Uncertainty Discourse and Applications

www.uda.reapress.com

Uncert. Disc. Appl.Vol. 1, No. 1 (2024) 121–139.

Paper Type: Original Article

Soft expert approach in rough fuzzy set and its application in MCDM problem

Srinivasan Vijayabalaji[1], **Shanmugam Kalaiselvan**[2], **Bijan Davvaz**[3] **and Said Broumi**[4]

[1,2]Department of Mathematics, University College of Engineering Panruti, Tamilnadu, India; balaji1977harshini@gmail.com; kselvan44@gmail.com.

[3]Department of Mathematical Sciences, Yazd University, Yazd, Iran; davvaz@yazd.ac.ir.

[4]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco; broumisaid785@gmail.com.

Citation:

Received: 14 July 2024 Revised: 21 August 2024 Accepted: 16 October 2024	Vijayabalaji, S., Kalaiselvan, S., Davvaz, B., & Broumi, S. (2024). Soft expert approach in rough fuzzy set and its application in MCDM problem. *Uncertainty Discourse and Applications,1*(1), 121-139

Abstract

One of the interesting part in the study of uncertainty is to learn about their fusion models. In particular while studying about fuzzy set and rough set one may be interested to know about their joint models. Rough fuzzy sets and fuzzy rough sets are those types. After the introduction of soft sets many research developments started emerging both in the theoretical and application prospective manner. Though this theory sounds good it has its own limitation in describing expert opinion. To overcome this difficulty the novel idea of soft expert set was being developed. This paper attempts to inter-relate soft expert set with rough fuzzy set in theoretical aspect. An approach to decision-making situation based on the soft expert rough fuzzy set model is also given in a lucid manner.

Keywords: Fuzzy set, SE-set, SEA-space, SER-approximations.

1|Introduction

A significant way to deal with uncertainty by defining two approximations over universe set was introduced in Pawlak [15]. The novel theory bears the name as rough set and has wide applications in various domains

Corresponding Author:balaji1977harshini@gmail.com

[15, 17, 25]. Zhang and Zhu [25] constructed a rough logic system RSL, based on rough sets and extensional regular double stone algebra. Pawlak and Slowinski [17] reviewed the methodology of the rough set analysis of multi-attribute decision problems.

Following Zadeh's [23] presentation of the concept of fuzzy set, various studies on the generalization of fuzzy set were being done for the past five decades. Xiao et al. [21] introduced fuzzy setting of soft modules and extended to fuzzy soft exactness. The most up-to-date information on this topic was provided by Mordeson et al. [14]. Many researchers have incorporated the generalization of fuzzy set concept. Atanassov [4] developed the concept of intuitionistic fuzzy set. Gorzalczany [8] introduced and discussed a method of approximate inference which operates on the extension of the concept of an interval-valued fuzzy set. Jiang et al. [9] presented an adjustable approach to fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets. Majumdar and Samanta [11] defined generalized fuzzy soft sets and studied some of their properties. They applied generalized fuzzy soft sets in decision making problems and medical diagnosis problems.

The notion of soft sets, a new mathematical tool for dealing with uncertainty was introduced by Molodtsov [13]. He has demonstrated a number of applications based on this principle. Soft set theory research and its applications in numerous fields are swiftly progressing. The smoothness of functions, Riemann integration, Perron integration, operations research, probability, measurement and game theory are just a few of the disciplines where soft sets could be useful. Aktas and Çağman [1] established soft group theory which expands the concept of group to incorporate soft set algebraic structures.

Dubois and Prade [5] made a significant approach by combining fuzzy set and rough set. In particular they developed two models namely rough fuzzy sets and fuzzy rough sets. They analyzed several properties pertaining to those hybrid models. [1, 6, 22] exhibits the inter-relation between the three theories. Feng along with Davvaz [7] made a significant contribution in combining soft sets and rough sets. They developed soft-rough fuzzy sets that generalizes Dubois model. Meng et al. [12] proposed a new soft rough set model and its properties are derived. The notion of soft rough fuzzy lattices (ideals, filters) over lattices was introduced by Zhu and Hu [26], is an extended notion of soft rough lattices (ideals, filters) and rough fuzzy lattices (ideals, filters) over lattices. Sun and Ma [19] proposed a new concept of soft fuzzy rough set by combining the fuzzy soft set with the traditional fuzzy rough set. Subsequently, they gave an approach to decision making problem based on soft fuzzy rough set model by analyzing the limitations. The concepts of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets are introduced by Zhang et al. [24] and developed an approach to intuitionistic fuzzy soft rough sets based on decision making problem.

Expert opinion is one of the key factor in all decision making situations. Alkhazeleh and Salleh [2] tried to combine soft set with experts opinion and as a result they generalized soft sets to soft expert sets. The novelty of their model is that it gives good results in decision making situations that involves more than one expert. Alkhazaleh and Salleh [3] generalized the concept of soft expert set to fuzzy soft expert set (FSE-set) and discussed a mapping on fuzzy soft expert classes along with its properties. Kalaiselvan and Vijayabalaji [10] defined the concept of a soft expert symmetric group (SES-group) as a generalization of the symmetric group and soft expert set. The application of a SES-group in MCDM situations is also presented in the same paper. Vijayabalaji et al. [20] introduced the idea of pull back and push out on soft module homomorphism. Hybrid structures of soft modules and rough modules namely soft-rough modules (respectively modified soft-rough modules) over soft-rough rings (respectively modified soft-rough rings) are also introduced. They completed with the illustration of a decision making problem in modified soft-rough module environment.

Motivated by the above hybrid models, this paper moves towards combining soft expert set with rough fuzzy set. In section 3 we provide three definitions for agree soft expert rough fuzzy set and disagree soft expert rough fuzzy set, by taking lower agree, upper agree, lower disagree and upper disagree soft expert rough approximations of fuzzy set ξ with respect to soft expert approximation space $SE = (U, (\aleph, \mathcal{V}))$. In section 4, the application of soft expert rough fuzzy set in decision making situation is presented. An algorithm on soft expert rough fuzzy set with supporting example is provided in the same section. Conclusion and direction of future research is given in section 5.

2|Preliminary Concepts

In this In this section we recall some familiar concepts which will be needed in the sequel.

Throughout this paper, let U be an universe, $\mathcal{Z} = E \times X \times O$ and $\mathcal{V} \subseteq \mathcal{Z}$, where E, X and $O = \{0 = \text{agree}, 1 = \text{disagree}\}$ be the collection of parameters, experts(agents) and opinions respectively. $P(U)$ denote the power set of U. $\digamma(U)$ denotes the collection of fuzzy subset of U. $(\aleph, \mathcal{V})_U$ denotes that $(\aleph, \mathcal{V})$ is SE-set in U.

