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A Fuzzy Goal Programming Approach for Solving Multi-Objective Minimum Cost Flow Problems with Possibilistic Coefficients

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Abstract

This paper studies a multi-objective Minimum Cost Flow (MCF) with possibilistic objective function coefficients. A necessary and sufficient condition for investigating the α-possibly optimal solution is established. A fuzzy Goal Programming (GP) approach is applied to obtain the α-parametric optimal compromise solution. The parametric study under the concept of α-possibly optimal solution is analyzed without differentiability. Finally, a numerical example is given for the paper to clarify the methodology.

Keywords: Minimum cost flow, Multi-objective optimization, Possibilistic variables, Fuzzy goal programming approach, α-possibly optimal solution, Goal programming, Compromise solution, Parametric analysis.

1 | Introduction

Minimum Cost Flow (MCF) problems serve as a pivotal concept within the broader scope of network flow problems. The fundamental objective of an MCF problem is to optimize the transportation of commodities across a capacitated network, ensuring the cost associated with this flow is minimized. This intricate task involves orchestrating the movement of goods from suppliers at certain nodes to meet the demands at other specified nodes. The applicability of MCF extends far beyond its immediate definition, influencing a myriad of network-related challenges. Notably, MCF finds relevance in diverse problem domains such as maximum

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flow, assignment, shortest path determination, transportation logistics, transshipment quandaries, multi-stage production inventory planning, nurse scheduling, project assignment, mold allocation, college course assignment, and even the optimization of automobile routing [1]–[4]. A noteworthy contribution to the understanding and resolution of MCF problems comes from the work of Hu et al. [5]. They introduced an algorithm characterized by its adept handling of complementarity slackness at each iteration. Employing a dual approach, the algorithm updates node potentials iteratively, pinpointing augmenting paths and enhancing the efficiency of the solution-seeking process.

This intricate dance of nodes, capacities, and costs within the framework of MCF not only addresses the immediate challenge of cost-effective commodity flow but also lays the groundwork for addressing a spectrum of interconnected network problems. As we delve deeper into this work, we'll explore not only the nuances of MCF but also its broader implications and the innovative approaches researchers are taking to tackle its complexities. The journey begins with a thorough exploration of the background and significance of MCF in the realm of optimization and decision-making.

The exploration of multi-objective MCF problems has been a subject of significant inquiry within the academic landscape, as evidenced by the contributions of various scholars. Kumar and Kaur [6] and Hamacher et al. [7] stand among those who have delved into this intricate realm, paving the way for a deeper understanding of optimization challenges involving multiple objectives.

Bazaraa et al. [8] made strides in applying parametric analysis, particularly in the context of large-scale linear programming. The study by Steuer [9] took a unique approach by decomposing the parametric space of convex combination parametric programming, offering insights through thorough parametric analysis. Luhandjula [10] contributed to the field by examining multi-objective linear problems with coefficients represented by possibilistic data, adding a layer of complexity to the understanding of these optimization challenges.

The Multi-Objective Transportation Problem (MOTP) has also garnered attention from researchers. Lee and Moore [11] and Hemaida and Kwak [12] applied Goal Programming (GP) techniques to navigate the complexities of MOTP, seeking satisfactory solutions. However, Tamiz et al. [13] and Romero [14] critically discussed the limitations of GP, shedding light on areas where improvements and alternative approaches were needed. In pursuing robust solutions, many researchers turned to fuzzy programming approaches for solving MOTP. Li and La [15], Abd El-Wahed [16],Bit et al. [17], Chanas et al. [18], Ehrgott and Verma [19], Chalam [20] and Lai and Hwang [21] are among those who embraced the versatility of fuzzy programming, contributing to a growing body of knowledge in this domain. Adding a layer of novelty, Cui et al. [22] proposed a groundbreaking general MCF model. This model was designed not only to optimize the distribution pattern of evacuation flow and rescue flow on the same network but also to introduce the concept of conflict cost. This innovative approach extends the traditional understanding of MCF problems, addressing real-world scenarios with additional complexity.

This paper introduces the multi-objective MCF problem, incorporating variables with possibilistic characteristics. The parametric investigation associated with the α-possibly optimal solution is outlined and established without the requirement for differentiability.

The rest of the paper is outlined as follows.

