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Intuitionistic Fuzzy Soft Expert Graphs with Application

Vakkas Uluçay[1,*] , Memet Şahin[2]

[1] Department of Mathematics, Kilis 7 Aralık University, Kilis-Turkey; vakkas.ulucay@kilis.edu.tr.

[2] Department of Mathematics, Gaziantep University, Gaziantep-Turkey; mesahin@gantep.edu.tr.

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Abstract

Many fields deal with uncertain data. Classical mathematical tools are unable to solve uncertain data in many situations. There are several theories, viz., the theory of probability, fuzzy set, and intuitionistic fuzzy set, for dealing with uncertainties, but they have their own difficulties. The reason for the difficulties is the inadequacy of the parameterization tools of the theories. The parameterization tools of soft set theory enhance the flexibility of its application. This paper introduces the concept of intuitionistic fuzzy soft expert graph, union, and the intersection of intuitionistic fuzzy soft expert graph. Finally, the new concept is the intuitionistic fuzzy soft expert graph-based multi-criteria decision-making method.

Keywords: Graph, soft expert set, Intuitionistic fuzzy soft set, Intuitionistic fuzzy soft expert set, Intuitionistic fuzzy soft expert graph.

1 | Introduction

The concept of fuzzy set theory was introduced by Zadeh [1] to solve difficulties in dealing with uncertainties. Since then, the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real-life problems involving ambiguous and uncertain environments. Atanassov [2] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh's [1] fuzzy set. Molodtsov [3] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Molodtsov's [3] soft sets give us a new technique for dealing with uncertainty from the viewpoint of parameters. Maji et al. [4] proposed soft sets. Alkhazaleh and Salleh [5], [6] defined the concept of a soft expert set. Graph theory has now become a major branch of applied mathematics and is generally regarded as a branch of combinatorics. The graph is a widely used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization, and computer science. When the relations between nodes (or vertices) in

Corresponding Author: vakkas.ulucay@kilis.edu.tr

problems are indeterminate, the fuzzy graphs and their extensions [7]–[13]. The above theories have been applied to many areas, including real decision-making problems [14]-[32].

Using examples to make the concept easier, we have discussed different operations defined on intuitionistic fuzzy soft expert graphs. Isotonic fuzzy soft expert graphs are pictorial representations in which each vertex and edge is an element of intuitionistic fuzzy soft sets. This paper has been arranged as the following.

Section 2 presents some basic concepts about graphs and fuzzy soft sets, which will be employed in later sections. Section 3 presents the concept of intuitionistic fuzzy soft expert graphs, and some of their fundamental properties have been studied. In Section 4, we present an application of intuitionistic fuzzy soft expert graphs in decision-making, and then an illustrative example is given. Finally, the conclusions are drawn in Section 5.

2 | Preliminaries

Definition 1 ([33]). A fuzzy graph is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V, and μ is a symmetric fuzzy relation onσ. i.e. $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(uv) \le \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. $\sigma$is called the fuzzy vertex set of V, and μ is called the fuzzy edge set of E.

Definition 2 ([33]). The fuzzy subgraph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma, \mu)$, if $\tau(u) \le \sigma(u)$ for all $u \in V$ and $\rho(u.v) \le \mu(u, v)$ for all $u, v \in V$.

Definition 3 ([34]). An intuitionistic fuzzy graph is of the form $G = (V, E)$ where

I. $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, (i = 1,2, \dots, n)$,

II. $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \ge max[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E, (i, j = 1,2, \dots, n)$.

3 | Intuitionistic Fuzzy Soft Expert Graphs

Let $\mathcal{V}$ be a universe, $\mathcal{Y}$ a set of parameters, $\mathcal{X}$ a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = \mathcal{Y} \times \mathcal{X} \times O$ and $A \subseteq Z$.