Definition 2.1. [13] Let U be an universe. Consider a nonempty set A, $A \subseteq E$, where E be a set of parameters. The pair $(\aleph, A)$ is said to be soft set over U, if $\aleph : A \to P(U)$ is a function.
Definition 2.2. [15] Let $(U, \mathcal{R})$ be a Pawlak approximation space (PA-space). $\mathcal{R}$ will generate a partition $U/\mathcal{R} = \{[\varrho]_\mathcal{R} | \varrho \in U\}$ on U, where $[\varrho]_\mathcal{R}$ is the equivalence class with respect to $\mathcal{R}$ containing ϱ. These equivalence classes are referred to as $\mathcal{R}$-elementary sets. For each $Y \subseteq U$, the lower and upper approximations of Y with respect to $(U, \mathcal{R})$ are denoted by $\underline{\mathcal{R}}(Y)$ and $\overline{\mathcal{R}}(Y)$ respectively, given by,
$\underline{\mathcal{R}}(Y) = \{\varrho \in U | [\varrho]_\mathcal{R} \subseteq Y\}$,
$\overline{\mathcal{R}}(Y) = \{\varrho \in U | [\varrho]_\mathcal{R} \cap Y \neq \emptyset\}$.
Then Y is said to be definable, if $\underline{\mathcal{R}}(Y) = \overline{\mathcal{R}}(Y)$; otherwise Y is said to be a rough set.
Definition 2.3. [7] Let $(U, \mathcal{R})$ be a PA-space and $\mathfrak{S} = (\aleph, A)_U$ be a soft set, the lower and upper rough approximations of $(\aleph, A)$ in $(U, \mathcal{R})$ are soft sets over U, respectively denoted by $\mathcal{R}_*(\mathfrak{S}) = (\aleph_*, A)$ and $\mathcal{R}^*(\mathfrak{S}) = (\aleph^*, A)$ are given by,
$\aleph_*(\kappa) = \underline{\mathcal{R}}(\aleph(\kappa)) = \{\varrho \in U | [\varrho]_\mathcal{R} \subseteq \aleph(\kappa)\}$,
$\aleph^*(\kappa) = \overline{\mathcal{R}}(\aleph(\kappa)) = \{\varrho \in U | [\varrho]_\mathcal{R} \cap \aleph(\kappa) \neq \emptyset\}$, for all $\kappa \in A$.
Then $\mathfrak{S}$ is said to be definable, if $\mathcal{R}_*(\mathfrak{S}) = \mathcal{R}^*(\mathfrak{S})$; otherwise $\mathfrak{S}$ is known as rough soft set.
Definition 2.4. [5] Let $(U, \mathcal{R})$ be a PA-space and ξ being the set of all fuzzy sets in $P(U)$, the lower and upper rough approximations of ξ in $(U, \mathcal{R})$ are fuzzy subsets in U, respectively denoted by $\underline{\mathcal{R}}(\xi)$ and $\overline{\mathcal{R}}(\xi)$, given by,
$\underline{\mathcal{R}}_(\xi)(\varrho) = \wedge\{\xi(\vartheta) | \vartheta \subseteq [\varrho]_\mathcal{R}\}$,
$\overline{\mathcal{R}}_(\xi)(\varrho) = \vee\{\xi(\vartheta) | \vartheta \subseteq [\varrho]_\mathcal{R}\}$, for all $\varrho \in U$.
Then ξ is said to be definable, if $\underline{\mathcal{R}}(\xi) = \overline{\mathcal{R}}(\xi)$; otherwise ξ is said to be a rough fuzzy set.
Definition 2.5. [5] For given fuzzy approximation space $(U, \mathcal{R})$, the fuzzy lower and fuzzy upper approximations of ξ with respect to $(U, \mathcal{R})$ are fuzzy sets in U, respectively denoted by $\underline{\mathcal{R}}(\xi)$ and $\overline{\mathcal{R}}(\xi)$, given by,
$\underline{\mathcal{R}}_(\xi)(\varrho) = \wedge\{\xi(\vartheta) \vee (1 - \mathcal{R}(\varrho, \vartheta)\}$
$\overline{\mathcal{R}}_(\xi)(\varrho) = \vee\{\xi(\vartheta) \wedge (1 - \mathcal{R}(\varrho, \vartheta))\}$, for all $\varrho \in U$.
The pair $(\underline{\mathcal{R}}_(\xi), \overline{\mathcal{R}}_(\xi))$ is said to be a fuzzy rough set.
Definition 2.6. [7] Let $\mathfrak{S}_1 = (\aleph, A)$ be a soft set over U, the pair $\mathfrak{S} = (U, \mathfrak{S}_1)$ be a soft approximation space. Based on $\mathfrak{S}$, the lower and upper soft rough approximations of X are respectively denoted by $\underline{apr}_\mathfrak{S}(X)$ and $\overline{apr}_\mathfrak{S}(X)$, given by,
$\underline{apr}_\mathfrak{S}(X) = \{\varrho \in U | \exists\vartheta \in A(\varrho \in \aleph(\vartheta) \subseteq X)\}$,
$\overline{apr}_\mathfrak{S}(X) = \{\varrho \in U | \exists\vartheta \in A(\varrho \in \aleph(\vartheta), \aleph(\vartheta) \cap X \neq \emptyset)\}$, for every $X \subseteq U$.
Then X is said to be soft definable, if $\underline{apr}_\mathfrak{S}(X) = \overline{apr}_\mathfrak{S}(X)$; otherwise X is known as soft rough set.
Definition 2.7. [7] Let $\xi \in \digamma(U)$ be a fuzzy set, $\mathfrak{S}_1 = (\aleph, A)$ be a full soft set over U and $\mathfrak{S} = (U, \mathfrak{S}_1)$ be a soft approximation space. Then the lower and upper soft rough approximations of ξ with respect to $\mathfrak{S}$ are fuzzy sets in U, respectively denoted by $\underline{sap}_\mathfrak{S}(\xi)$ and $\overline{sap}_\mathfrak{S}(\xi)$, given by,
$\underline{sap}_\mathfrak{S}(\xi)(\varrho) = \wedge\{\xi(\vartheta) | \exists\gamma \in A(\{\varrho, \vartheta\} \subseteq \aleph(\gamma))\}$,
$\overline{sap}_\mathfrak{S}(\xi)(\varrho) = \vee\{\xi(\vartheta) | \exists\gamma \in A(\{\varrho, \vartheta\} \subseteq \aleph(\gamma))\}$, for all $\varrho \in U$.
Then ξ is said to be soft definable, if $\underline{sap}_\mathfrak{S}(\xi) = \overline{sap}_\mathfrak{S}(\xi)$; otherwise ξ is said to be a soft rough fuzzy set.
Definition 2.8. [2] Let $\mathcal{V} \subseteq \mathcal{Z}$, the pair $(\aleph, \mathcal{V})_U$ is known as soft expert set (SE-set), if $\aleph : \mathcal{V} \to P(U)$is a function.
Definition 2.9. [3] Let $\mathcal{V} \subseteq \mathcal{Z}$, if $\digamma_\xi : \mathcal{V} \to I^U$ is a function, then the pair $(\digamma_\xi, \mathcal{V})_U$ is known as fuzzy soft expert set (FSE-set), (i.e) a FSE-set $(\digamma_\xi, \mathcal{V})_U = \{(a, k_{\xi_{\digamma(a)}(k)})/k \in U, a \in \mathcal{V}\}$.

3|Soft expert rough fuzzy set

This section exhibits our new approach that combines soft-rough fuzzy set and soft expert set (SE-set) namely soft expert rough fuzzy set and related results.

Definition 3.1. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the soft expert approximation space (SEA-space) $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in F(U)$, the lower agree, upper agree, lower disagree and upper disagree SER-approximations of ξ with respect to SE are fuzzy sets in U, respectively denoted by $\underline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^p_0}_{SE}(\xi)$, given by,
$\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho) = \bigwedge\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\}$,
$\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho) = \bigvee\{\xi(\varrho) \vee \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\}$,
$\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho) = \bigwedge\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 0) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 0)\}$,
$\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho) = \bigvee\{\xi(\varrho) \vee \xi(\vartheta) | \exists (e_i, p_j, 0) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 0)\}$, for all $\varrho \in U$.
Then (i) ξ is said to be agree soft expert definable, if $\underline{\mathfrak{S}^p_1}_{SE}(\xi) = \overline{\mathfrak{S}^p_1}_{SE}(\xi)$; otherwise ξ is called an agree soft expert rough fuzzy set.

(ii) ξ is said to be disagree soft expert definable, if $\underline{\mathfrak{S}^p_0}_{SE}(\xi) = \overline{\mathfrak{S}^p_0}_{SE}(\xi)$; otherwise ξ is said to be a disagree soft expert rough fuzzy set.
Example 3.2. Let $U = \{\varrho_1, \varrho_2, \cdots, \varrho_6\}$, $E = \{\kappa_1, \kappa_2, \kappa_3\}$, $P = \{ep_1, ep_2\}$, $O = \{1, 0\}$ and $\mathcal{V} = \{(\kappa_1, ep_1, 1)$, $(\kappa_1, ep_1, 0)$, $(\kappa_1, ep_2, 1)$, $(\kappa_1, ep_2, 0)$, $(\kappa_2, ep_1, 1)$, $(\kappa_2, ep_1, 0)$, $(\kappa_2, ep_2, 1)$, $(\kappa_2, ep_2, 0)$, $(\kappa_3, ep_1, 1)$, $(\kappa_3, ep_1, 0)$, $(\kappa_3, ep_2, 1)$, $(\kappa_3, ep_2, 0)\} \subset E \times P \times O$. The SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ is defined by
$\aleph(\kappa_1, ep_1, 1) = \{\varrho_1, \varrho_4, \varrho_6\}$,
$\aleph(\kappa_1, ep_1, 0) = \{\varrho_2, \varrho_3, \varrho_5\}$,
$\aleph(\kappa_1, ep_2, 1) = \{\varrho_4, \varrho_5\}$,
$\aleph(\kappa_1, ep_2, 0) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_6\}$,
$\aleph(\kappa_2, ep_1, 1) = \{\varrho_2, \varrho_4, \varrho_5, \varrho_6\}$,
$\aleph(\kappa_2, ep_1, 0) = \{\varrho_1, \varrho_3\}$,
$\aleph(\kappa_2, ep_2, 1) = \{\varrho_3, \varrho_5\}$,
$\aleph(\kappa_2, ep_2, 0) = \{\varrho_1, \varrho_2, \varrho_4, \varrho_6\}$,
$\aleph(\kappa_3, ep_1, 1) = \{\varrho_1, \varrho_2, \varrho_6\}$,
$\aleph(\kappa_3, ep_1, 0) = \{\varrho_3, \varrho_4, \varrho_5\}$,
$\aleph(\kappa_3, ep_2, 1) = \{\varrho_1, \varrho_4, \varrho_6\}$,
$\aleph(\kappa_3, ep_2, 0) = \{\varrho_2, \varrho_3, \varrho_5\}$,
as in Table 1.

Table 1. Soft expert set

$(\aleph, \mathcal{V})$	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5	ϱ_6
$(\kappa_1, ep_1, 1)$	1	0	0	1	0	1
$(\kappa_1, ep_1, 0)$	0	1	1	0	1	0
$(\kappa_1, ep_2, 1)$	0	0	0	1	1	0
$(\kappa_1, ep_2, 0)$	1	1	1	0	0	1
$(\kappa_2, ep_1, 1)$	0	1	0	1	1	1
$(\kappa_2, ep_1, 0)$	1	0	1	0	0	0
$(\kappa_2, ep_2, 1)$	0	0	1	0	1	0
$(\kappa_2, ep_2, 0)$	1	1	0	1	0	1
$(\kappa_3, ep_1, 1)$	1	1	0	0	0	1
$(\kappa_3, ep_1, 0)$	0	0	1	1	1	0
$(\kappa_3, ep_2, 1)$	1	0	0	1	0	1
$(\kappa_3, ep_2, 0)$	0	1	1	0	1	0