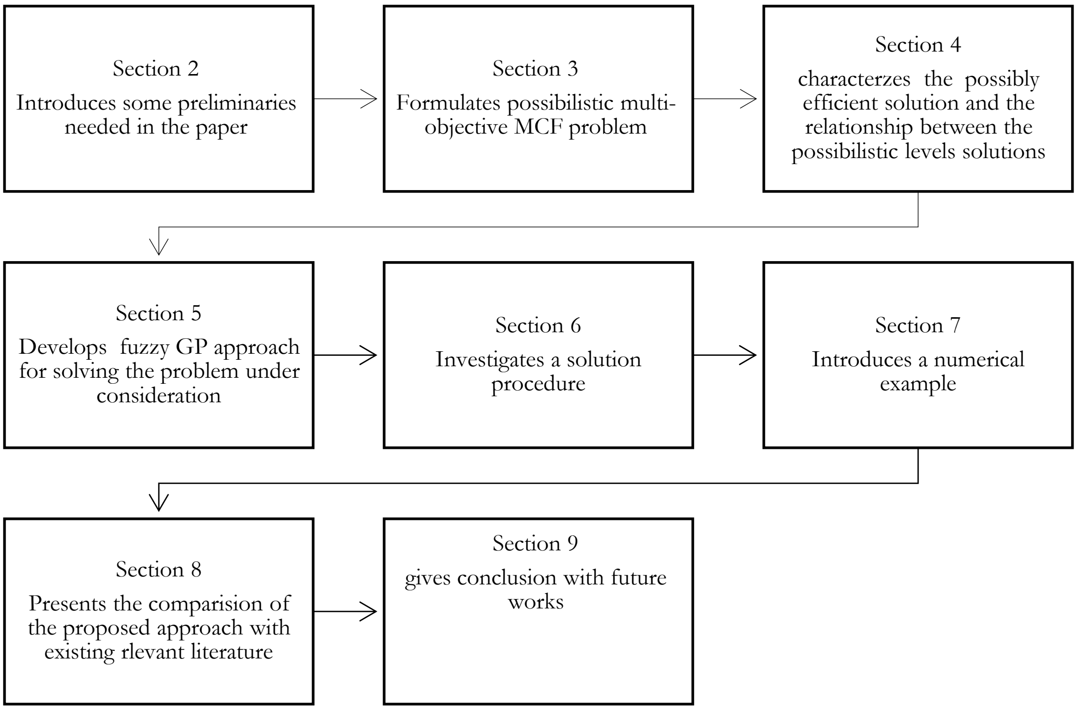

Fig. 1. Layout of remaining paper.

2 | Background

This section introduces fundamental concepts and outcomes associated with possibilistic variables, including their α-level set, possibilistic distribution, and the supporting elements.

Definition 1 ([23]**,** [24]**).** A possibilistic variable, denoted as u on V, is a variable characterized by a possibility distribution $\mu_u(v)$. This characterization implies that if u is a variable with values in V, then the associated μ_u corresponding to u can be conceptualized as a fuzzy constraint. The distribution is represented by a possibility distribution function $\mu_u: V \longrightarrow [0,1]$. For each $v \in V$, this function expresses the degree of compatibility of u with the realization $v \in V$. In the scenario where V is a Cartesian product of $V_1, V_2, \dots, V_n$, the n-ary possibility distribution is formulated as $\mu_u(v) = (\mu_{u_1}(v_1), \mu_{u_2}(v_2), \dots, \mu_{u_n}(v_n))$.

Definition 2. The α-level set of possibilistic variable u is

$$u_\alpha = \{v \in V: \mu_u(v) \geq \alpha\}.$$

Definition 3 ([23]**).** A possibility distribution μ_u on V is said to be convex if

$$\mu_u(\gamma v^1 + (1-\gamma)v^2) \geq \min\left(\mu_u(v^1), \mu_u(v^2)\right);\ \text{for all } v^1,\ v^2 \in V, \gamma \in [0,1].$$

Definition 4 ([23]**).** The support of a possibilistic variable u is

$$\text{Supp}\ (u) = \left\{v \in V: \sup_{v \in N_\delta(y)} \mu_u(v) > 0; \text{for all } \delta > 0\right\}, \text{where } N_\delta(y) = \{v \in V \in: \|v - y\| < \delta\}.$$

3 | Problem Statement and Solution Concepts

Consider the following multi-objective MCF problem with a possibilistic framework, as introduced by Shih and Lee in 1999 (Poss MOMCF):

$$\min \tilde{F}_r(x, \tilde{c}^r) = \sum_{(i,j) \in M} \tilde{c}^r_{ij}\, x_{ij}\,, r = 1, 2, \dots, K,$$

Subject to

$$x \in X = \left\{ \begin{array}{ll} \sum_{j:(i,j)\in M} x_{ij} - \sum_{k:(k,i)\in M} x_{li} = b(i); & \text{for all } i \in V, \\ x_{ij} \in U_{ij}; \text{for all } (i,j) \in M, x_{ij} \geq 0; & \text{for all } (i,j) \in M \end{array} \right\}.$$

Here,

M: the set of arcs (i, j).