Definition 4. Let $G^* = (\mathcal{V}, \mathbb{E})$ be a simple graph, and A be the set of parameters. Let $IFSE(\mathcal{V})$ be the set of all intuitionistic sets in $\mathcal{V}$. By an intuitionistic fuzzy soft expert graph *IFSEG*, we mean a 4-tuple $G = (G^*, A, f, g)$ where $f: A \to IFSE(\mathcal{V})$, $g: A \to IFSE(\mathcal{V} \times \mathcal{V})$ defined as $f(\alpha) = f_\alpha = \{\langle x, \mu_{f_\alpha}(x), \vartheta_{f_\alpha}(x)\rangle : x \in \mathcal{V}\}$ and $g(\alpha) = g_\alpha = \{\langle (x, y), \mu_{f_\alpha}(x, y), \vartheta_{f_\alpha}(x, y)\rangle : (x, y) \in \mathcal{V} \times \mathcal{V}\}$ are intuitionistic fuzzy soft sets over $\mathcal{V}$ and $\mathcal{V} \times \mathcal{V}$, respectively, such that

$$\mu_{g_\alpha}(x, y) \le \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\},$$
$$\vartheta_{g_\alpha}(x, y) \le \min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\},$$

For all $(x, y) \in \mathcal{V} \times \mathcal{V}$ and $\alpha \in A$. We can also denote an *IFSEG* by $G = (G^*, A, f, g) = \{IFSE(\alpha): \alpha \in A\}$ which is a parameterized family of graphs $IFSE(\alpha)$ we call them intuitionistic fuzzy soft expert graphs.

Example 1. Suppose that $G^* = (\mathcal{V}, \mathbb{E})$ be a simple graph with $\mathcal{V} = \{x_1, x_2, x_3\}, \mathcal{Y} = \{e_1, e_2, e_3\}$ be a set of parameters and $\mathcal{X} = \{p\}$ be a set of experts. An IFSEG is given in *Table 1* below and $\mu_{g_\alpha}(x_i, x_j) = 0$ and $\vartheta_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$.

Table 1. Intuitionistic fuzzy soft expert graph.

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.5,0.5)	(0,1)	(0.3,0.7)
$(e_2, p, 1)$	(0.4,0.6)	(0.1,0.9)	(0.2,0.8)
$(e_3, p, 1)$	(0.6,0.4)	(0.8,0.2)	(0.7,0.3)
$(e_1, p, 0)$	(0.9,0.1)	(0.6,0.4)	(0.5,0.5)
$(e_2, p, 0)$	(0.3,0.7)	(0.5,0.5)	(0.2,0.8)
$(e_3, p, 0)$	(0.2,0.8)	(0.3,0.7)	(0.4,0.6)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,1)	(0,1)	(0.2,0.8)
$(e_2, p, 1)$	(0.1,0.9)	(0.5,0.5)	(0.9,0.1)
$(e_3, p, 1)$	(0.4,0.6)	(1,0)	(0.7,0.3)
$(e_1, p, 0)$	(0.6,0.4)	(0,1)	(0.6,0.4)
$(e_2, p, 0)$	(0.3,0.7)	(0.1,0.9)	(0.3,0.7)
$(e_3, p, 0)$	(0.2,0.8)	(0.8,0.2)	(1,0)

Definition 5. An IFSEG $G = (G^*, A^1, f^1, g^1)$ is called an IFSE subgraph of $G = (G^*, A, f, g)$ if

I. $A^1 \subseteq A$

II. ${f_\alpha}^1 \subseteq f$, that is, $\mu_{{f^1}_\alpha}(x) \leq \mu_{f_\alpha}(x), \vartheta_{{f^1}_\alpha}(x) \leq \vartheta_{f_\alpha}(x)$.

III. ${g_\alpha}^1 \subseteq g$, that is, $\mu_{{f^1}_\alpha}(x, y) \leq \mu_{f_\alpha}(x, y), \vartheta_{{f^1}_\alpha}(x, y) \leq \vartheta_{f_\alpha}(x, y)$; for all $\alpha \in A^1$.

Example 2. Suppose that $G^* = (\mathcal{V}, \mathbb{E})$ be a simple graph with $\mathcal{V} = \{x_1, x_2, x_3\}, \mathcal{Y} = \{e_1\}$ be a set of parameters and $\mathcal{X} = \{p\}$ be a set of experts. An IFSE subgraph of *Example 1* is given in *Table 2* below and $\mu_{g_\alpha}(x_i, x_j) = 0$ and $\vartheta_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in \mathcal{V} \times \mathcal{V} \backslash \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$.