Let $\xi \in F(U)$ is defined by $\xi(\varrho_1) = 0.1, \xi(\varrho_2) = 0.8, \xi(\varrho_3) = 0.9, \xi(\varrho_4) = 0.3, \xi(\varrho_5) = 0.7, \xi(\varrho_6) = 0.5$. Then the SER-approximations of ξ with respect to SE is

$$\begin{aligned}
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_1) &= \bigwedge\Big\{0.1 \wedge 0.3, 0.1 \wedge 0.5, 0.1 \wedge 0.8, 0.1 \wedge 0.5, 0.1 \wedge 0.3, 0.1 \wedge 0.5\Big\} \\
&= \bigwedge\Big\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\Big\} \\
&= 0.1 \\
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_2) &= \bigwedge\Big\{0.8 \wedge 0.3, 0.8 \wedge 0.7, 0.8 \wedge 0.5, 0.8 \wedge 0.1, 0.8 \wedge 0.5\Big\} \\
&= \bigwedge\Big\{0.3, 0.7, 0.5, 0.1, 0.5\Big\} \\
&= 0.1 \\
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_3) &= \bigwedge\Big\{0.9 \wedge 0.7\Big\} \\
&= \bigwedge\Big\{0.7\Big\} \\
&= 0.7 \\
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_4) &= \bigwedge\Big\{0.3 \wedge 0.1, 0.3 \wedge 0.5, 0.3 \wedge 0.7, 0.3 \wedge 0.8, 0.3 \wedge 0.7, 0.3 \wedge 0.5, \\
&\qquad 0.3 \wedge 0.1, 0.3 \wedge 0.5\Big\} \\
&= \bigwedge\Big\{0.1, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3\Big\} \\
&= 0.1 \\
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_5) &= \bigwedge\Big\{0.7 \wedge 0.3, 0.7 \wedge 0.8, 0.7 \wedge 0.3, 0.7 \wedge 0.5, 0.7 \wedge 0.9\Big\} \\
&= \bigwedge\Big\{0.3, 0.7, 0.3, 0.5, 0.7\Big\} \\
&= 0.3 \\
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_6) &= \bigwedge\Big\{0.5 \wedge 0.1, 0.5 \wedge 0.3, 0.5 \wedge 0.8, 0.5 \wedge 0.3, 0.5 \wedge 0.7, 0.5 \wedge 0.1, \\
&\qquad 0.5 \wedge 0.8, 0.5 \wedge 0.1, 0.5 \wedge 0.3\Big\} \\
&= \bigwedge\Big\{0.1, 0.3, 0.5, 0.3, 0.5, 0.1, 0.5, 0.1, 0.3\Big\} \\
&= 0.1 \\
\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_1) &= \bigvee\Big\{0.1 \vee 0.3, 0.1 \vee 0.5, 0.1 \vee 0.8, 0.1 \vee 0.5, 0.1 \vee 0.3, 0.1 \vee 0.5\Big\} \\
&= \bigvee\Big\{0.3, 0.5, 0.8, 0.5, 0.3, 0.5\Big\} \\
&= 0.8 \\
\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_2) &= \bigvee\Big\{0.8 \vee 0.3, 0.8 \vee 0.7, 0.8 \vee 0.5, 0.8 \vee 0.1, 0.8 \vee 0.5\Big\} \\
&= \bigvee\Big\{0.8, 0.8, 0.8, 0.8, 0.8\Big\} \\
&= 0.8 \\
\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_3) &= \bigvee\Big\{0.9 \vee 0.7\Big\} \\
&= \bigvee\Big\{0.9\Big\} \\
&= 0.9 \\
\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_4) &= \bigvee\Big\{0.3 \vee 0.1, 0.3 \vee 0.5, 0.3 \vee 0.7, 0.3 \vee 0.8, 0.3 \vee 0.7, 0.3 \vee 0.5, \\
&\qquad 0.3 \vee 0.1, 0.3 \vee 0.5\Big\} \\
&= \bigvee\Big\{0.3, 0.5, 0.7, 0.8, 0.7, 0.5, 0.3, 0.5\Big\} \\
&= 0.8 \\
\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_5) &= \bigvee\Big\{0.7 \vee 0.3, 0.7 \vee 0.8, 0.7 \vee 0.3, 0.7 \vee 0.5, 0.7 \vee 0.9\Big\} \\
&= \bigvee\Big\{0.7, 0.8, 0.7, 0.7, 0.9\Big\}
\end{aligned}$$

Since $\underline{\mathfrak{S}^p_1}_{SE}(\xi) \neq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$. Hence ξ is an agree soft expert rough fuzzy set.

$$\begin{aligned}
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_1) &= \bigwedge\Big\{0.1\wedge 0.8, 0.1\wedge 0.9, 0.1\wedge 0.5, 0.1\wedge 0.9, 0.1\wedge 0.8, 0.1\wedge 0.3\Big\}\\
&\quad\; 0.1\wedge 0.5\Big\}\\
&= \bigwedge\Big\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\Big\}\\
&= 0.1\\
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_2) &= \bigwedge\Big\{0.8\wedge 0.9, 0.8\wedge 0.7, 0.8\wedge 0.1, 0.8\wedge 0.9, 0.8\wedge 0.5, 0.8\wedge 0.1\Big\}\\
&\quad\; 0.8\wedge 0.3, 0.8\wedge 0.5, 0.8\wedge 0.9, 0.8\wedge 0.7\Big\}\\
&= \bigwedge\Big\{0.8, 0.7, 0.1, 0.8, 0.5, 0.1, 0.3, 0.5, 0.8, 0.7\Big\}\\
&= 0.1\\
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_3) &= \bigwedge\Big\{0.9\wedge 0.8, 0.9\wedge 0.7, 0.9\wedge 0.1, 0.9\wedge 0.8, 0.9\wedge 0.5, 0.9\wedge 0.1,\Big\}\\
&\quad\; 0.9\wedge 0.3, 0.9\wedge 0.7, 0.9\wedge 0.8, 0.9\wedge 0.7\Big\}\\
&= \bigwedge\Big\{0.8, 0.7, 0.1, 0.8, 0.5, 0.1, 0.3, 0.7, 0.8, 0.7\Big\}\\
&= 0.1\\
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_4) &= \bigwedge\Big\{0.3\wedge 0.1, 0.3\wedge 0.8, 0.3\wedge 0.5, 0.3\wedge 0.9, 0.3\wedge 0.7\Big\}\\
&= \bigwedge\Big\{0.1, 0.3, 0.3, 0.3, 0.3\Big\}\\
&= 0.1\\
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_5) &= \bigwedge\Big\{0.7\wedge 0.8, 0.7\wedge 0.9, 0.7\wedge 0.9, 0.7\wedge 0.3, 0.7\wedge 0.8, 0.7\wedge 0.9\Big\}\\
&= \bigwedge\Big\{0.7, 0.7, 0.7, 0.3, 0.7, 0.7\Big\}\\
&= 0.3\\
\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_6) &= \bigwedge\Big\{0.5\wedge 0.1, 0.5\wedge 0.8, 0.5\wedge 0.9, 0.5\wedge 0.1, 0.5\wedge 0.8, 0.5\wedge 0.3\Big\}\\
&= \bigwedge\Big\{0.1, 0.5, 0.5, 0.1, 0.5, 0.3\Big\}\\
&= 0.1\\
\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_1) &= \bigvee\Big\{0.1\vee 0.8, 0.1\vee 0.9, 0.1\vee 0.5, 0.1\vee 0.9, 0.1\vee 0.8, 0.1\vee 0.3\Big\}\\
&\quad\; 0.1\vee 0.5\Big\}\\
&= \bigvee\Big\{0.8, 0.9, 0.5, 0.9, 0.8, 0.3, 0.5\Big\}\\
&= 0.9\\
\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_2) &= \bigvee\Big\{0.8\vee 0.9, 0.8\vee 0.7, 0.8\vee 0.1, 0.8\vee 0.9, 0.8\vee 0.5, 0.8\vee 0.1\Big\}\\
&\quad\; 0.8\vee 0.3, 0.8\vee 0.5, 0.8\vee 0.9, 0.8\vee 0.7\Big\}\\
&= \bigvee\Big\{0.9, 0.8, 0.8, 0.9, 0.8, 0.8, 0.8, 0.8, 0.9, 0.8\Big\}\\
&= 0.9\\
\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_3) &= \bigvee\Big\{0.9\vee 0.8, 0.9\vee 0.7, 0.9\vee 0.1, 0.9\vee 0.8, 0.9\vee 0.5, 0.9\vee 0.1,\Big\}\\
&\quad\; 0.9\vee 0.3, 0.9\vee 0.7, 0.9\vee 0.8, 0.9\vee 0.7\Big\}\\
&= \bigvee\Big\{0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9\Big\}\\
&= 0.9\\
\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_4) &= \bigvee\Big\{0.3\vee 0.1, 0.3\vee 0.8, 0.3\vee 0.5, 0.3\vee 0.9, 0.3\vee 0.7\Big\}\\
&= \bigvee\Big\{0.3, 0.8, 0.5, 0.9, 0.7\Big\}
\end{aligned}$$

Since $\underline{\mathfrak{S}^p_0}_{SE}(\xi) \neq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$. Hence ξ is a disagree soft expert rough fuzzy set.

Theorem 3.3. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in \digamma(U)$, we have

(i) $\underline{\mathfrak{S}^p_1}_{SE}(\xi)(u) = \bigwedge\limits_{u \in \aleph(e_i, p_j, 1)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$,

(ii) $\overline{\mathfrak{S}^p_1}_{SE}(\xi)(u) = \bigvee\limits_{u \in \aleph(e_i, p_j, 1)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$.

(iii) $\underline{\mathfrak{S}^p_0}_{SE}(\xi)(u) = \bigwedge\limits_{u \in \aleph(e_i, p_j, 0)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 0)} \{\xi(\alpha)\}$,

(iv) $\overline{\mathfrak{S}^p_0}_{SE}(\xi)(u) = \bigvee\limits_{u \in \aleph(e_i, p_j, 0)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 0)} \{\xi(\alpha)\}$.

Proof: (i) Let $(e_i, p_j, 1) \in \mathcal{V}$, $u \in \aleph(e_i, p_j, 1)$, then for every $\alpha \in \aleph(e_i, p_j, 1)$, we have $\{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)$, so $\xi(\alpha) \geq \wedge\{\xi(\alpha) : \exists(e_i, p_j, 1) \in \mathcal{V}, \{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)\} \geq \wedge\{\xi(u) \wedge \xi(\alpha) : \exists(e_i, p_j, 1) \in \mathcal{V}, \{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)\} = \underline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$. Consequently $\bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\} \geq \underline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$ and so

$\bigwedge\limits_{u \in \aleph(e_i, p_j, 1)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\} \geq \underline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$.