V: the set of nodes.

x_{ij}: the decision variable representing the flow through the arc (i, j).

$U_{ij} = [l_{ij}, u_{ij}]$: capacity of arc (i, j).

$\tilde{c}^r_{ij}$: the possibilistic penalty per unit of flow through the arc (i, j) in the $\tilde{c}^r_{ij}$ objective function $r = 1, 2, \ldots, K$.

$b(i)$ denotes the net flow generated at node i, where positive, zero, or negative values classify node i as a supply node, transshipment node, or demand node, respectively.

It is important to note that the parameters $\tilde{c}^K_{ij}$ are vectors of possibilistic variables on $\mathbb{R}$, characterized by possibility distributions $\mu_{\tilde{c}^r_{ij}}$. It is assumed that all possibility distributions in the Poss MOMCF problem are convex cones with compact supports, denoted as $u_0 = \text{supp}(u)$.

Definition 5. $x^* \in G$ is an $\alpha -$ possibly efficient solution for Poss MOMCF if there is no $x \in G$ such that

$$\text{Poss}\left(\begin{array}{c} F_1(x, \tilde{c}^1) \leq F_1(\hat{x}, \tilde{c}^1), F_2(x, \tilde{c}^2) \leq F_2(\hat{x}, \tilde{c}^2), \\ \ldots, F_{r-1}(x, \tilde{c}^{r-1}) \leq F_{r-1}(\hat{x}, \tilde{c}^{r-1}), F_r(x, \tilde{c}^r) \leq F_r(\hat{x}, \tilde{c}^r), \\ F_{r+1}(x, \tilde{c}^{r+1}) \leq F_{r+1}(\hat{x}, \tilde{c}^{r+1}), \ldots, F_K(x, \tilde{c}^K) \leq F_K(\hat{x}, \tilde{c}^K) \end{array} \right) \geq \alpha. \quad (1)$$

Based on the extension principle, we have

$$\text{Poss}\left(\begin{array}{c} F_1(x, \tilde{c}^1) \leq F_1(\hat{x}, \tilde{c}^1), F_2(x, \tilde{c}^2) \leq F_2(\hat{x}, \tilde{c}^2), \\ \ldots, F_{r-1}(x, \tilde{c}^{r-1}) \leq F_{r-1}(\hat{x}, \tilde{c}^{r-1}), F_r(x, \tilde{c}^r) \leq F_r(\hat{x}, \tilde{c}^r), \\ F_{r+1}(x, \tilde{c}^{r+1}) \leq F_{r+1}(\hat{x}, \tilde{c}^{r+1}), \ldots, F_K(x, \tilde{c}^K) \leq F_K(\hat{x}, \tilde{c}^K) \end{array} \right)$$

$$= \underset{(c^1, c^2, \ldots, c^K)\in E}{\text{Sup}} \min\left(\begin{array}{c} \mu_{\tilde{c}^1}(c^1), \mu_{\tilde{c}^2}(c^2), \ldots, \mu_{\tilde{c}^{r-1}}(c^{r-1}), \\ \mu_{\tilde{c}^r}(c^r), \ \mu_{\tilde{c}^{r+1}}(c^{r+1}), \ldots, \ \mu_{\tilde{c}^K}(c^K) \end{array} \right), \quad (2)$$

where

$$E = \left\{ \begin{array}{c} (c^1, c^2, \ldots, c^K): F_1(x, \tilde{c}^1) \leq F_1(\hat{x}, \tilde{c}^1), F_2(x, \tilde{c}^2) \leq F_2(\hat{x}, \tilde{c}^2), \\ \ldots, F_{r-1}(x, \tilde{c}^{r-1}) \leq F_{r-1}(\hat{x}, \tilde{c}^{r-1}), F_r(x, \tilde{c}^r) \leq F_r(\hat{x}, \tilde{c}^r), \\ F_{r+1}(x, \tilde{c}^{r+1}) \leq F_{r+1}(\hat{x}, \tilde{c}^{r+1}), \ldots, F_K(x, \tilde{c}^K) \leq F_K(\hat{x}, \tilde{c}^K) \end{array} \right\}, \quad (3)$$

$\mu_{\tilde{c}^r}, r = 1, 2, \ldots, K$ are arcs $K(i, j)$ possibly distributions.

4 | Characterization of α-Possibly Efficient Solution for the Poss MOMCF Problem

For investigating the $\alpha -$possibly efficient solutions for the Poss MOMCF problem, let us consider the $\alpha -$ parametric multi-objective MCF problem.