Table 2. IFSE subgraph.

f^1	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.7)	(0,1)	(0.3,0.7)
$(e_1, p, 0)$	(0.2,0.8)	(0.1,0.9)	(0.2,0.8)
g^1	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,1)	(0,1)	(0.2,0.8)
$(e_1, p, 0)$	(0.1,0.9)	(0.5,0.5)	(0.6,0.4)

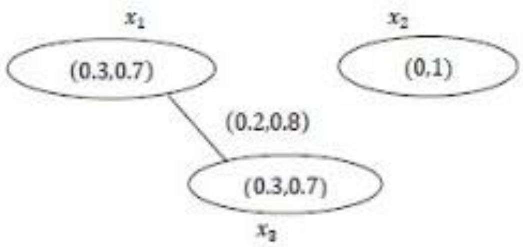

Fig. 1. $IFSE(e_1, p, 1)$ **corresponding to** $(e_1, p, 1)$.

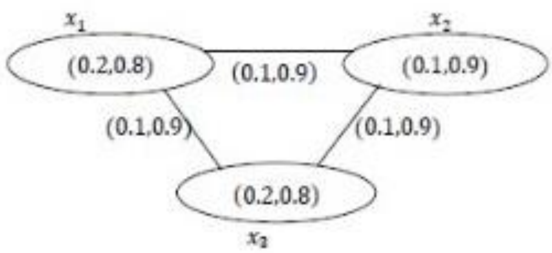

Fig. 2. IFSE$(e_1, p, 0)$ corresponding to $(e_1, p, 0)$.

Definition 6. An IFSE subgraph $G = (G^*, A^1, f^1, g^1)$ is said to be spanning IFSE subgraph of $G = (G^*, A, f, g)$ if ${f_\alpha}^1(x) = f(x)$; for all $x \in \mathcal{V}, \alpha \in A^1$.

Definition 7. An agree-IFSE graph $G_1 = (G^*, A, f_1, g_1)$ over $G^* = (\mathcal{V}, \mathbb{E})$ is an IFSE subgraph of $G = (G^*, A, f, g)$ defined as follows:

$$G_1 = (G^*, A, f_1, g_1) = \{f_1(\alpha), g_1(\alpha): \alpha \in \mathbb{E} \times \mathcal{X} \times \{1\}\}.$$

Example 3. Consider *Example 1*, then, the agree-IFSE graph $G_1 = (G^*, A, f_1, g_1)$ over $G^* = (\mathcal{V}, \mathbb{E})$.

Table 3. The agree-IFSEG.

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.5,0.5)	(0,1)	(0.3,0.7)
$(e_2, p, 1)$	(0.4,0.6)	(0.1,0.9)	(0.2,0.8)
$(e_3, p, 1)$	(0.6,0.4)	(0.8,0.2)	(0.7,0.3)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,1)	(0,1)	(0.2,0.8)
$(e_2, p, 1)$	(0.1,0.9)	(0.5,0.5)	(0.9,0.1)
$(e_3, p, 1)$	(0.4,0.6)	(1,0)	(0.7,0.3)

Definition 8. An disagree-IFSEG $G_0 = (G^*, A, f_0, g_0)$ over $G^* = (\mathcal{V}, \mathbb{E})$ is an IFSE subgraph of $G = (G^*, A, f, g)$ defined as follows:

$$G_0 = (G^*, A, f_0, g_0) = \{f_0(\alpha), g_0(\alpha): \alpha \in \mathbb{E} \times \mathcal{X} \times \{0\}\}.$$

Example 4. Consider *Example 1*, then the disagree-IFSEG $G_0 = (G^*, A, f_0, g_0)$ over $G^* = (\mathcal{V}, \mathbb{E})$.

Table 4. The disagree-IFSEG.

f	x_1	x_2	x_3
$(e_1, p, 0)$	(0.9,0.1)	(0.6,0.4)	(0.5,0.5)
$(e_2, p, 0)$	(0.3,0.7)	(0.5,0.5)	(0.2,0.8)
$(e_3, p, 0)$	(0.2,0.8)	(0.3,0.7)	(0.4,0.6)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	(0.6,0.4)	(0,1)	(0.6,0.4)
$(e_2, p, 0)$	(0.3,0.7)	(0.1,0.9)	(0.3,0.7)
$(e_3, p, 0)$	(0.2,0.8)	(0.8,0.2)	(1,0)