Conversely, let $(e_i, p_j, 1) \in \mathcal{V}$ such that $\{u, w\} \subseteq \aleph(e_i, p_j, 1)$, it follows that both $\xi(u)$ and $\xi(w)$ are greater than or equal to $\bigwedge\limits_{u \in \aleph(e_i, p_j, 1)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$, so $\xi(u) \wedge \xi(w) \geq \bigwedge\limits_{u \in \aleph(e_i, p_j, 1)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$.
Hence $\underline{\mathfrak{S}^p_1}_{SE}(\xi)(u) \geq \bigwedge\limits_{u \in \aleph(e_i, p_j, 1)} \bigwedge\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$.

(ii) Let $(e_i, p_j, 1) \in \mathcal{V}$, $u \in \aleph(e_i, p_j, 1)$, then for every $\alpha \in \aleph(e_i, p_j, 1)$, we have $\{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)$, so $\xi(\alpha) \leq \vee\{\xi(\alpha) : \exists(e_i, p_j, 1) \in \mathcal{V}, \{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)\} \leq \vee\{\xi(u) \vee \xi(\alpha) : \exists(e_i, p_j, 1) \in \mathcal{V}, \{u, \alpha\} \subseteq \aleph(e_i, p_j, 1)\} = \overline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$. Consequently $\bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\} \leq \overline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$ and so

$\bigvee\limits_{u \in \aleph(e_i, p_j, 1)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\} \leq \overline{\mathfrak{S}^p_1}_{SE}(\xi)(u)$.

Conversely, let $(e_i, p_j, 1) \in \mathcal{V}$ such that $\{u, w\} \subseteq \aleph(e_i, p_j, 1)$, it follows that both $\xi(u)$ and $\xi(w)$ are less than or equal to $\bigvee\limits_{u \in \aleph(e_i, p_j, 1)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$, so $\xi(u) \vee \xi(w) \leq \bigvee\limits_{u \in \aleph(e_i, p_j, 1)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$.
Hence $\overline{\mathfrak{S}^p_1}_{SE}(\xi)(u) \leq \bigvee\limits_{u \in \aleph(e_i, p_j, 1)} \bigvee\limits_{\alpha \in \aleph(e_i, p_j, 1)} \{\xi(\alpha)\}$.

Proof of (iii) and (iv) is similar to proof (i) and (ii). □

Theorem 3.4. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi, \psi \in \digamma(U)$, we have

(i)$\underline{\mathfrak{S}^p_1}_{SE}(\emptyset) = \overline{\mathfrak{S}^p_1}_{SE}(\emptyset) = \underline{\mathfrak{S}^p_0}_{SE}(\emptyset) = \overline{\mathfrak{S}^p_0}_{SE}(\emptyset) = \emptyset$,

(ii) $\underline{\mathfrak{S}^p_1}_{SE}(U) = \overline{\mathfrak{S}^p_1}_{SE}(U) = \underline{\mathfrak{S}^p_0}_{SE}(U) = \overline{\mathfrak{S}^p_0}_{SE}(U) = U$,

(iii)If $\psi \subseteq \xi$, then $\underline{\mathfrak{S}^p_1}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_1}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^p_0}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^p_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^p_0}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$,

(iv) $\underline{\mathfrak{S}^p_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^p_1}_{SE}(\xi))^c$, $\overline{\mathfrak{S}^p_1}_{SE}(\xi^c) = (\underline{\mathfrak{S}^p_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^p_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^p_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^p_0}_{SE}(\xi))^c$,

(v) $\underline{\mathfrak{S}^p_1}_{SE}(\xi \cap \psi) = \underline{\mathfrak{S}^p_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^p_1}_{SE}(\psi)$,

(vi) $\underline{\mathfrak{S}^p_1}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^p_1}_{SE}(\xi) \cup \underline{\mathfrak{S}^p_1}_{SE}(\psi)$,

(vii) $\overline{\mathfrak{S}^p_1}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi) \cap \overline{\mathfrak{S}^p_1}_{SE}(\psi)$,

(viii) $\overline{\mathfrak{S}^p_1}_{SE}(\xi \cup \psi) = \overline{\mathfrak{S}^p_1}_{SE}(\xi) \cup \overline{\mathfrak{S}^p_1}_{SE}(\psi)$,

(ix) $\underline{\mathfrak{S}^p_0}_{SE}(\xi \cap \psi) = \underline{\mathfrak{S}^p_0}_{SE}(\xi) \cap \underline{\mathfrak{S}^p_0}_{SE}(\psi)$,

(x) $\underline{\mathfrak{S}^p_0}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^p_0}_{SE}(\xi) \cup \underline{\mathfrak{S}^p_0}_{SE}(\psi)$,

(xi) $\overline{\mathfrak{S}^p_0}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi) \cap \overline{\mathfrak{S}^p_0}_{SE}(\psi)$,

(xii)$\overline{\mathfrak{S}^p_0}_{SE}(\xi \cup \psi) = \overline{\mathfrak{S}^p_0}_{SE}(\xi) \cup \overline{\mathfrak{S}^p_0}_{SE}(\psi)$.

Proof: The proof of $(i), (ii)$ and (iii) are straight forward.
(iv) For all $\varrho \in U$, we have $(\underline{\mathfrak{S}^p_1}_{SE}(\xi))^c(\varrho) = 1 - \wedge\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \vee\{1 - (\xi(\varrho) \wedge \xi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \vee\{(1 - \xi(\varrho)) \vee (1 - \xi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \vee\{\xi^c(\varrho) \vee \xi^c(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \overline{\mathfrak{S}^p_1}_{SE}(\xi^c)(\varrho)$.

Hence $(\underline{\mathfrak{S}^p_1}_{SE}(\xi))^c = \overline{\mathfrak{S}^p_1}_{SE}(\xi^c)$.

By similar way one can prove $\underline{\mathfrak{S}^p_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^p_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^p_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^p_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^p_0}_{SE}(\xi))^c$.
(v) For all $\varrho \in U$, we have $\underline{\mathfrak{S}^p_1}_{SE}(\xi \cap \psi)(\varrho) = \wedge\{(\xi \cap \psi)(\varrho) \wedge (\xi \cap \psi)(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \wedge\{(\xi(\varrho) \wedge \psi(\varrho)) \wedge (\xi(\vartheta) \wedge \psi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \wedge\{(\xi(\varrho) \wedge \xi(\vartheta)) \wedge (\psi(\varrho) \wedge \psi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \wedge\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \wedge \wedge\{\psi(\varrho) \wedge \psi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} = \underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho) \wedge \underline{\mathfrak{S}^p_1}_{SE}(\psi)(\varrho)$.

Hence $\underline{\mathfrak{S}^p_1}_{SE}(\xi \cap \psi) = \underline{\mathfrak{S}^p_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^p_1}_{SE}(\psi)$.
The proof of $(vi) - (xii)$ are similar to the proof of (v). □

Definition 3.5. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in \digamma(U)$, the lower agree, upper agree, lower disagree and upper disagree SER-approximations of ξ with respect to SE are fuzzy sets in U, which are respectively denoted by $\underline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^q_0}_{SE}(\xi)$ given by

$$\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\alpha) = \bigvee_{\alpha \in \aleph(e_i, p_j, 1)} \bigwedge_{\beta \in \aleph(e_i, p_j, 1)} \{\xi(\beta)\},$$
$$\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\alpha) = \bigwedge_{\alpha \in \aleph(e_i, p_j, 1)} \bigvee_{\beta \in \aleph(e_i, p_j, 1)} \{\xi(\beta)\},$$
$$\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\alpha) = \bigvee_{\alpha \in \aleph(e_i, p_j, 0)} \bigwedge_{\beta \in \aleph(e_i, p_j, 0)} \{\xi(\beta)\},$$
$$\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\alpha) = \bigwedge_{\alpha \in \aleph(e_i, p_j, 0)} \bigvee_{\beta \in \aleph(e_i, p_j, 0)} \{\xi(\beta)\}, \text{ for all } \alpha \in U.$$

Then (i) ξ is said to be agree soft expert definable, if $\underline{\mathfrak{S}^q_1}_{SE}(\xi) = \overline{\mathfrak{S}^q_1}_{SE}(\xi)$; otherwise ξ is called an agree soft expert rough fuzzy set.

(ii) ξ is said to be disagree soft expert definable, if $\underline{\mathfrak{S}^q_0}_{SE}(\xi) = \overline{\mathfrak{S}^q_0}_{SE}(\xi)$; otherwise ξ is known as disagree soft expert rough fuzzy set.