(α-PMOMCF)

$$\min F_r(x, c^r) = \sum_{(i,j)\in M} c_{ij}^r x_{ij}, r = 1, 2, \dots, K,$$

Subject to

$$x \in X = \left\{ \begin{array}{ll} \sum_{j:(i,j)\in M} x_{ij} - \sum_{k:(k,i)\in M} x_{li} = b(i); & \text{for all } i \in V, \\ x_{ij} \in U_{ij}; \text{for all } (i,j) \in M, x_{ij} \geq 0; \text{for all } (i,j) \in M, c_{ij}^r \in \left(\tilde{c}_{ij}^r\right)_\alpha & \end{array} \right\}, \quad (4)$$

where, $\left(\tilde{c}_{ij}^r\right)_\alpha$ is the α −cut of the possibilistic variable c_{ij}^r. Based on the convexity assumption $\mu_{\tilde{c}_{ij}^r}\left(c_{ij}^r\right)$, (i,j) is arc, $r = \overline{1,K}$ are real intervals denoted by $\left[\left(c_{ij}^{r-}\right)_\alpha, \left(c_{ij}^{r+}\right)_\alpha\right]$. Let φ_α^r be the set of arcs (i,j) with $c_{ij}^r \in \left[\left(c_{ij}^{r-}\right)_\alpha, \left(c_{ij}^{r+}\right)_\alpha\right], r = \overline{1,K}$. The α −PMOMCF problem can be rewritten as

$$\min F_r(x, c^r) = \sum_{(i,j)\in M} c_{ij}^r x_{ij}, r = 1, 2, \dots, K, \quad (5)$$

Subject to

$$x \in X, \text{and } c^r \in \varphi_\alpha^r, r = \overline{1,K}.$$

Problem (5) can be rewritten as

$$\min F_r(x, c^r) = \sum_{(i,j)\in M} (c_{ij}^{r-}(\tau) + \tau c_{ij}^{r-}(\tau))x_{ij}, r = 1, 2, \dots, K, \quad (6)$$

Subject to

$$x \in X, \text{and } c^r \in \varphi_\alpha^r, r = \overline{1,K}, \qquad \tau \in [0,1].$$

Definition 6. $x^* \in G$ is an α −parametric efficient solution for α-PMOMCF problem if there is no $x \in G$ and $c^r \in \varphi_\alpha^r$ such that $F_r(x, c^r) \leq F_r(x^*, c^r)$; for all $r = \overline{1,K}$ and $F_r(x, c^r) < F_r(x^*, c^r)$ holds for at least one r.

Definition 7 ([25]**).** A feasible vector $Y^\circ \in X$ is said to be α −parametric compromise solution of α −PMOMCF if and only if $Y^\circ \in H$ and $F(Y) \leq \wedge_{Y\in X} F(Y)$, where $\wedge$ stands for the minimum, and H is the set of α − parametric efficient solutions.

Definition 8 ([26]**).** When the α-parametric compromise solution aligns with the preferences of decision-makers, it is termed an α-preferred parametric compromise solution.

Theorem 1. $x^* \in G$ is an α −possibly efficient solution for the Poss MOMCF problem if and only if α −parametric efficient solution for α-PMOMCF problem.

Proof: see [23].

5 | Fuzzy GP Approach for Solving Problem (5)

Based on the three concepts of fuzzy goals (G), fuzzy constraints (C), and fuzzy decision (D) introduced by Bellman and Zadeh [27], the fuzzy decision is defined as

$$D = C \cap G. \quad (7)$$

Then,

$$\mu_D(x) = \min\left(\mu_C(x), \mu_G(x)\right). \quad (8)$$

With the *Membership Function (8)*, let us describe the fuzzy goals for the problem under study. The linear Membership Function (MP) [28] is given by

$$\mu_r(F_r(x,c^r)) = \begin{cases} 0, & F_r(x,c^r) \le L_r, \\ \dfrac{U_r - F_r(x,c^r)}{U_r - L_r}, & L_r < F_r(x,c^r) < U_r, \\ 1, & F_r(x,c^r) \ge U_r, \end{cases} \quad (9)$$

where L_r, and U_r are the lower and upper bounds of $F_r(x,c^r)$, $L_r \neq U_r$, and can be calculated as

$$L_r = \min_x F_r(x,c^r),\ U_r = \max_x F_r(x,c^r), \quad r = 1,2,\dots,K. \quad (10)$$