Definition 9. The union of two-IFSEGs $G^1 = (G^*, A^1, f^1, g^1)$ and $G^2 = (G^*, A^2, f^2, g^2)$ is denoted by $G = (G^*, A, f, g)$ with $A = A^1 \cup A^2$ where the membership and non membership of the union are as follows

$$\mu_{f_\alpha}(x) = \begin{cases} \mu_{f_\alpha{}^1}(x) = \begin{cases} \text{if } e \in A^1 - A^2 \text{and } x \in \mathcal{V}^1 - \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^1 - A^2 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^1 \cap A^2 \text{and } x \in \mathcal{V}^1 - \mathcal{V}^2. \end{cases} \\ \mu_{f_\alpha{}^2}(x) = \begin{cases} \text{if } e \in A^2 - A^1 \text{and } x \in \mathcal{V}^2 - \mathcal{V}^1 \text{or,} \\ \text{if } e \in A^2 - A^1 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^2 \cap A^1 \text{and } x \in \mathcal{V}^2 - \mathcal{V}^1. \end{cases} \\ \max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\} \{\text{if } e \in A^1 \cap A^2 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2\}, \\ 0, \quad \text{otherwise.} \end{cases}$$

$$\vartheta_{f_\alpha}(x) = \begin{cases} w_{f_\alpha{}^1}(x) = \begin{cases} \text{if } e \in A^1 - A^2 \text{and } x \in \mathcal{V}^1 - \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^1 - A^2 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^1 \cap A^2 \text{and } x \in \mathcal{V}^1 - \mathcal{V}^2. \end{cases} \\ w_{f_\alpha{}^2}(x) = \begin{cases} \text{if } e \in A^2 - A^1 \text{and } x \in \mathcal{V}^2 - \mathcal{V}^1 \text{ or,} \\ \text{if } e \in A^2 - A^1 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2 \text{or,} \\ \text{if } e \in A^2 \cap A^1 \text{and } x \in \mathcal{V}^2 - \mathcal{V}^1. \end{cases} \\ \min\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^2}(x)\right\} \{\text{if } e \in A^1 \cap A^2 \text{and } x \in \mathcal{V}^1 \cap \mathcal{V}^2\}, \\ 0, \quad \text{otherwise.} \end{cases}$$

$$\mu_{g_\alpha}(x,y)=\begin{cases}\mu_{g_\alpha{}^1}(x,y)=\begin{cases}\text{if } e\in A^1-A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2)\text{or},\\ \text{if } e\in A^1-A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)\text{or},\\ \text{if } e\in A^1\cap A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2).\end{cases}\\ \mu_{g_\alpha{}^2}(x,y)=\begin{cases}\text{if } e\in A^2-A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)-(\mathcal{V}^1\times\mathcal{V}^1)\text{or},\\ \text{if } e\in A^2-A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)\cap(\mathcal{V}^1\times\mathcal{V}^1)\text{or},\\ \text{if } e\in A^2\cap A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)-(\mathcal{V}^1\times\mathcal{V}^1).\end{cases}\\ \max\{\mu_{g_\alpha{}^1}(x,y),\mu_{g_\alpha{}^2}(x,y)\}\begin{Bmatrix}\text{if } e\in A^1\cap A^2 \text{and}(x,y)\\ \in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)\end{Bmatrix},\\ 0,\quad \text{otherwise}.\end{cases}$$

$$\vartheta_{g_\alpha}(x,y)=\begin{cases}w_{g_\alpha{}^1}(x,y)=\begin{cases}\text{if } e\in A^1-A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2)\text{or},\\ \text{if } e\in A^1-A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)\text{or},\\ \text{if } e\in A^1\cap A^2 \text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2).\end{cases}\\ w_{g_\alpha{}^2}(x,y)=\begin{cases}\text{if } e\in A^2-A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)-(\mathcal{V}^1\times\mathcal{V}^1)\text{or},\\ \text{if } e\in A^2-A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)\cap(\mathcal{V}^1\times\mathcal{V}^1)\text{or},\\ \text{if } e\in A^2\cap A^1 \text{and}(x,y)\in(\mathcal{V}^2\times\mathcal{V}^2)-(\mathcal{V}^1\times\mathcal{V}^1).\end{cases}\\ \min\{w_{g_\alpha{}^1}(x,y),w_{g_\alpha{}^2}(x,y)\}\begin{Bmatrix}\text{if } e\in A^1\cap A^2 \text{and}(x,y)\\ \in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)\end{Bmatrix},\\ 0,\quad \text{otherwise}.\end{cases}$$

Proposition 1. The union $G=(G^*,A,f,g)$ of two IFSEGs $G^1=(G^*,A^1,f^1,g^1)$ and $G^2=(G^*,A^2,f^2,g^2)$ is an IFSEG.