Example 3.6. Let $\mathfrak{S} = (\aleph, \mathcal{V})$ be a SE-set with fuzzy set $\xi \in F(U)$ as in Example **??**. Then the SER-approximations of ξ with respect to SE is

$$\begin{aligned}
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_1) &= \vee\{0.1, 0.1, 0.1\} = 0.1,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_1) &= \wedge\{0.5, 0.8, 0.5\} = 0.5,\\
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_2) &= \vee\{0.3, 0.1\} = 0.3,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_2) &= \wedge\{0.8, 0.8\} = 0.8,\\
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_3) &= \vee\{0.9\} = 0.9,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_3) &= \wedge\{0.9\} = 0.9,\\
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_4) &= \vee\{0.1, 0.3, 0.3, 0.1\} = 0.3,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_4) &= \wedge\{0.5, 0.7, 0.8, 0.5\} = 0.5,\\
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_5) &= \vee\{0.3, 0.3, 0.7\} = 0.7,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_5) &= \wedge\{0.7, 0.8, 0.9\} = 0.7,\\
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_6) &= \vee\{0.1, 0.3, 0.1, 0.1\} = 0.3,\\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_6) &= \wedge\{0.5, 0.8, 0.8, 0.5\} = 0.5,
\end{aligned}$$

Since $\underline{\mathfrak{S}^q_1}_{SE}(\xi) \neq \overline{\mathfrak{S}^q_1}_{SE}(\xi)$. Hence ξ is an agree soft expert rough fuzzy set.

$$\begin{aligned}
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_1) &= \vee\{0.1, 0.1, 0.1\} = 0.1,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_1) &= \wedge\{0.9, 0.9, 0.8\} = 0.8,\\
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_2) &= \vee\{0.7, 0.1, 0.1, 0.7\} = 0.7,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_2) &= \wedge\{0.9, 0.9, 0.8, 0.9\} = 0.8,\\
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_3) &= \vee\{0.7, 0.1, 0.1, 0.3, 0.7\} = 0.7,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_3) &= \wedge\{0.9, 0.9, 0.9, 0.9, 0.9\} = 0.9,\\
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_4) &= \vee\{0.1, 0.3\} = 0.3,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_4) &= \wedge\{0.8, 0.9\} = 0.8,\\
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_5) &= \vee\{0.7, 0.3, 0.7\} = 0.7,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_5) &= \wedge\{0.9, 0.9, 0.9\} = 0.9,\\
\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_6) &= \vee\{0.1, 0.1\} = 0.1,\\
\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_6) &= \wedge\{0.9, 0.8\} = 0.8.
\end{aligned}$$

Since $\underline{\mathfrak{S}^q_0}_{SE}(\xi) \neq \overline{\mathfrak{S}^q_0}_{SE}(\xi)$. Hence ξ is a disagree soft expert rough fuzzy set.

Theorem 3.7. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi, \psi \in F(U)$, then

(q1)$\underline{\mathfrak{S}^q_1}_{SE}(\emptyset) = \overline{\mathfrak{S}^q_1}_{SE}(\emptyset) = \underline{\mathfrak{S}^q_0}_{SE}(\emptyset) = \overline{\mathfrak{S}^q_0}_{SE}(\emptyset) = \emptyset$,

(q2)$\underline{\mathfrak{S}^q_1}_{SE}(U) = \overline{\mathfrak{S}^q_1}_{SE}(U) = \underline{\mathfrak{S}^q_0}_{SE}(U) = \overline{\mathfrak{S}^q_0}_{SE}(U) = U$,

(q3)If $\psi \subseteq \xi$, then $\underline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi)$,

(q4) $\underline{\mathfrak{S}^q_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^q_1}_{SE}(\xi))^c$, $\overline{\mathfrak{S}^q_1}_{SE}(\xi^c) = (\underline{\mathfrak{S}^q_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^q_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^q_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^q_0}_{SE}(\xi))^c$,

(q5)$\underline{\mathfrak{S}^q_1}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^q_1}_{SE}(\psi)$,

(q6) $\underline{\mathfrak{S}^q_1}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi) \cup \underline{\mathfrak{S}^q_1}_{SE}(\psi)$, (q7) $\overline{\mathfrak{S}^q_1}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi) \cap \overline{\mathfrak{S}^q_1}_{SE}(\psi)$,

(q8) $\overline{\mathfrak{S}^q_1}_{SE}(\xi \cup \psi) \supseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi) \cup \overline{\mathfrak{S}^q_1}_{SE}(\psi)$,

(q9) $\underline{\mathfrak{S}^q_0}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi) \cap \underline{\mathfrak{S}^q_0}_{SE}(\psi)$,

(q10) $\underline{\mathfrak{S}^q_0}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi) \cup \underline{\mathfrak{S}^q_0}_{SE}(\psi)$,

(q11) $\overline{\mathfrak{S}^q_0}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi) \cap \overline{\mathfrak{S}^q_0}_{SE}(\psi)$,

(q12) $\overline{\mathfrak{S}^q_0}_{SE}(\xi \cup \psi) \supseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi) \cup \overline{\mathfrak{S}^q_0}_{SE}(\psi)$,

Proof: The proof of $(q1), (q2)$ and $(q3)$ are straight forward.
$(q4)$ For all $\alpha \in U$, we have

$$\begin{aligned}
(\underline{\mathfrak{S}^q_1}_{SE}(\xi))^c(\alpha) &= 1 - \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\xi(\beta)\} \\
&= \bigwedge_{\alpha \in \aleph(e_i,p_j,1)} (1 - \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\xi(\beta)\}) \\
&= \bigwedge_{\alpha \in \aleph(e_i,p_j,1)} \bigvee_{\beta \in \aleph(e_i,p_j,1)} \{1 - \xi(\beta)\} \\
&= \bigwedge_{\alpha \in \aleph(e_i,p_j,1)} \bigvee_{\beta \in \aleph(e_i,p_j,1)} \{\xi^c(\beta)\} \\
&= \overline{\mathfrak{S}^q_1}_{SE}(\xi^c)(\alpha).
\end{aligned}$$

Hence $(\underline{\mathfrak{S}^q_1}_{SE}(\xi))^c(\alpha) = \overline{\mathfrak{S}^q_1}_{SE}(\xi^c)(\alpha)$. By similar way one can prove $\underline{\mathfrak{S}^q_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^q_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^q_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^q_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^q_0}_{SE}(\xi))^c$,
$(q5)$ For all $\alpha \in U$, we have

$$\begin{aligned}
\underline{\mathfrak{S}^q_1}_{SE}(\xi \cap \psi)(\alpha) &= \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{(\xi \cap \psi)(\beta)\} \\
&= \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{(\xi(\beta) \wedge \psi(\beta))\} \\
&= \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \{ \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\xi(\beta)\} \wedge \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\psi(\beta)\}\} \\
&= \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\xi(\beta)\} \wedge \bigvee_{\alpha \in \aleph(e_i,p_j,1)} \bigwedge_{\beta \in \aleph(e_i,p_j,1)} \{\psi(\beta)\} \\
&\leq \underline{\mathfrak{S}^q_1}_{SE}(\xi)(\alpha) \wedge \underline{\mathfrak{S}^q_1}_{SE}(\psi)(\alpha).
\end{aligned}$$

Hence $\underline{\mathfrak{S}^q_1}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^q_1}_{SE}(\psi)$.
The proof of $(q6) - (q12)$ are similar to the proof of $(q5)$. □

Theorem 3.8. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in F(U)$, then $\underline{\mathfrak{S}^p_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$.

Proof: For all $\varrho \in U$,

$$\begin{aligned}
\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho) &= \bigwedge_{\varrho \in \aleph(e_i,p_j,1)} \bigwedge_{\vartheta \in \aleph(e_i,p_j,1)} \{\xi(\vartheta)\} \\
&\leq \bigvee_{\varrho \in \aleph(e_i,p_j,1)} \bigwedge_{\vartheta \in \aleph(e_i,p_j,1)} \{\xi(\vartheta)\} \\
&= \underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho) \text{ and} \\
\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho) &= \bigwedge_{\varrho \in \aleph(e_i,p_j,1)} \bigvee_{\vartheta \in \aleph(e_i,p_j,1)} \{\xi(\vartheta)\} \\
&\leq \bigvee_{\varrho \in \aleph(e_i,p_j,1)} \bigvee_{\vartheta \in \aleph(e_i,p_j,1)} \{\xi(\vartheta)\} \\
&= \overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho).
\end{aligned}$$

Hence $\underline{\mathfrak{S}^p_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi)$ and $\overline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$. Similarly $\underline{\mathfrak{S}^p_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$. $\square$

Corollary3.9. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\psi \in F(U)$, then $\underline{\mathfrak{S}^p_1}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \psi \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\psi)$ and $\underline{\mathfrak{S}^p_0}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \psi \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\psi)$.

Proof: Let $(e_i, p_j, 1) \in \mathcal{V}$ such that $u \in \aleph(e_i, p_j, 1)$, then $\bigwedge_{v \in \aleph(e_i,p_j,1)} \{\psi(v)\} \leq \psi(u)$, implies $\underline{\mathfrak{S}^q_1}_{SE}(\psi)(u) \leq \psi(u)$, so $\underline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \psi$. Similarly $\bigvee_{v \in \aleph(e_i,p_j,1)} \{\psi(v)\} \geq \psi(u)$, it follows that $\overline{\mathfrak{S}^q_1}_{SE}(\psi)(u) \geq \psi(u)$, so $\overline{\mathfrak{S}^q_1}_{SE}(\psi) \supseteq \psi$. Hence from Theorem **??** $\underline{\mathfrak{S}^p_1}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \psi \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\psi)$. Similarly $\underline{\mathfrak{S}^p_0}_{SE}(\psi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \psi \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\psi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\psi)$. $\square$

Definition 3.10. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in F(U)$, then the lower agree, upper agree, lower disagree and upper disagree SER-approximations of ξ with respect to SE are fuzzy sets in U, respectively denoted by $\underline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi)$, given by
$\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) = \vee\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\}$,
$\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) = \wedge\{\xi(\varrho) \vee \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\}$,
$\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho) = \vee\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 0) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 0)\}$,
$\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho) = \wedge\{\xi(\varrho) \vee \xi(\vartheta) | \exists (e_i, p_j, 0) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 0)\}$, for all $\varrho \in U$.
Then (i) ξ is said to be agree soft expert definable, if $\underline{\mathfrak{S}^r_1}_{SE}(\xi) = \overline{\mathfrak{S}^r_1}_{SE}(\xi)$; otherwise ξ is called an agree soft expert rough fuzzy set.