By applying the *Fuzzy Decision (8)* and *Membership Function (9)*, α −PMOMCF can be rewritten as

$$\begin{aligned} &\max \min_{r=1,K} \left(\mu_r(F_r(x,c^r))\right), \\ &\text{Subject to} \\ &x \in X, \text{and } c^r \in \varphi_\alpha^r, r = \overline{1,K}. \end{aligned} \quad (11)$$

Problem (11) can be converted into well-defined linear programming using the auxiliary variable ϑ as

$$\begin{aligned} &\max \vartheta, \\ &\text{Subject to} \\ &\vartheta \le \mu_r(F_r(x,c^r)), r = \overline{1,K}, \\ &x \in X, \text{and } c^r \in \varphi_\alpha^r, r = \overline{1,K}. \end{aligned} \quad (12)$$

In order to formulate *Problem (12)* as a GP [29], we introduce the negative and positive deviational variables.

$$F_r(x,c^r) - v_r^+ + v_r^- = G_r, r = \overline{1,K}, \quad (13)$$

where G_r is the aspiration level of the objective function r. Now, *Problem (12)* is reformulated as a mixed integer GP problem as

$$\begin{aligned} &\max \vartheta, \\ &\text{Subject to} \\ &\vartheta \le \mu_r(F_r(x,c^r)), r = \overline{1,K}, \\ &x \in X, \text{and } c^r \in \varphi_\alpha^r, r = \overline{1,K}, \\ &F_r(x,c^r) - v_r^+ + v_r^- = G_r, \\ &v_r^-, v_r^+ \ge 0, r = \overline{1,K}, 0 \le \vartheta \le 1. \end{aligned} \quad (14)$$

6 | Solution Procedure

In this section, the solution procedure for addressing the Poss MOMCF problem is outlined through the following steps:

Step 1. Begin by considering the Poss MOMCF problem.

Step 2. Solve each one of the objective functions and continue this process K times. If all the resulting solutions are equal, select one of them and go to Step 5.

Step 3. Define each objective's MP and establish the aspiration level.

Step 4. Formulate *Problem (13)*, then solve it using computer packages like GAMS.

Step5. Conclude the process and determine the stability set of the first kind $S(x^\circ, c^\circ)$ as

$$\omega_{ij}\left(c_{ij}^r - \left(c_{ij}^{r+}\right)_\alpha\right) = 0; \text{ for all arc } (i,j), r = \overline{1,K},$$
$$\varphi_{ij}\left(\left(c_{ij}^{r-}\right)_\alpha - c_{ij}^r\right) = 0; \text{ for all arc } (i,j), r = \overline{1,K}.$$

7 | Numerical Example

Consider the following problem:

$$\min F_1(x, c^1) = \sum_{(i,j)\in M} \tilde{c}^1_{ij} x_{ij},$$

$$\min F_2(x, c^2) = \sum_{(i,j)\in M} \tilde{c}^2_{ij} x_{ij},$$

Subject to

$$x_{12} + x_{13} = 10,$$
$$x_{24} + x_{25} - x_{12} = 0,$$
$$x_{34} + x_{35} - x_{13} = 20, \quad (15)$$
$$x_{45} - x_{24} - x_{34} = -15,$$
$$-x_{25} - x_{35} - x_{45} = -15,$$
$$x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10],$$
$$x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50].$$

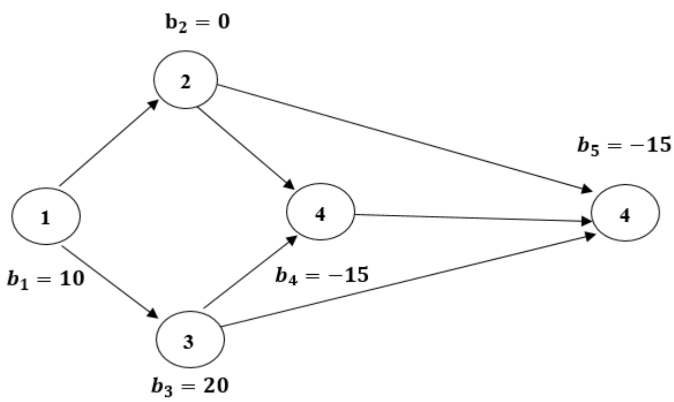

Fig. 2. A network with 5 nodes and 7 arcs [6].