Proof:

I. if $e\in A^1-A^2$ $\text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2)$, then $\mu_{g_\alpha}(x,y)=\mu_{g_\alpha{}^1}(x,y)\le\min\{\mu_{f_\alpha{}^1}(x),\mu_{f_\alpha{}^1}(y)\}=\min\{\mu_{f_\alpha{}^1}(x),\mu_{f_\alpha{}^1}(y)\}$, so $\mu_{g_\alpha}(x,y)\le\min\{\mu_{f_\alpha}(x),\mu_{f_\alpha}(y)\}$,

Also, $\vartheta_{g_\alpha}(x,y)=\vartheta_{g_\alpha{}^1}(x,y)\le\min\{\vartheta_{f_\alpha{}^1}(x),\vartheta_{f_\alpha{}^1}(y)\}=\min\{\vartheta_{f_\alpha{}^1}(x),\vartheta_{f_\alpha{}^1}(y)\}$.

So $\vartheta_{g_\alpha}(x,y)\le\min\{\vartheta_{f_\alpha}(x),\vartheta_{f_\alpha}(y)\}$.

Now $w_{g_\alpha}(x,y)=w_{g_\alpha{}^1}(x,y)\ge\max\{w_{f_\alpha{}^1}(x),w_{f_\alpha{}^1}(y)\}=\max\{w_{f_\alpha{}^1}(x),w_{f_\alpha{}^1}(y)\}$.

So $w_{g_\alpha}(x,y)\ge\max\{w_{f_\alpha}(x),w_{f_\alpha}(y)\}$.

Similarly if $\{e\in A^1-A^2\text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)\}$, or if $\{e\in A^1\cap A^2\text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)-(\mathcal{V}^2\times\mathcal{V}^2)\}$, we can show the same as done above.

II. if $e\in A^1\cap A^2$ $\text{and}(x,y)\in(\mathcal{V}^1\times\mathcal{V}^1)\cap(\mathcal{V}^2\times\mathcal{V}^2)$, then

$$\begin{aligned}\mu_{g_\alpha}(x,y)&=\max\left\{\mu_{f_\alpha{}^1}(x),\mu_{f_\alpha{}^1}(y)\right\}\\ &\le\max\left\{\min\left\{\mu_{f_\alpha{}^1}(x),\mu_{f_\alpha{}^1}(y)\right\},\min\left\{\mu_{f_\alpha{}^2}(x),\mu_{f_\alpha{}^2}(y)\right\}\right\}\\ &\le\min\left\{\max\left\{\mu_{f_\alpha{}^1}(x),\mu_{f_\alpha{}^2}(x)\right\},\max\left\{\mu_{f_\alpha{}^1}(y),\mu_{f_\alpha{}^2}(y)\right\}\right\}\\ &=\min\{\mu_{f_\alpha}(x),\mu_{f_\alpha}(y)\}.\end{aligned}$$

Also

$$\begin{aligned}\vartheta_{g_\alpha}(x,y)&=\min\left\{\vartheta_{f_\alpha{}^1}(x),w\vartheta_{f_\alpha{}^1}(y)\right\}\\ &\ge\min\left\{\max\left\{w\vartheta_{f_\alpha{}^1}(x),\vartheta_{f_\alpha{}^1}(y)\right\},\max\left\{\vartheta_{f_\alpha{}^2}(x),\vartheta_{f_\alpha{}^2}(y)\right\}\right\}\\ &\ge\max\left\{\min\left\{\vartheta_{f_\alpha{}^1}(x),\vartheta_{f_\alpha{}^2}(x)\right\},\min\left\{\vartheta_{f_\alpha{}^1}(y),\vartheta_{f_\alpha{}^2}(y)\right\}\right\}\\ &=\max\{\vartheta_{f_\alpha}(x),\vartheta_{f_\alpha}(y)\}.\end{aligned}$$

Hence, the union $G=G^1\cup G^2$ is an IFSEG.