(ii) ξ is said to be disagree soft expert definable, if $\underline{\mathfrak{S}^r_0}_{SE}(\xi) = \overline{\mathfrak{S}^r_0}_{SE}(\xi)$; otherwise ξ is called a disagree soft expert rough fuzzy set.

Example 3.11. Let $\mathfrak{S} = (\aleph, \mathcal{V})$ be a SE-set with fuzzy set $\xi \in F(U)$ as in Example **??**. Then the SER-approximations of ξ with respect to SE is

$$
\begin{aligned}
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_1) &= \bigvee \Big\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\Big\} \\
&= 0.1 \\
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_2) &= \bigvee \Big\{0.3, 0.7, 0.5, 0.1, 0.5\Big\} \\
&= 0.7 \\
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_3) &= \bigvee \Big\{0.7\Big\} \\
&= 0.7 \\
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_4) &= \bigvee \Big\{0.1, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3\Big\} \\
&= 0.3 \\
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_5) &= \bigvee \Big\{0.3, 0.7, 0.3, 0.5, 0.7\Big\} \\
&= 0.7 \\
\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_6) &= \bigvee \Big\{0.1, 0.3, 0.5, 0.3, 0.5, 0.1, 0.5, 0.1, 0.3\Big\} \\
&= 0.5 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_1) &= \bigwedge \Big\{0.3, 0.5, 0.8, 0.5, 0.3, 0.5\Big\} \\
&= 0.3 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_2) &= \bigwedge \Big\{0.8, 0.8, 0.8, 0.8, 0.8\Big\} \\
&= 0.8 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_3) &= \bigwedge \Big\{0.9\Big\} \\
&= 0.9 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_4) &= \bigwedge \Big\{0.3, 0.5, 0.7, 0.8, 0.7, 0.5, 0.3, 0.5\Big\} \\
&= 0.3 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_5) &= \bigwedge \Big\{0.7, 0.8, 0.7, 0.7, 0.9\Big\} \\
&= 0.7 \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_6) &= \bigwedge \Big\{0.5, 0.5, 0.8, 0.5, 0.7, 0.5, 0.8, 0.5, 0.5\Big\} \\
&= 0.5
\end{aligned}
$$

Since $\underline{\mathfrak{S}^r_1}_{SE}(\xi) \neq \overline{\mathfrak{S}^r_1}_{SE}(\xi)$. Hence ξ is an agree soft expert rough fuzzy set.

$$\begin{aligned}
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_1) &= \bigvee\Big\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\Big\}\\
&= 0.1\\
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_2) &= \bigvee\Big\{0.8, 0.7, 0.1, 0.8, 0.5, 0.1, 0.3, 0.5, 0.8, 0.7\Big\}\\
&= 0.8\\
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_3) &= \bigvee\Big\{0.8, 0.7, 0.1, 0.8, 0.5, 0.1, 0.3, 0.7, 0.8, 0.7\Big\}\\
&= 0.8\\
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_4) &= \bigvee\Big\{0.1, 0.3, 0.3, 0.3, 0.3\Big\}\\
&= 0.3\\
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_5) &= \bigvee\Big\{0.7, 0.7, 0.7, 0.3, 0.7, 0.7\Big\}\\
&= 0.7\\
\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_6) &= \bigvee\Big\{0.1, 0.5, 0.5, 0.1, 0.5, 0.3\Big\}\\
&= 0.5\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_1) &= \bigwedge\Big\{0.8, 0.9, 0.5, 0.9, 0.8, 0.3, 0.5\Big\}\\
&= 0.3\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_2) &= \bigwedge\Big\{0.9, 0.8, 0.8, 0.9, 0.8, 0.8, 0.8, 0.8, 0.9, 0.8\Big\}\\
&= 0.8\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_3) &= \bigwedge\Big\{0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9\Big\}\\
&= 0.9\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_4) &= \bigwedge\Big\{0.3, 0.8, 0.5, 0.9, 0.7\Big\}\\
&= 0.3\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_5) &= \bigwedge\Big\{0.8, 0.9, 0.9, 0.7, 0.8, 0.9\Big\}\\
&= 0.7\\
\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_6) &= \bigwedge\Big\{0.5, 0.8, 0.9, 0.5, 0.8, 0.5\Big\}\\
&= 0.5
\end{aligned}$$

Since $\underline{\mathfrak{S}^r_0}_{SE}(\xi) \neq \overline{\mathfrak{S}^r_0}_{SE}(\xi)$. Hence ξ is a disagree soft expert rough fuzzy set.

Theorem 3.12. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi, \psi \in F(U)$, then

(r1) $\underline{\mathfrak{S}^r_1}_{SE}(\emptyset) = \overline{\mathfrak{S}^r_1}_{SE}(\emptyset) = \underline{\mathfrak{S}^r_0}_{SE}(\emptyset) = \overline{\mathfrak{S}^r_0}_{SE}(\emptyset) = \emptyset$,

(r2) $\underline{\mathfrak{S}^r_1}_{SE}(U) = \overline{\mathfrak{S}^r_1}_{SE}(U) = \underline{\mathfrak{S}^r_0}_{SE}(U) = \overline{\mathfrak{S}^r_0}_{SE}(U) = U$,

(r3) If $\xi \subseteq \psi$, then $\underline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\psi)$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^r_1}_{SE}(\psi)$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\psi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^r_0}_{SE}(\psi)$,

(r4) $\underline{\mathfrak{S}^r_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^r_1}_{SE}(\xi))^c$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi^c) = (\underline{\mathfrak{S}^r_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^r_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^r_0}_{SE}(\xi))^c$,

(r5) $\underline{\mathfrak{S}^r_1}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^r_1}_{SE}(\psi)$

(r6) $\underline{\mathfrak{S}^r_1}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi) \cup \underline{\mathfrak{S}^r_1}_{SE}(\psi)$

(r7) $\overline{\mathfrak{S}^r_1}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^r_1}_{SE}(\xi) \cap \overline{\mathfrak{S}^r_1}_{SE}(\psi)$

(r8) $\overline{\mathfrak{S}^r_1}_{SE}(\xi \cup \psi) \supseteq \overline{\mathfrak{S}^r_1}_{SE}(\xi) \cup \overline{\mathfrak{S}^r_1}_{SE}(\psi)$

(r9) $\underline{\mathfrak{S}^r_0}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi) \cap \underline{\mathfrak{S}^r_0}_{SE}(\psi)$

(r10) $\underline{\mathfrak{S}^r_0}_{SE}(\xi \cup \psi) \supseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi) \cup \underline{\mathfrak{S}^r_0}_{SE}(\psi)$

(r11) $\overline{\mathfrak{S}^r_0}_{SE}(\xi \cap \psi) \subseteq \overline{\mathfrak{S}^r_0}_{SE}(\xi) \cap \overline{\mathfrak{S}^r_0}_{SE}(\psi)$

(r12) $\overline{\mathfrak{S}^r_0}_{SE}(\xi \cup \psi) \supseteq \overline{\mathfrak{S}^r_0}_{SE}(\xi) \cup \overline{\mathfrak{S}^r_0}_{SE}(\psi)$

Proof: The proof of $(r1), (r2)$ and $(r3)$ are straight forward.
$(r4)$ For all $\varrho \in U$, we have

$$\begin{aligned}
(\underline{\mathfrak{S}^r_1}_{SE}(\xi))^c(\varrho) &= 1 - \vee\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \wedge\{1 - (\xi(\varrho) \wedge \xi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \wedge\{(1 - \xi(\varrho)) \vee (1 - \xi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \wedge\{\xi^c(\varrho) \vee \xi^c(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \overline{\mathfrak{S}^r_1}_{SE}(\xi^c)(\varrho).
\end{aligned}$$

Hence $(\underline{\mathfrak{S}^r_1}_{SE}(\xi))^c = \overline{\mathfrak{S}^r_1}_{SE}(\xi^c)$. By similar way one can prove $\underline{\mathfrak{S}^r_1}_{SE}(\xi^c) = (\overline{\mathfrak{S}^r_1}_{SE}(\xi))^c$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi^c) = (\overline{\mathfrak{S}^r_0}_{SE}(\xi))^c$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi^c) = (\underline{\mathfrak{S}^r_0}_{SE}(\xi))^c$,
$(r5)$ For all $\varrho \in U$, we have

$$\begin{aligned}
\underline{\mathfrak{S}^r_1}_{SE}(\xi \cap \psi)(\varrho) &= \vee\{(\xi \cap \psi)(\varrho) \wedge (\xi \cap \psi)(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \vee\{(\xi(\varrho) \wedge \psi(\varrho)) \wedge (\xi(\vartheta) \wedge \psi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \vee\{(\xi(\varrho) \wedge \xi(\vartheta)) \wedge (\psi(\varrho) \wedge \psi(\vartheta)) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&\leq \vee\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \wedge \\
&\qquad \vee \{\psi(\varrho) \wedge \psi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&\leq \underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) \wedge \underline{\mathfrak{S}^r_1}_{SE}(\psi)(\varrho).
\end{aligned}$$

Hence $\underline{\mathfrak{S}^r_1}_{SE}(\xi \cap \psi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi) \cap \underline{\mathfrak{S}^r_1}_{SE}(\psi)$.
The proof of $(r6) - (r12)$ are similar to the proof of $(r5)$. □

Theorem 3.13. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in F(U)$, then $\underline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi)$.