The possibilistic variables $\tilde{c}^1_{ij}$, and $\tilde{c}^2_{ij}$ are represented by the possibility distributions $\mu_{\tilde{c}^1_{ij}}(.)$, and $\mu_{\tilde{c}^2_{ij}}(.)$ in *Figs. 1* and *2*. The supports of the possibilistic variables $\tilde{c}^1_{ij}$, and $\tilde{c}^2_{ij}$ are [3,12], and [2, 10]. Hence, for the parametric functions $0 \leq \tau \leq 1$, the supports are

$\text{Supp}\,(\tilde{c}^1_{12}) = 3 + 4\tau, \quad \mu_{\tilde{c}^1_{12}}(3) = \mu_{\tilde{c}^1_{12}}(7) = 0,$

$\text{Supp}\,(\tilde{c}^1_{13}) = 6 - 2\tau, \quad \mu_{\tilde{c}^1_{13}}(4) = \mu_{\tilde{c}^1_{13}}(6) = 0,$

$\text{Supp}\,(\tilde{c}^1_{24}) = 5 + 4\tau, \quad \mu_{\tilde{c}^1_{24}}(5) = \mu_{\tilde{c}^1_{24}}(9) = 0,$

$\text{Supp}\,(\tilde{c}^1_{25}) = 7 + \tau, \quad \mu_{\tilde{c}^1_{13}}(7) = \mu_{\tilde{c}^1_{13}}(8) = 0,$

$\text{Supp}\,(\tilde{c}^1_{34}) = 9 + 2\tau, \quad \mu_{\tilde{c}^1_{34}}(9) = \mu_{\tilde{c}^1_{34}}(11) = 0,$

$\text{Supp}\,(\tilde{c}^1_{35}) = 10 + 2\tau, \quad \mu_{\tilde{c}^1_{35}}(10) = \mu_{\tilde{c}^1_{35}}(12) = 0,$

$\text{Supp}\,(\tilde{c}^1_{45}) = 1 + \tau, \quad \mu_{\tilde{c}^1_{45}}(1) = \mu_{\tilde{c}^1_{45}}(2) = 0,$

$\text{Supp}\,(\tilde{c}^2_{12}) = 13 - 4\tau, \quad \mu_{\tilde{c}^2_{12}}(13) = \mu_{\tilde{c}^2_{12}}(9) = 0,$

$\text{Supp}\,(\tilde{c}^2_{13}) = 4 + 4\tau, \quad \mu_{\tilde{c}^2_{13}}(4) = \mu_{\tilde{c}^2_{13}}(8) = 0,$

$\text{Supp}\,(\tilde{c}^2_{24}) = 7 - 4\tau, \quad \mu_{\tilde{c}^2_{24}}(7) = \mu_{\tilde{c}^2_{24}}(3) = 0,$

$\text{Supp}\,(\tilde{c}^2_{25}) = 4 + 4\tau, \quad \mu_{\tilde{c}^2_{25}}(4) = \mu_{\tilde{c}^2_{25}}(8) = 0,$

$\text{Supp}\,(\tilde{c}^2_{34}) = 7 - 2\tau, \quad \mu_{\tilde{c}^2_{34}}(5) = \mu_{\tilde{c}^2_{34}}(7) = 0,$

$\text{Supp}\left(\tilde{c}_{35}^{2}\right) = 4 + 2\tau, \qquad \mu_{\tilde{c}_{35}^{2}}(4) = \mu_{\tilde{c}_{35}^{2}}(6) = 0,$
$\text{Supp}\left(\tilde{c}_{45}^{2}\right) = 10 - 4\tau, \qquad \mu_{\tilde{c}_{45}^{2}}(6) = \mu_{\tilde{c}_{45}^{2}}(10) = 0.$

$\text{Min } F_1(\tau) = (3 + 4\tau)x_{12} + (6 - 2\tau)x_{13} + (5 + 4\tau)x_{24} + (7 + \tau)x_{25} + (9 + 2\tau)x_{34}$
$+ (10 + 2\tau)x_{35} + (1 + \tau)x_{45},$
$\text{Min } F_2(\tau) = (13 - 4\tau)x_{12} + (4 + 4\tau)x_{13} + (7 - 4\tau)x_{24} + (4 + 4\tau)x_{25} + (7 - 2\tau)x_{34}$
$+ (4 + 2\tau)x_{35} + (10 - 4\tau)x_{45},$
Subject to
$x_{12} + x_{13} = 10,$
$x_{24} + x_{25} - x_{12} = 0,$ **(16)**
$x_{34} + x_{35} - x_{13} = 20,$
$x_{45} - x_{24} - x_{34} = -15,$
$-x_{25} - x_{35} - x_{45} = -15,$
$x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10],$
$x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50]$, and $\tau \in [0,1]$.
$L_1 = 265, \quad U_1 = 375, L_2 = 250, \quad U_1 = 425.$