Definition 10. The intersection of two IFSEGs $G^1 = (G^{1^*}, A^1, f^1, g^1)$ and $G^2 = (G^{2^*}, A^2, f^2, g^2)$ is denoted by $G = (G^*, A, f, g)$ with $A = A^1 \cap A^2, \mathcal{V} = \mathcal{V}^1 \cap \mathcal{V}^2$ and the membership and non membership of the intersection are as follows:

$$\mu_{f_\alpha} = \begin{cases} \mu_{f_\alpha{}^1}(x) \text{if } e \in A^1 - A^2, \\ \mu_{f_\alpha{}^2}(x) \text{if } e \in A^2 - A^1, \\ \min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\} \text{if } e \in A^1 \cap A^2. \end{cases}$$

$$\vartheta_{f_\alpha} = \begin{cases} \vartheta_{f_\alpha{}^1}(x) \text{if } e \in A^1 - A^2, \\ \vartheta_{f_\alpha{}^2}(x) \text{if } e \in A^2 - A^1, \\ \max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\} \text{if } e \in A^1 \cap A^2. \end{cases}$$

$$\mu_{g_\alpha} = \begin{cases} \mu_{g_\alpha{}^1}(x,y) \text{if } e \in A^1 - A^2, \\ \mu_{g_\alpha{}^2}(x,y) \text{if } e \in A^2 - A^1, \\ \min\{\mu_{g_\alpha{}^1}(x,y), \mu_{g_\alpha{}^2}(x,y)\} \text{if } e \in A^1 \cap A^2. \end{cases}$$

$$\vartheta_{g_\alpha} = \begin{cases} \vartheta_{g_\alpha{}^1}(x,y) \text{if } e \in A^1 - A^2, \\ \vartheta_{g_\alpha{}^2}(x,y) \text{if } e \in A^2 - A^1, \\ \max\{\vartheta_{g_\alpha{}^1}(x,y), \vartheta_{g_\alpha{}^2}(x,y)\} \text{if } e \in A^1 \cap A^2. \end{cases}$$

Proposition 2. The intersection $G = (G^*, A, f, g)$ of two IFSEGs $G^1 = (G^*, A^1, f^1, g^1)$ and $G^2 = (G^*, A^2, f^2, g^2)$ is an IFSEG where $A = A^1 \cap A^2, \mathcal{V} = \mathcal{V}^1 \cap \mathcal{V}^2$.

Proof:

I. if $e \in A^1 - A^2$, then $\mu_{g_\alpha}(x,y) = \mu_{g_\alpha{}^1}(x,y) \le \min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\} = \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$,

so $\mu_{g_\alpha}(x,y) \le \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$.

Now $\vartheta_{g_\alpha}(x,y) = \vartheta_{g_\alpha{}^1}(x,y) \ge \max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\} = \max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$, so $\vartheta_{g_\alpha}(x,y) \ge \max\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\}$.

Similarly, if $\{e \in A^1 - A^2\}$ we can show the same as done above.

II. if $e \in A^1 \cap A^2$, then

$$\begin{aligned} \mu_{g_\alpha}(x,y) &= \max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\} \\ &\le \min\left\{\min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}, \min\left\{\mu_{f_\alpha{}^2}(x), \mu_{f_\alpha{}^2}(y)\right\}\right\} \\ &\le \min\left\{\min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\}, \min\left\{\mu_{f_\alpha{}^1}(y), \mu_{f_\alpha{}^2}(y)\right\}\right\} \\ &= \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}. \end{aligned}$$

Now

$$\begin{aligned} \vartheta_{g_\alpha}(x,y) &= \max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\} \\ &\ge \max\left\{\max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}, \max\left\{\vartheta_{f_\alpha{}^2}(x), \vartheta_{f_\alpha{}^2}(y)\right\}\right\} \\ &\ge \max\left\{\max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\}, \max\left\{\vartheta_{f_\alpha{}^1}(y), \vartheta_{f_\alpha{}^2}(y)\right\}\right\} \\ &= \max\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\}. \end{aligned}$$

Hence the intersection $G = G^1 \cap G^2$ is an IFSEG.