Proof: For all $\varrho \in U$,

$$\begin{aligned}
\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho) &= \bigvee_{\varrho \in \aleph(e_i, p_j, 1)} \bigwedge_{l \in \aleph(e_i, p_j, 1)} \{\xi(l)\} \\
&\leq \vee\{\xi(\varrho) \wedge \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&= \underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) \text{ and} \\
\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) &= \wedge\{\xi(\varrho) \vee \xi(\vartheta) | \exists (e_i, p_j, 1) \in \mathcal{V}, \{\varrho, \vartheta\} \subseteq \aleph(e_i, p_j, 1)\} \\
&\leq \bigwedge_{\varrho \in \aleph(e_i, p_j, 1)} \bigvee_{l \in \aleph(e_i, p_j, 1)} \{\xi(l)\} \\
&= \overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho).
\end{aligned}$$

Hence $\underline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi)$. Similarly $\underline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi)$. □

Corollary 3.14. For a given SE-set $\mathfrak{S} = (\aleph, \mathcal{V})$ over U with the SEA-space $SE = (U, \mathfrak{S})$ along with fuzzy set $\xi \in F(U)$, then $\underline{\mathfrak{S}^p_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \xi \subseteq \overline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$ and $\underline{\mathfrak{S}^p_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \xi \subseteq \overline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$.

Proof: Let $(e_i, p_j, 1) \in \mathcal{V}$ such that $\varrho \in \aleph(e_i, p_j, 1)$, then $\xi(\varrho) \wedge \xi(\vartheta) \leq \xi(\varrho)$, for all $\vartheta \in \aleph(e_i, p_j, 1)$, implies $\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) \leq \xi(\varrho)$, so $\underline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \xi$. Similarly $\xi(\varrho) \vee \xi(\vartheta) \geq \xi(\varrho)$, for all $\vartheta \in \aleph(e_i, p_j, 1)$, it follows that $\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho) \geq \xi(\varrho)$, so $\overline{\mathfrak{S}^r_1}_{SE}(\xi) \supseteq \xi$. Hence from Theorem **??** and **??** $\underline{\mathfrak{S}^p_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \xi \subseteq \overline{\mathfrak{S}^r_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_1}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_1}_{SE}(\xi)$. Similarly $\underline{\mathfrak{S}^p_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \underline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \xi \subseteq \overline{\mathfrak{S}^r_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^q_0}_{SE}(\xi) \subseteq \overline{\mathfrak{S}^p_0}_{SE}(\xi)$. □

4|Application of soft expert rough fuzzy set

In this section, an approach to decision-making situation based on the soft expert rough fuzzy set model is established.

The algorithm is as follows.
Algorithm
Step 1: Input the soft expert set $\mathfrak{S} = (\aleph, \mathcal{V})_U$ over the universe U and the fuzzy set $\xi \in F(U)$.
Step 2: Compute the SEA-space $SE = (U, \mathfrak{S})$, $\underline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_0}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_0}_{SE}(\xi)$, $\underline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi)$.
Step 3: Find $a_k = \sum_{i \in \{p,q,r\}} (\underline{\mathfrak{S}^i_1}_{SE}(\xi)(\varrho_k) + \overline{\mathfrak{S}^i_1}_{SE}(\xi)(\varrho_k))$.
Step 4: Find $d_k = \sum_{i \in \{p,q,r\}} (\underline{\mathfrak{S}^i_0}_{SE}(\xi)(\varrho_k) + \overline{\mathfrak{S}^i_0}_{SE}(\xi)(\varrho_k))$.
Step 5: Compute $s_k = a_k - d_k$.
Step 6: Find ω for which $s_\omega = \max s_k$.
Step 7: Conclusion ϱ_ω is the optimal choice. If ω has more than one value, then any one of them is can be chosen.

Decision-making problem using this algorithm.
Using this algorithm, we can find the best choice for the company to fill the vacancy for a position. To exhibit the novelty of the above algorithm we provide an example below.
Example 4.1. Problem statement
Suppose a company wants to recruit a person for one vacant position. The company short-listed four candidates (say $\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5$) and they have to select one person among them.

Step 1:Consider a set of parameters $E = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5\}$, where the parameters $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ represent the characteristics or qualities that the candidates are assessed on, namely "experience", "excellent", "attitude", "professionalism" and "technical knowledge" respectively. The hiring committee consists of the manager (ep_1), head of the department (ep_2) and director (ep_3) of the company, this committee is represented by $X = \{ep_1, ep_2, ep_3\}$ (a set of experts), the set of opinions of the hiring committee members is represented by a set $O = \{1 = agree, 0 = disagree\}$. To verify their certificates

TABLE 2. Tabular representation of soft expert set $(\aleph, \mathcal{V})_U$

$(\aleph, \mathcal{V})$	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
$(\kappa_1, ep_1, 1)$	1	0	0	1	0
$(\kappa_1, ep_1, 0)$	0	1	1	0	1
$(\kappa_1, ep_2, 1)$	0	1	1	0	1
$(\kappa_1, ep_2, 0)$	1	0	0	1	0
$(\kappa_1, ep_3, 1)$	0	0	0	1	1
$(\kappa_1, ep_3, 0)$	1	1	1	0	0
$(\kappa_2, ep_1, 1)$	1	0	1	1	0
$(\kappa_2, ep_1, 0)$	0	1	0	0	1
$(\kappa_2, ep_2, 1)$	0	1	1	1	1
$(\kappa_2, ep_2, 0)$	1	0	0	0	0
$(\kappa_2, ep_3, 1)$	1	0	0	1	1
$(\kappa_2, ep_3, 0)$	0	1	1	0	0
$(\kappa_3, ep_1, 1)$	1	1	0	1	0
$(\kappa_3, ep_1, 0)$	0	0	1	0	1
$(\kappa_3, ep_2, 1)$	1	0	1	0	1
$(\kappa_3, ep_2, 0)$	0	1	0	1	0
$(\kappa_3, ep_3, 1)$	1	0	0	0	1
$(\kappa_3, ep_3, 0)$	0	1	1	1	0
$(\kappa_4, ep_1, 1)$	1	0	0	1	1
$(\kappa_4, ep_1, 0)$	0	1	1	0	0
$(\kappa_4, ep_2, 1)$	0	1	0	1	1
$(\kappa_4, ep_2, 0)$	1	0	1	0	0
$(\kappa_4, ep_3, 1)$	0	1	0	1	1
$(\kappa_4, ep_3, 0)$	1	0	1	0	0
$(\kappa_5, ep_1, 1)$	1	1	0	1	0
$(\kappa_5, ep_1, 0)$	0	0	1	0	1
$(\kappa_5, ep_2, 1)$	1	1	1	0	1
$(\kappa_5, ep_2, 0)$	0	0	0	1	0
$(\kappa_5, ep_3, 1)$	0	0	1	1	1
$(\kappa_5, ep_3, 0)$	1	1	0	0	0

(1)

and other supporting documents, the hiring committee constructs the following SE-set $(\aleph, \mathcal{V})$ over U is as follows,

$$\begin{aligned}
(\aleph, \mathcal{V}) = \big\{&((\kappa_1, ep_1, 1), \{\varrho_1, \varrho_4\}), ((\kappa_1, ep_2, 1), \{\varrho_2, \varrho_3, \varrho_5\}),\\
&((\kappa_1, ep_3, 1), \{\varrho_4, \varrho_5\}), ((\kappa_2, ep_1, 1), \{\varrho_1, \varrho_3, \varrho_4\}),\\
&((\kappa_2, ep_2, 1), \{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}), ((\kappa_2, ep_3, 1), \{\varrho_1, \varrho_4, \varrho_5\}),\\
&((\kappa_3, ep_1, 1), \{\varrho_1, \varrho_2, \varrho_4\}), ((\kappa_3, ep_2, 1), \{\varrho_1, \varrho_3, \varrho_5\}),\\
&((\kappa_3, ep_3, 1), \{\varrho_1, \varrho_5\}), ((\kappa_4, ep_1, 1), \{\varrho_1, \varrho_4, \varrho_5\}),\\
&((\kappa_4, ep_2, 1), \{\varrho_2, \varrho_4, \varrho_5\}), ((\kappa_4, ep_3, 1), \{\varrho_2, \varrho_4, \varrho_5\}),\\
&((\kappa_5, ep_1, 1), \{\varrho_1, \varrho_2, \varrho_4\}), ((\kappa_5, ep_2, 1), \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}),\\
&((\kappa_5, ep_3, 1), \{\varrho_3, \varrho_4, \varrho_5\}), ((\kappa_1, ep_1, 0), \{\varrho_2, \varrho_3, \varrho_5\}),\\
&((\kappa_1, ep_2, 0), \{\varrho_1, \varrho_4\}), ((\kappa_1, ep_3, 0), \{\varrho_1, \varrho_2, \varrho_3\}),\\
&((\kappa_2, ep_1, 0), \{\varrho_2, \varrho_5\}), ((\kappa_2, ep_2, 0), \{\varrho_1\}),\\
&((\kappa_2, ep_3, 0), \{\varrho_2, \varrho_3\}), ((\kappa_3, ep_1, 0), \{\varrho_3, \varrho_5\}),\\
&((\kappa_3, ep_2, 0), \{\varrho_2, \varrho_4\}), ((\kappa_3, ep_3, 0), \{\varrho_2, \varrho_3, \varrho_4\}),\\
&((\kappa_4, ep_1, 0), \{\varrho_2, \varrho_3\}), ((\kappa_4, ep_2, 0), \{\varrho_1, \varrho_3\}),\\
&((\kappa_4, ep_3, 0), \{\varrho_1, \varrho_3\}), ((\kappa_5, ep_1, 0), \{\varrho_3, \varrho_5\}),\\
&((\kappa_5, ep_2, 0), \{\varrho_4\}), ((\kappa_5, ep_3, 0), \{\varrho_1, \varrho_2\})\big\}.
\end{aligned}$$