At τ =0, the GP for the problem becomes

$\max \vartheta$
Subject to
⬚ $3x_{12} + 6x_{13} + 5x_{24} + 7x_{25} + 9x_{34} + 10x_{35} + x_{45} + \vartheta 110 \leq 375,$
⬚ $13x_{12} + 4x_{13} + 7x_{24} + 4x_{25} + 7x_{34} + 4x_{35} + 10x_{45} + \vartheta 175 \leq 425,$
$x_{12} + x_{13} = 10,$
$x_{24} + x_{25} - x_{12} = 0,$
$x_{34} + x_{35} - x_{13} = 20,$ **(17)**
$x_{45} - x_{24} - x_{34} = -15,$
$-x_{25} - x_{35} - x_{45} = -15,$
$x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10],$
$x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50],$
⬚ $3x_{12} + 6x_{13} + 5x_{24} + 7x_{25} + 9x_{34} + 10x_{35} + x_{45} - v_1^+ + v_1^- = 265,$
⬚ $13x_{12} + 4x_{13} + 7x_{24} + 4x_{25} + 7x_{34} + 4x_{35} + 10x_{45} - v_2^+ + v_2^- = 250,$
$v_1^+, v_1^-, v_2^+, v_2^- =$ and $\vartheta \in [0,1].$

Using the GINO software, the optimal compromise solution is

$\mathring{x}_{12} = \mathring{x}_{24} = 8.56,\ \mathring{x}_{13} = 1.44, \mathring{x}_{25} = \ \mathring{x}_{45} = 0,\ \mathring{x}_{34} = 6.44, \mathring{x}_{35} = 15, v_1^+ = 20.11,$
$v_1^- = v_2^- = 0, v_2^+ = 32,\ \vartheta^{\circ} = 0.82.$

To determine the stability set $S\left(\mathring{x}_{12}, \mathring{x}_{24}, \mathring{x}_{13}, \mathring{x}_{25},\ \mathring{x}_{45}, \mathring{x}_{34}, \mathring{x}_{35}, c_{ij}^{1^{\circ}}, c_{ij}^{2^{\circ}}\right)$,

$\omega_{12}^1(3 - (c_{12}^{1+})_0) = 0, \omega_{13}^1(6 - (c_{13}^{1+})_0) = 0, \omega_{24}^1(5 - (c_{24}^{1+})_0) = 0, \omega_{25}^1\left(7 - \left(c_{25}^{1+}\right)_0\right) = 0,$

$\omega_{34}^1(9 - (c_{34}^{1+})_0) = 0, \omega_{35}^1\left(10 - \left(c_{35}^{1+}\right)_0\right) = 0, \omega_{45}^1\left(1 - \left(c_{45}^{1+}\right)_0\right) = 0, \omega_{12}^2(13 - (c_{12}^{2+})_0) = 0,$

$\omega_{13}^2(4 - (c_{13}^{2+})_0) = 0, \omega_{24}^2(7 - (c_{24}^{2+})_0) = 0, \omega_{25}^2\left(4 - \left(c_{25}^{2+}\right)_0\right) = 0, \omega_{34}^2(7 - (c_{34}^{2+})_0) =$

$0, \omega_{35}^2\left(4 - \left(c_{35}^{2+}\right)_0\right) = 0, \omega_{45}^2\left(10 - \left(c_{45}^{2+}\right)_0\right) = 0$

$\omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1, \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 \geq 0.$

We get $I_1 \subseteq \{1, 2\}$.

For $I_1 = \emptyset, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1, \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 = 0$. Then

$$S_{I_1}\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right)$$
$$=\left\{\begin{array}{l} c_{ij}^{r}: (c_{12}^{1+})_0 \geq 3, (c_{13}^{1+})_0 \geq 6, (c_{24}^{1+})_0 \geq 5, \\ \left(c_{25}^{1+}\right)_0 \geq 7, (c_{34}^{1+})_0 \geq 9, \left(c_{35}^{1+}\right)_0 \geq 10, \left(c_{45}^{1+}\right)_0 \geq 1, (c_{12}^{2+})_0 \geq 13, \\ (c_{13}^{2+})_0 \geq 4, (c_{24}^{2+})_0 \geq 7, \left(c_{25}^{2+}\right)_0 \geq 4, (c_{34}^{2+})_0 \geq 7, \left(c_{35}^{2+}\right)_0 \geq 4, \left(c_{45}^{2+}\right)_0 \geq 10 \end{array}\right\}.$$