4 | Applications of IFSEG

Assume that a company wants to fill a position to be chosen by an expert committee. Suppose that $G^* = (\mathcal{V}, \mathbb{E})$ be a simple graph with $\mathcal{V} = \{x_1, x_2, x_3\}, Y = \{e_1\}$ be a set of parameters for computer knowledge. Let $X = \{p, q\}$ be a set of two expert committee members. An IFSEG is given in *Table 5* below and $\mu_{g_\alpha}(x_i, x_j) = 0$

and $\vartheta_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$. After serious deliberation, the committee constructs the following IFSEG.

Table 5. IFSEG.

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.6,0.7)	(0,1)	(0.2,0.3)
$(e_1, q, 1)$	(0.2,0.6)	(0.5,0.4)	(0.2,0.5)
$(e_1, p, 0)$	(0.1,0.4)	(0.6,0.3)	(0.3,0.9)
$(e_1, q, 0)$	(0.1,0.1)	(0.6,0.3)	(0.5,0.8)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0.5,0.4)	(0.1,0,6)	(0.1,0.7)
$(e_1, q, 1)$	(0.1,0.1)	(0.2,0.2)	(0.1,0.4)
$(e_1, p, 0)$	(0.1,0.2)	(0.2,0.1)	(0,0.2)
$(e_1, q, 0)$	(0.2,0.2)	(0.3,0.2)	(0.4,0.2)

The company may follow the following algorithm to fill the position:

I. Input the IFSEG.

II. Find the mean of each IFSE edge according to the relationship among criteria for each alternative.

III. Find an agree-IFSEG and a disagree- IFSEG.

IV. Find $C_j = \sum_i x_{ij}$ for agree- IFSEG.

V. Find $K_j = \sum_i x_{ij}$ for disagree- IFSEG.

VI. Find $S_j = C_j - K_j$.

VII. Find r, for which $s_r = \max s_j$, where,s_r is the optimal choice object. If r has more than one value, then the company could choose any one of them using its option.

IFSE edges according to the relationship among criteria for each alternative.

Table 6. IFSEGs.

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0.5,0.4)	(0.1,0,6)	(0.1,0.7)
$(e_1, q, 1)$	(0.1,0.1)	(0.2,0.2)	(0.1,0.4)
$(e_1, p, 0)$	(0.1,0.2)	(0.2,0.1)	(0,0.2)
$(e_1, q, 0)$	(0.2,0.2)	(0.3,0.2)	(0.4,0.2)

Table 7 presents the agree-IFSEG by using the mean of each IFSEG.

Table 7. Tabular presentation of the agree-IFSEG.

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,45	0,35	0,4
$(e_1, q, 1)$	0,1	0,2	0,25

Table 8 presents the disagree-IFSEG, respectively, by using the mean of each IFSEG.

Table 8. Tabular presentation of the disagree-NSEG.

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,15	0,15	0,1
$(e_1, q, 0)$	0,2	0,25	0,3

$C_j = \sum_i x_{ij}$ for agree-IFSEG.

Table 9. Sum of agree-IFSEG.

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,45	0,35	0,4
$(e_1, q, 1)$	0,1	0,2	0,25
$C_j = \sum_i x_{ij}$	0,55	0,55	0,65

$K_j = \sum_i x_{ij}$ for disagree-IFSEG.

Table 10. Sum of disagree-IFSEG.

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,15	0,15	0,1
$(e_1, q, 0)$	0,2	0,25	0,3
$K_j = \sum_i x_{ij}$	0,35	0,4	0,4

From *Tables 9* and *10*, we can compute the values of $S_j = C_j - K_j$ as in *Table 11*.

Table 11. $S_j = C_j - K_j$.

j	X	C_j	K_j	S_j
1	x_1	0,55	0,35	0,20
2	x_2	0,55	0,4	0,15
3	x_3	0,65	0,4	0,25

Since max $S_j = 0{,}25$, hence the committee will choose candidate x_3 with a master's degree for the job.

5 | Conclusion

In this paper, we have introduced the concept of IFSEG, union and the intersection of them has been explained with an example which has broader application in the field of modern sciences and technology, especially in research areas of computer science, including database theory, data mining, neural networks, expert systems, cluster analysis, control theory, and image capturing.

Author Contribution

M. S. research design, conceptualization, and validation. V. U. data gathering, computing, and editing. S. A. E Methodology, visualization and formal analysis. The authors have read and agreed to the published version of the manuscript.

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All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors of this article are members of the UDA editorial board and did not participate in the editorial review or decision to publish this work. They have declared that they have no conflicts of interest related to this work.

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