The fuzzy set $\xi \in F(U)$ is defined by $\xi(\varrho_1) = 0.9, \xi(\varrho_2) = 0.2, \xi(\varrho_3) = 0.3, \xi(\varrho_4) = 0.7, \xi(\varrho_5) = 0.8$.
Step 2: Compute the SEA-space $SE = (U, \mathfrak{S})$, $\underline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^p_0}_{SE}(\xi)$, $\overline{\mathfrak{S}^p_0}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^q_0}_{SE}(\xi)$, $\overline{\mathfrak{S}^q_0}_{SE}(\xi)$, $\underline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\overline{\mathfrak{S}^r_1}_{SE}(\xi)$, $\underline{\mathfrak{S}^r_0}_{SE}(\xi)$ and $\overline{\mathfrak{S}^r_0}_{SE}(\xi)$, as in Tables 3 and 4

Table 3. Agree soft expert rough approximations

U	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
$\underline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_k)$	0.2	0.2	0.2	0.2	0.2
$\underline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_k)$	0.8	0.2	0.2	0.7	0.8
$\underline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_k)$	0.8	0.2	0.3	0.7	0.8
$\overline{\mathfrak{S}^r_1}_{SE}(\xi)(\varrho_k)$	0.9	0.3	0.3	0.7	0.8
$\overline{\mathfrak{S}^q_1}_{SE}(\xi)(\varrho_k)$	0.9	0.8	0.8	0.8	0.8
$\overline{\mathfrak{S}^p_1}_{SE}(\xi)(\varrho_k)$	0.9	0.9	0.9	0.9	0.9
a_k	4.5	2.6	2.8	4.0	4.3

Step 3: Now $a_k = \sum_{i \in \{p,q,r\}} (\underline{\mathfrak{S}^i_1}_{SE}(\xi)(\varrho_k) + \overline{\mathfrak{S}^i_1}_{SE}(\xi)(\varrho_k))$ is calculated as in table 3.
$a_1 = 4.5$,
$a_2 = 2.6$,
$a_3 = 2.8$,
$a_4 = 4.0$,
$a_5 = 4.3$.

Table 4. Disagree soft expert rough approximations

U	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
$\underline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_k)$	0.2	0.2	0.2	0.2	0.2
$\underline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_k)$	0.7	0.2	0.3	0.7	0.3
$\underline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_k)$	0.7	0.2	0.3	0.7	0.3
$\overline{\mathfrak{S}^r_0}_{SE}(\xi)(\varrho_k)$	0.9	0.3	0.3	0.7	0.8
$\overline{\mathfrak{S}^q_0}_{SE}(\xi)(\varrho_k)$	0.9	0.3	0.3	0.7	0.8
$\overline{\mathfrak{S}^p_0}_{SE}(\xi)(\varrho_k)$	0.9	0.9	0.9	0.9	0.8
d_k	4.3	2.1	2.3	3.9	3.2

Step 4: Also $d_k = \sum_{i \in \{p,q,r\}} (\underline{\mathfrak{S}^i_0}_{SE}(\xi)(\varrho_k) + \overline{\mathfrak{S}^i_0}_{SE}(\xi)(\varrho_k))$ is calculated as in Table 4.
$d_1 = 4.3$,
$d_2 = 2.1$,
$d_3 = 2.3$,
$d_4 = 3.9$,
$d_5 = 3.2$.
Step 5: Computing $s_k = a_k - d_k$,
$s_1 = a_1 - d_1 = 4.5 - 4.3 = 0.2$,
$s_2 = a_2 - d_2 = 2.6 - 2.1 = 0.5$,

$s_3 = a_3 - d_3 = 2.8 - 2.3 = 0.5$,
$s_4 = a_4 - d_4 = 4.0 - 3.9 = 0.1$,
$s_5 = a_5 - d_5 = 4.2 - 3.2 = 1.1$.
Step 6: Since $\max s_k = s_5$, thus $\omega = 5$.
Step 7: Hence ϱ_5 is the optimal choice. So the company will choose the candidate ϱ_5 applied for the position.

5|Conclusion

Based on fuzzy set theory, SE-set theory and rough set theory models soft expert rough fuzzy set is introduced and its properties are derived. An interesting algorithm on soft expert rough fuzzy set is provided with illustrative example. As a further research we plan to extend the idea of soft expert rough fuzzy set to soft expert rough intuitionistic fuzzy set, soft expert rough interval-valued fuzzy set, soft expert rough interval-valued fuzzy matrix, etc.

Acknowledgments

The authors would like to express their sincere thanks to the anonymous referees for their valuable comments and corrections.

Author Contribution

Bijan Davvaz and Said Broumi: methodology, software, and editing. Srinivasan Vijayabalaji and Shanmugam Kalaiselvan: conceptualization, writing and editing. All authors have read and agreed to the published version of the manuscript.

Funding

The authors declare that no external funding or support was received for the research presented in this paper, including administrative, technical, or in-kind contributions. Data Availability All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings. Funders played no role in the study's design, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results.

References

[1] H. Aktaş & N. Çağman, Soft sets and soft groups, Information Sciences, 177(13) (2007), 2726–2735, DOI: https://doi.org/10.1016/j.ins.2006.12.008.

[2] S. Alkhazaleh & A. R. Salleh, Soft expert sets, Advances in Decision Sciences, 2011 (2011), 757868, DOI: https://doi.org/10.1155/2011/757868.

[3] S. Alkhazaleh & A. R. Salleh, Fuzzy soft expert set and its application, Applied Mathematics, 5 (2014), 1349–1368, DOI: DOI: 10.4236/am.2014.59127.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1) (1986), 87–96, DOI: https://doi.org/10.1016/S0165-0114(86)80034-3.

[5] D. Dubois & H. Prade, Rough fuzzy sets and fuzzy rough sets, International Journal of General Systems, 17(2-3) (1990), 191–209, DOI: https://doi.org/10.1080/03081079008935107.

[6] F. Feng, Y.B. Jun, X. Liu & L. Li, An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics, 234(1) (2010), 10–20, DOI: https://doi.org/10.1016/j.cam.2009.11.055.

[7] F. Feng, C. Li, B. Davvaz & M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing, 14(9) (2010), 899-911, DOI: https://doi.org/10.1007/s00500-009-0465-6.

[8] M.B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems, 21(1) (1987), 1–17, DOI: https://doi.org/10.1016/0165-0114(87)90148-5.

[9] Y. Jiang, Y. Tang & Q. Chen, An adjustable approach to intuitionistic fuzzy soft sets based decision making, Applied Mathematical Modelling, 35(2) (2011), 824–836, DOI: https://doi.org/10.1016/j.apm.2010.07.038

[10] S. Kalaiselvan & S. Vijayabalaji, Soft expert symmetric group and its application in MCDM problem, Symmetry, 14(12) (2022), 2685, DOI: https://doi.org/10.3390/sym14122685.

[11] P. Majumdar & S.K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications, 59(4) (2010), 1425-1432, DOI: https://doi.org/10.1016/j.camwa.2009.12.006.

[12] D. Meng, X. Zhang & K. Qin, Soft rough fuzzy sets and soft fuzzy rough sets, Computers and Mathematics with Applications, 62(12) (2011), 4635-4645, DOI: https://doi.org/10.1016/j.camwa.2011.10.049.

[13] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37(4-5) (1999), 19–31, DOI: https://doi.org/10.1016/S0898-1221(99)00056-5.

[14] J.N. Mordeson, K.R. Bhutani & A. Rosenfeld, Fuzzy group theory, Springer-Verlag Berlin Heidelberg, 2005.

[15] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences, 11(5) (1982), 341–356, DOI: https://doi.org/10.1007/BF01001956.

[16] Z. Pawlak, Rough sets: Theoretical aspects of reasoning about data, Kluwer Academic Publishers, 1991.

[17] Z. Pawlak & R. Sowinski, Rough set approach to multi-attribute decision analysis, European Journal of Operational Research, 72(3) (1994), 443–459, DOI: https://doi.org/10.1016/0377-2217(94)90415-4.

[18] A. Sezgin & A.O. Atagün, On operations of soft sets, Computers and Mathematics with Applications, 61 (2011), 1457-1467, DOI: https://doi.org/10.1016/j.camwa.2011.01.018.

[19] B. Sun & W. Ma, Soft fuzzy rough sets and its application in decision making, Artificial Intelligence Review, 41(1) (2014), 67-80, DOI: https://doi.org/10.1007/s10462-011-9298-7.

[20] S. Vijayabalaji, S. Kalaiselvan, N. Thillaigovindan & B. Davvaz, Modified soft-rough modules and their approximations, Journal of Multiple-Valued Logic and Soft Computing, 41(6) (2023), 509–535.

[21] G. Xiao, D. Xiang & J. Zhan, Fuzzy soft modules, East Asian Mathematical Journal, 28(1) (2012), 1–11.

[22] X.B. Yang, T.Y. Lin, J.Y. Yang, Y. Li & D.J. Yu, Combination of interval-valued fuzzy set and soft set, Computers and Mathematics with Applications, 58(3) (2009), 521–527, DOI: https://doi.org/10.1016/j.camwa.2009.04.019.

[23] L.A. Zadeh, Fuzzy sets, Information and Control, 8(3) (1965), 338–353, DOI: https://doi.org/10.1016/S0019-9958(65)90241-X.

[24] H. Zhang, L. Shu & S. Liao, Intuitionistic fuzzy soft rough set and its application in decision making, Abstract and Applied Analysis, 2014 (2014), 287314, DOI: https://doi.org/10.1155/2014/287314.

[25] X. Zhang & F. Zhu, Rough logic system RSL and fuzzy logic system Luk, Journal of the University of Electron Science and Technology of China, 40(2) (2011), 296–302.

[26] K.Y. Zhu & B.Q. Hu, A new study on soft rough fuzzy lattices (ideals, filters) over lattices, Journal of Intelligent and Fuzzy Systems, 33(4) (2017), 2391–2402, DOI: 10.3233/JIFS-17520.