For $I_2 = \{1\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 > 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 = 0$. Then

$$S_{I_2}\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right)$$
$$=\left\{\begin{array}{l} c_{ij}^{r}: (c_{12}^{1+})_0 = 3, (c_{13}^{1+})_0 = 6, (c_{24}^{1+})_0 = 5, \\ \left(c_{25}^{1+}\right)_0 = 7, (c_{34}^{1+})_0 = 9, \left(c_{35}^{1+}\right)_0 = 10, \left(c_{45}^{1+}\right)_0 = 1, (c_{12}^{2+})_0 \geq 13, \\ (c_{13}^{2+})_0 \geq 4, (c_{24}^{2+})_0 \geq 7, \left(c_{25}^{2+}\right)_0 \geq 4, (c_{34}^{2+})_0 \geq 7, \left(c_{35}^{2+}\right)_0 \geq 4, \left(c_{45}^{2+}\right)_0 \geq 10 \end{array}\right\}.$$

For $I_3 = \{2\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 = 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 > 0$. Then

$$S_{I_3}\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right)$$
$$=\left\{\begin{array}{l} c_{ij}^{r}: (c_{12}^{1+})_0 \geq 3, (c_{13}^{1+})_0 \geq 6, (c_{24}^{1+})_0 \geq 5, \\ \left(c_{25}^{1+}\right)_0 \geq 7, (c_{34}^{1+})_0 \geq 9, \left(c_{35}^{1+}\right)_0 \geq 10, \left(c_{45}^{1+}\right)_0 \geq 1, (c_{12}^{2+})_0 = 13, \\ (c_{13}^{2+})_0 = 4, (c_{24}^{2+})_0 = 7, \left(c_{25}^{2+}\right)_0 = 4, (c_{34}^{2+})_0 = 7, \left(c_{35}^{2+}\right)_0 = 4, \left(c_{45}^{2+}\right)_0 = 10 \end{array}\right\}.$$

For $I_4 = \{1,2\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 > 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 > 0$. Then

$$S_{I_4}\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right) =$$
$$\left\{\begin{array}{l} c_{ij}^{r}: (c_{12}^{1+})_0 = 3, (c_{13}^{1+})_0 = 6, (c_{24}^{1+})_0 = 5, \\ \left(c_{25}^{1+}\right)_0 = 7, (c_{34}^{1+})_0 = 9, \left(c_{35}^{1+}\right)_0 = 10, \left(c_{45}^{1+}\right)_0 = 1, (c_{12}^{2+})_0 = 13, \\ (c_{13}^{2+})_0 = 4, (c_{24}^{2+})_0 = 7, \left(c_{25}^{2+}\right)_0 = 4, (c_{34}^{2+})_0 = 7, \left(c_{35}^{2+}\right)_0 = 4, \left(c_{45}^{2+}\right)_0 = 10 \end{array}\right\}.$$

Hence

$$S\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right) =$$
$$\cup_{P=1}^{4} S_{I_p}\left(\mathring{x}_{12},\mathring{x}_{24},\mathring{x}_{13},\mathring{x}_{25},\ \mathring{x}_{45}\ ,\mathring{x}_{34},\mathring{x}_{35},c_{ij}^{1^\circ},c_{ij}^{2^\circ}\right).$$

8 | Discussion

This section compares the proposed approach with some existing literature to illustrate its advantages. *Table 1* investigates this comparison in the case of some parameters.

Table 1. Comparisons of different researcher's contributions.

Author's Name	α −Efficient Solution	α −Parametric Compromise Solution	Fuzzy GP	Stability Set of the First Kind	Environment
Ghatee and Hashemi [30]	×	×	×	×	Fuzzy
Bustos et al. [31]	×	×	×	×	Stochastic
Alharbi et al. [32]	√	√	√	√	Fuzzy
Proposed approach	√	√	√	√	Possibilistic

9 | Conclusion

In this study, we introduced a multi-objective MCF problem featuring possibilistic variables. To address this problem, a fuzzy GP approach was employed, offering the advantage of accommodating conflicting goals and facilitating consideration of the decision environment. The GAMS software was utilized to obtain the solution, providing a robust computational framework. The parametric study associated with the α-possibly optimal solution was defined and determined without requiring differentiability. Looking ahead, future endeavors may involve expanding this investigation to include other fuzzy-like structures, such as interval-valued fuzzy sets, Neutrosophic sets, Pythagorean fuzzy sets, Spherical fuzzy sets, etc. This extension could benefit from additional in-depth discussions and insightful comments, contributing to a more comprehensive understanding of the problem landscape.

Author Contribution

R. S. K. research design, conceptualization, and validation. H. W. Kh. data gathering, computing, and editing. R. S. K Methodology, visualization and formal analysis. The authors have read and agreed to the published version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

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