism. Our findings demonstrated the ability of neutrosophic Kripke structures to handle instances where classical logic fails, such as those characterized by inadequate knowledge, contradicting evidence, or dynamic changes over time. The capacity to express and reason about possibilities within a neutrosophic Kripke framework allows academics and practitioners to approach real-world problems with greater flexibility and accuracy.

In this article, we have studied the integration of neutrosophic logic and Kripke models with possibility theory. Our work can be seen as an initial and innovative step that could stimulate new insights in this direction, as there is a lack of relevant literature on this topic. For this reason, we aimed to examine the fundamental properties of such an integrated theory. As a result, possible future work could focus on improving and enhancing neutrosophic Kripke structures as formalism for possibility theory. This includes resolving possible constraints, investigating new aspects of neutrosophic logic, and designing algorithms for tasks such as reasoning and decision-making. Domain-specific research may assess the formalism's effectiveness in fields such as healthcare and finance, while integrating it with machine learning approaches may improve model resilience.

Comparative research with other formalisms, such as fuzzy logic and dempster-shafer theory, would provide information about relative strengths. Extending the application to dynamic systems and temporal reasoning, as well as conducting thorough case studies and empirical validations, will help demonstrate the real-world usefulness of the proposed formalism. Moreover, developing algorithms to automate this framework could enhance its applicability in real-time decision-making systems. Potential applications could also be explored in finance, risk management, and cognitive computing, where nuanced treatments of uncertainty are crucial. Finally, supporting multidisciplinary cooperation can improve our knowledge of neutrosophic Kripke structures and encourage their use across fields. Pursuing these pathways will contribute to the continued progress of the formalism, guaranteeing its relevance and application in tackling a wide range of uncertainty and possibility-related concerns.

Author Contributions

Antonios Paraskevas conceptualized the study, developed the theoretical framework for Neutrosophic Possibility Theory, and explored its modal perspectives and practical applications. He performed the literature review, conducted the analysis, and prepared the manuscript for publication.

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Data Availability

The paper is theoretical in nature, and no empirical data were used. All relevant details and derivations are included in the manuscript.

Conflicts of Interest

The author declares no conflicts of interest concerning the publication of this work.

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